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Research on the Best Shooting State Based on the "Three Forces" Model

Xuguang Liu[1], Ruqing Zhao[2], Qifei Chen[2], Ming Shi[3], Ziling Xing[2] and Yanan Zhang[4,*]

[1]Department of Physical Education, Nanjing University of Information Science & Technology, Nanjing, 210044, China
[2]Changwang School of Honors, Nanjing University of Information Science & Technology, Nanjing, 210044, China
[3]School of Automation, Nanjing University of Information Science & Technology, Nanjing, 210044, China
[4]Engineering Training Center, Nanjing University of Information Science & Technology, Nanjing, 210044, China
[*]Corresponding Author: Yanan Zhang. Email: phyzyn@nuist.edu.cn
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Abstract: The shooting state during shooting refers to the basketball's shooting speed, shooting angle and the ball's rotation speed. The basketball flight path is also related to these factors. In this paper, based on the three forces of Gravity, Air Resistance and Magnus Force, the "Three Forces" model is established, the Kinetic equations are derived, the basketball flight trajectory is solved by simulation, and the best shot state when shooting is obtained through the shooting percentage. Compared with the "Single Force" model that only considers Gravity, the shooting percentage of the "Three Forces" model is higher. The reason is that the Magnus Force generated by considering the basketball rotation speed is considered. Although in the "Three Forces" model, the shot speed is faster and the shot is harder, the backspin will reduce the angle of the shot and achieve the goal of saving effort. By calculating the best shot state and giving the athlete's usual training state range, you can guide the training, thereby improving the athlete's shooting percentage during the game.

Keywords: Kinetic equations; trajectory; Magnus Force; best shooting state

1 Introduction

Most researches on basketball trajectory are qualitative analysis of improving shooting percentage, or analysis of trajectory equation based on the influence of gravity only after basketball is shot. References [1–3] analyzed the shooting process and various factors that affect the shooting percentage, and put forward four factors affecting the basketball shooting percentage: technical factors, physical fitness factors, psychological factors, and time and space factors. It is limited to qualitative analysis and does not give quantitative results. Reference [4] proposed that back spin can maximize the angle of incidence and increase the shooting percentage. Reference [5] considered the influence of forward spin and back spin on the basketball trajectory, and proposed that during forward spin or at the basket, forward spin can shorten the flight distance, speed up the fall, and increase the shooting percentage of hollow into the net; while through back spin, It can increase the stability and flight time of basketball in mid-to-long distance shooting, and increase the shooting percentage of hollow nets, but it also does not give a quantitative result. Reference [6] analyzed the impact of height, distance and shooting angle on basketball shooting percentage, and proposed that the increase in height, distance and shooting angle all have a great impact on shooting percentage.

Reference [7] used MATLAB software to analyze the basketball movement trajectory of the quadratic function, and used the control variable method to find the best range of four factors affecting the speed, the angle of the shot, the horizontal distance and the vertical height. But the research methods and conclusions are based on the assumption that basketball is only affected by gravity. Reference [8] gave the best angle of anti-angle deviation and the most labor-saving shooting angle through the variational method, and proposed that when the line of the shot point and the basket is exactly the bisector of the shot angle, the anti-angle deviation of the shot is the best. And it is the most labor-saving; but it does not consider the impact of resistance and the rotation of the ball, and also ignores the impact of the incident section of the basketball on the basketball's hoop. Reference [9] analyzed the incident angle of basketball, through the size of the incident section, the allowable error range for shooting at different incident angles was given, and pointed out that it is impossible for basketball to enter the basket when the incident angle is less than 33°, and pointed out that the shooting percentage can be increased by increasing the angle of incidence of the basketball by backspin. But it did not consider the impact of resistance.

According to the actual situation, this paper establishes the "Three Forces" mode. Through simulation calculation, analyzing the relationship between the flight trajectory of the back spin, forward spin and non-spin ball, and the shooting speed and angle. According to different shooters' own conditions, discuss the best shooting state and the range of shooting state during training. In order to verify the validity of the model, the shooting percentage of the "Three Forces" model and the "Single Force" model was compared by Monte Carlo method [10].

2 Analysis of the Forces on Basketball in Sports

In the standard shooting process, the athlete will use a series of continuous movements to transmit force from the legs, along the waist and the forearm to the wrist and fingers, and finally "pull out" the ball; experienced athlete will control the force of the wrist and fingers to makes the basketball spin so that the basketball can achieve a more stable flight or a longer flight time. The shooting percentage is affected by many factors [11], so it is necessary to carry out a theoretical analysis. This article makes a more detailed analysis of the force of basketball, as follows.

2.1 Basketball Force Analysis in the Air

For basketball beginners, the non-standard shooting action is a bit like shooting put. The fingers of the wrist directly push the ball out. In this case, the basketball is flying in the air without rotation or with a small rotation speed. During the actual training of professional basketball players, coaches all require basketball players to "pick" the basketball with their wrist fingers. That is, the wrist exerts force, the fingers should rub against the basketball, and the basketball has a certain rotation speed when flying in the air. At this time, it is obviously inappropriate to use basketball as a mass point to analyze its movement. According to rigid body mechanics, the basketball at this time can be regarded as a rigid body. Under the action of the friction between the fingers and the basketball, a rotational moment is generated on the basketball, which can be simplified as a rigid basketball body doing plane motion. The center of mass of the basketball is doing parabolic motion with resistance, and the basketball itself is also rotating around a certain diameter of the center of the ball.

In order to analyze the flight trajectory of basketball more accurately, it is assumed that the basketball is subjected to gravity, air resistance, and Magnus force during flight. In fluid mechanics, the air resistance of an object is related to the speed of the object. During the basketball flight after shooting, because its speed is relatively small, the resistance and speed received can be regarded as a proportional relationship [8], and the direction of resistance is opposite to the direction of speed [12]. which is

$$\vec{f} = -k\vec{v} \tag{1}$$

where k is the drag coefficient, which is related to the actual measured value in the field [13], and $k = 0.53$ in this article; v is the flying speed of the basketball.

Magnus force [9,12] refers to the rotation of the object to drive the flow of the surrounding airflow, so that the surrounding airflow velocity is different, thus generating a pressure difference, this pressure difference will produce a force perpendicular to the direction of movement of the object, namely Magnus force, its expression is

$$F = \frac{\pi}{2}\rho AR^3\omega \times v \tag{2}$$

where ρ is the fluid density, A is a constant related to the surface characteristics of the ball and the internal friction of the airflow. R, ω, v are the radius of the basketball, the rotational angular velocity and the flying speed of the basketball, respectively.

2.2 The dynamic equation of basketball flight

Through force analysis, the kinematics equation of basketball flying is established:

$$m\vec{a} = \vec{F} + \vec{G} + \vec{f} \tag{3}$$

$$\begin{cases} \vec{F} = \frac{\pi}{2}\rho AR^3\omega \times v \\ \vec{G} = m\vec{g} \\ \vec{f} = -k\vec{v} \end{cases} \tag{4}$$

In order to facilitate the analysis, the above equations are decomposed in the horizontal and vertical directions:

$$\begin{cases} \frac{d^2x}{dt^2} = -\frac{k}{m}\frac{dx}{dt} - \frac{b}{m}\omega\frac{dy}{dt} \\ \frac{d^2y}{dt^2} = -g - \frac{k}{m}\frac{dy}{dt} + \frac{b}{m}\omega\frac{dx}{dt} \end{cases} \tag{5}$$

where $b = \pi / 2\rho AR^3$. According to the above equation, the flight process of basketball from shot to hit is simulated, and various factors that affect the shooting percentage are analyzed.

3 Simulation verification

3.1 Discretization of equation

Discretizing the above Eq. (5), the following equation can be obtained:

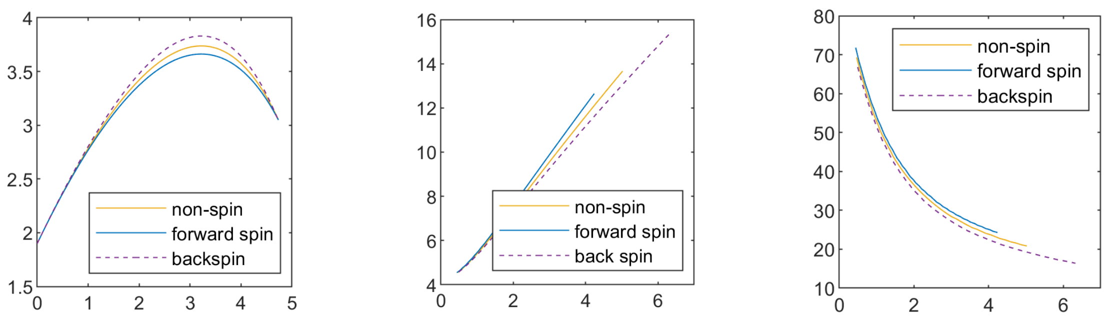
$$\begin{cases} \frac{x(t+2) - 2x(t+1) + x(t)}{\Delta t^2} = -\frac{k}{m}\frac{x(t+1) - x(t)}{\Delta t} - \frac{b\omega}{m}\frac{y(t+1) - y(t)}{\Delta t} \\ \frac{y(t+2) - 2y(t+1) + y(t)}{\Delta t^2} = -g - \frac{k}{m}\frac{y(t+1) - y(t)}{\Delta t} + \frac{b\omega}{m}\frac{x(t+1) - x(t)}{\Delta t} \end{cases} \tag{6}$$

Given the initial conditions and boundary conditions, the incident angle when the basketball flies to the hoop plane can be obtained. According to the calculation method [9], the projection of the basketball on the hoop plane is used to judge whether it equals the score.

Taking into account the players' actual conditions on the field, the highest point of the trajectory obtained by the ordinary differential equation is more than 1.5H, and the value is discarded. Among them, H represents the vertical height of the basket, and the international standard is 3.05 m.

3.2 The Effect of Spin on Flight

Using the model described above, the trajectories of the basketball's forward spin, no spin, and backspin and the relationship between the speed and the angle of the shot were calculated, and the results are shown in Fig. 1. Fig. 1a shows that the flight trajectory of the backspin ball is different from the non-spin and forward spin modes, and the angle of incidence is relatively large; Fig. 1b shows that the flying speed of the backspin ball is the smallest when flying the same distance. Fig. 1c shows that for the same distance, the shot angle required for backspin is the smallest. These characteristics consistent with experience can be explained by simple physical principles, so we won't repeat them in this article. According to published articles [9], the greater the incident angle when reaching the basket, the higher the shooting percentage. Spinning the ball after shooting will increase the angle of incidence. But does the higher the rotation speed, the higher the shooting percentage? When considering the physical consumption of athletes, should the shooting angle and shooting speed be slightly adjusted? Such research is rarely involved. In this paper, based on the actual force the basketball receives in flight, the "Three Forces" model is established for numerical simulation to solve the basketball flight trajectory. Based on guaranteeing the shooting percentage, discuss the relationship between shooting angle, shooting speed, and basketball rotation, and try to give theoretical guidance to actual basketball shooting training.



(a) (b) (c)

Figure 1: Comparison of backspin, forward spin and non-spin ball. (a) flight trajectory (b) flight speed (c) shot angle

3.3 The Comprehensive Influence of Three Factors on Shooting Percentage

3.3.1 Best Shooting Position

In reality, shooting, whether in training or on the field, is restricted by many factors. Compared with other studies, the model established in this paper considers more forces and is more in line with reality. Besides, considering only the impact of one factor on the shooting percentage cannot guide normal training. Because when shooting, there are usually many factors that affect the shooting percentage. Moreover, various factors will interfere with each other. This article establishes the "Three Forces" model, and sets the shooting speed, shooting angle, and basketball rotation angular speed as the parameters that affect the shooting percentage. Through calculation, we try to find the relationship between each other and the comprehensive impact on the shooting percentage. According to the analysis results, it is used to guide the normal training of basketball players, so that the players can practice in the most suitable range of various factors, and form a conditioned reflex to the body muscles. In this way, during the game, the athlete's shooting percentage can be improved.

Table 1: Shooting percentage and shooting status

Center point of the shot angle range	Center point of the shot speed range	Shooting Percentages
$\theta = 51^0$	$v = 9.6\text{m/s}$	79.36%
$\theta = 55^0$	$v = 10.1\text{m/s}$	98.55%
$\theta = 56^0$	$v = 10.2\text{m/s}$	98.38%

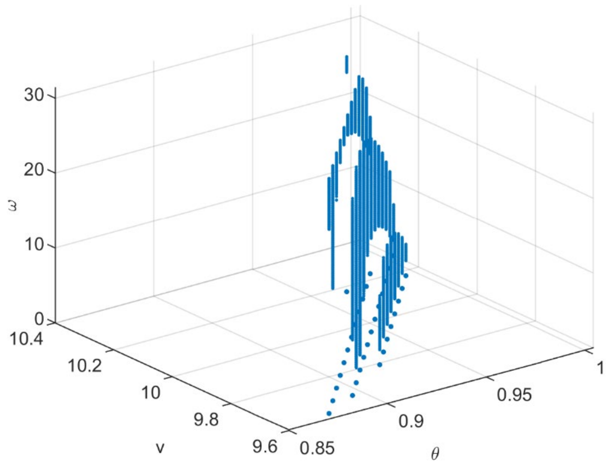


Figure 2: The shooting percentage of the shot

The conditions of the shooters are not all the same. This article mainly discusses the impact of shooting speed, shooting angle, and basketball rotation angle speed on shooting percentage. Therefore, we fix the shooting height of the shooter h = 1.9 m, and take the basket as the center point, and make a semicircle with the distance between the free-throw line and the basket as the radius. The shooter is located at any position of the semicircle (the distance between the shooter and the basket L = 4.325 m).

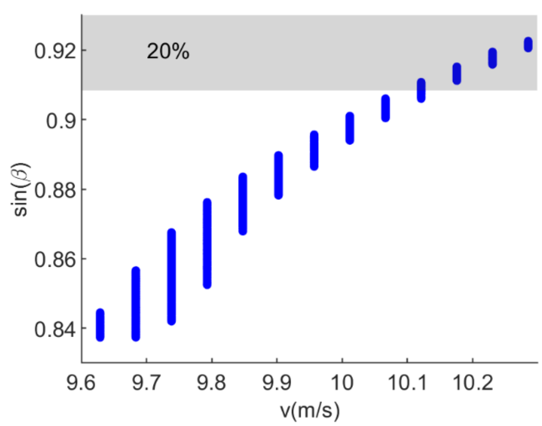


Figure 3: Finding the best shooting range

Substituting the initial state parameters (handling speed, shot angle, basketball rotation angular speed) into the dynamic Eq. (5) of 2.2, stipulating the boundary conditions (limiting the maximum flying height of the basketball and requiring the basket to be located on the basketball trajectory), according to 3.1 Eq. (6), find a series of incident angles that satisfy the equation, and use the method of literature [9] to judge whether it hit. Fig. 2 shows the possible values of the shooting speed, the shooting angle, and the rotational angular velocity when the distance between the shooting position and the basket frame is L = 4.325 m. In the picture, you can observe which shooting state will hit, but athletes cannot always shoot in a certain state, but can only practice more in a certain range. At the same time, as the shooting speed and shooting angle increase, the shooting percentage will increase, but this also means that the athlete's physical strength is consumed more. Can we make our shots more effortless based on ensuring a high shooting percentage? According to the calculation results, it can be found that when the speed increases to a certain extent, the increase in shooting speed brought about by the increase in shooting speed is limited, to find the most reasonable shooting state range. This article refers to the Scree method when screening the main influencing components in the principal component analysis [14]. Select the first 20% of the incident angle corresponding to all the shots that can be hit as the range of the best shot state. Within this range, any set (v, θ, ω) can hit the basket, but the lower the shot speed, the more energy will be saved. Therefore, the set of shot state parameter values with the smallest speed is the target found. The result is shown in Fig. 3.

Table 2: The effect of spin on the shooting percentage

Basketball spin state	Shot state	Average shooting percentage
Backspin $\omega = 25.1\text{rad/s}$	$\theta = 55^0$ $v = 9.9\sim10.1\text{m/s}$	41.50%
Forward spin $\omega = -25.1\text{rad/s}$	$\theta = 55^0$ $v = 9.9\sim10.1\text{m/s}$	36.38%
No spin $\omega = 0\text{rad/s}$	$\theta = 55^0$ $v = 9.9\sim10.1\text{m/s}$	40.70%

When the shooting distance is L = 4.325 m, the calculated best shooting speed value is v = 10.1 m/s. The meaning of this value is that if considering the “Three Forces”, the horizontal distance from the ring is 4.325 m, and the shooting height is h = 1.9 m, the trainer should take the speed v = 10.1 m/s as the center point, training within the range of $(v - v \times 0.05, v + v \times 0.05)$. And how does the shooting angle determine the range? According to the same method, find the best angle of shooting. That is, the training range of the shot angle is $(\theta - \theta \times 0.05, \theta + \theta \times 0.05)$.

Tab. 1 lists the comparison between the best shot speed calculated by the model and the shooting percentage of the other two speeds. The other two velocities are the minimum and maximum velocities of the same rotational angular velocity in the solution of the equation (shoots that hit the basket). Although the shot state in the table is the corresponding specific value, in the simulation calculation, the value is taken as the center, 5% of the shot state itself is used as the neighborhood, and the value range is constituted by a random normal distribution. This setting takes into account that even after years of training, athletes will not shoot at a certain speed and angle when shooting, but habitually use a certain range of shooting speed and angle to shoot the basketball spin speed to make shots. Projecting the three-dimensional map of Fig. 2 on the v and θ planes, it can be seen that the firing speed and the firing angle are approximately linear relationship. Therefore, only the firing speed is used as the comparison object in Tab. 1.

By comparison, it can be seen that when the angle is lower than the best shot angle calculated by the model, the shooting percentage is lower; while the shot state higher than this speed, the shooting percentage hardly changes. Therefore, athletes who train with the best shot state calculated by the model will obtain the greatest “price ratio”. Fig. 3 shows the calculation results of the fixed shooter’s height, the height of the shooting point, and the horizontal distance from the shooting point to the ring. When the shooting conditions change, the best training range for different shooting conditions can be found. It varies from person to person, and changes due to environmental changes.

3.3.2 Analysis of the Rotation Speed of Basketball

According to experience, basketball’s spin will increase the shooting percentage, but it is very rare to conduct a theoretical analysis of this and compare the spin and non-spin shooting percentages. In this article 3.2, we simply discuss that spin is beneficial to increase the angle of incidence and reduce the speed and angle of the shot, but whether the backspin can affect the shooting percentage at the same time with other factors has not yet been given a clear conclusion. The specific comparison results are given in Tab. 2. It can be seen that, compared with forwarding spin and no spin, backspin can indeed improve the shooting percentage. The lower shooting percentage obtained in the table is because the best shot without spin is used as the reference range, and the three adjacent ranges are selected to find the average shooting percentage. This comparison is just to show the impact of backspin on the shooting percentage. If calculated based on the best shot state of the backspin in Tab. 1, the shooting percentage will be significantly improved.

3.4 Comparative Analysis of "Three Forces" Model and "Single Force" Model

Only considering the influence of gravity on the basketball trajectory, the shooting trajectory must meet the following conditions:

$$\tan\theta = \frac{v_0^2}{gL}[1 \pm \sqrt{1 - \frac{2g}{v_0^2}(H - h + \frac{gL^2}{2v_0^2})}] \tag{7}$$

$$v_0 \geq \sqrt{g[H - h + \sqrt{L^2 + (H-h)^2}]} \tag{8}$$

Taking into account the impact of the ball [15] on basketball shooting, this paper compares the shooting percentage of the same shooting angle and shooting speed under the "Single Force" (only gravity) and "Three Forces" models. The results are shown in Tab. 3. In the analysis, the comparison between professional athletes and ordinary people in the two models is selected. The purpose is to prove that this model is also suitable for ordinary basketball enthusiasts for training guidance. Considering that professional athletes have been trained, the error of the shooting state is smaller than that of ordinary people. Obviously, the shooting percentage of the "Three Forces" model is much higher than that of the single force model. The decrease in a three-point shooting is due to the longer shooting distance, the natural increase in shooting speed, and the rapid increase in shooting power. At this time, whether it is a professional athlete or an ordinary basketball enthusiast, even if the shooting state fluctuates at 5% and 10%, the shooting speed range will be expanded, resulting in a lower shooting percentage, which is also in line with the actual situation.

Table 3: Comparison of shooting percentage between "Single Force" model and "Three Force" model

Free throw line position: 4.325 m	Three Forces model shooting percentage	Single Force model shooting percentage
Professional athletes: shooting state fluctuates by 5%	98.48%	60.74%
Ordinary people: The shot state fluctuates by 10%	68.17%	40.77%
Three-point line position: 6.75 m	Three Forces model shooting percentage	Single Force model shooting percentage
Professional athletes: shooting state fluctuates by 5%	62.76%	39.27%
Ordinary people: The shot state fluctuates by 10%	37.09%	30.84%

Why is the shooting percentage of the "Three Forces" model higher? It is because it not only considers the three forces acting on the basketball, which is more in line with the real situation. At the same time, the Magnus force in the "Three Forces" model is the force formed by the up and down pressure changes due to the rotation of the basketball. This is the purpose of comparing back spin, forward spin, and non-spin. Fig. 4 shows the comparison between the "Single Force" model and the "Three Forces" model in terms of firing speed and firing angle. Taking the situation of hitting the basket calculated by the model as a sample, the "Three Forces" model considers more power and is relatively fast and more laborious. However, because the backspin will reduce the angle of the shot and increase the angle of incidence, this feature can just make up for the physical energy consumed due to the high shot speed.

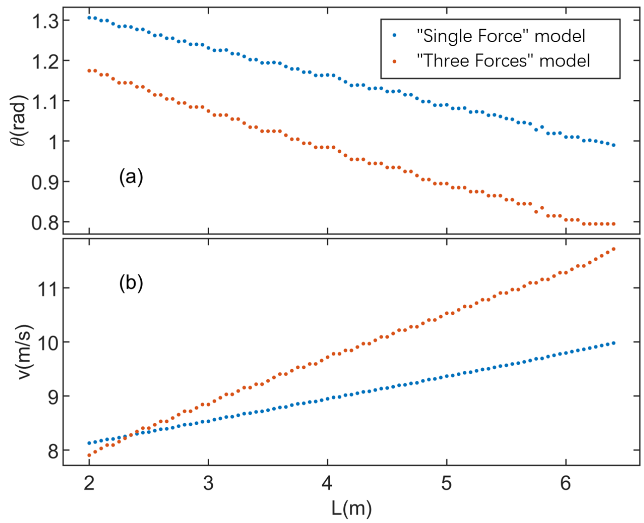


Figure 4: Comparison of shooting angle and speed between "Single Force" model and "Three Forces" model

4 Conclusion

The main purpose of basketball players' usual training is to produce conditioned reflexes of the muscles and to form a habit of shooting at any distance. If the training can be guided by scientific analysis, it will get twice the result with half the effort. This article establishes the "Three Forces" model, which truly reflects the forces on the basketball flight. Three parameters of shooting angle, shooting speed and basketball rotation speed are input into the model, and the best shooting state at different shooting distances is solved by the shooting percentage. Through the comparison, it can be seen that the shooting percentage is the highest in the range near the best shot state, and the most physical energy is saved. The Magnus force in the "Three Forces" model is generated by the rotation of the basketball, so the backspin shot can increase the shooting percentage. Comparing the "Three Forces" model with the "Single Force" model, the difference between the shooting speed and the shooting angle is accurately calculated, and the shooting percentage is also significantly improved.

In this paper, the determination of the best shooting state range is based on the model solution value, with 5% of the value as the neighborhood, and a random normal distribution is adopted. Naturally, the neighborhood range can also be relatively expanded, so that the training can be gradually reduced from a large range. This work is supported by the Innovation and Entrepreneurship Training Program of the Nanjing University of Information Technology. The main goal of the follow-up work is to design a sensor that can instantly measure the athlete's shooting speed, shooting angle, or basketball rotation speed. According to the theoretical results of the model, the athlete's training can be monitored and guided.

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