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Non-Linear Localization Algorithm Based on Newton Iterations

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Abstract: In order to improve the performance of time difference of arrival (TDOA) localization, a nonlinear least squares algorithm is proposed in this paper. Firstly, based on the criterion of the minimized sum of square error of time difference of arrival, the location estimation is expressed as an optimal problem of a non-linear programming. Then, an initial point is obtained using the semi-definite programming. And finally, the location is extracted from the local optimal solution acquired by Newton iterations. Simulation results show that when the number of anchor nodes is large, the performance of the proposed algorithm will be significantly better than that of semi-definite programming approach with the increase of measurement noise.

Keywords: Wireless localization; non-linear programming; time difference of arrival; newton Iteration

1 Introduction

Wireless positioning is to estimate the location of target nodes in a coordinate system by using some physical parameters of wireless signals. At present, wireless location can be used not only in emergency calls, but also in intelligent transportation, digital city and so on, and it has a wide range of applications [1–3].

According to the physical parameters used to localization, the algorithms can be divided into four basic types: Received signal strength (RSS) localization [4], time of arrival (TOA) localization [5], time difference of arrival (TDOA) localization, and angle of arrival (AOA) localization [6]. Among all the four kinds of localization, the TDOA has the higher precision than RSS and lower hardware complexity compared to the AOA, so it is used widely. And to the best of our knowledge, the current research about the TDOA algorithm can be divided into three categories: The first is the least squares positioning algorithm [7–9]; the second is the convex programming localization algorithm [10]; the third is the Taylor series localization algorithm [11].

In order to improve the performance of location algorithm of TDOA, a non-linear least squares (NLLS) method is proposed in this paper. Firstly, based on the criterion of the minimized sum of square error of the squared range difference from target to different anchor nodes, the location estimation is expressed as an optimal problem of a non-linear programming; then, an initial point is obtained using the semi-definite programming; and finally, the location is extracted from the local optimal solution acquired by Newton iterations. Simulation results show that when the number of anchor nodes is large, the performance of the proposed algorithm will be significantly better than that of semi-definite programming approach with the increase of measurement noise.

2 Problem Statements

In this paper, we only take the two-dimensional coordinate system is consideration. Assuming that there are $N+1\left(N\geq 3\right)$ anchor nodes, which are not in the same line, with known coordinates

$(x_i, y_i), i = 0, \cdots, N$. We let the anchor node labeled 0 be the reference node, and the other be the ordinary nodes. We also assume that there is a target node with unknown coordinates (x, y). It is worth noting that although this paper studies the location algorithm in two-dimensional space, it is also suitable to three-dimensional space.

Through TDOA measurement, we can get the time difference t_{i0} between signals from the target node to the common anchor node i and to the reference node. The time difference multiplied by the propagation speed of the signal, we can get the distance difference d_{i0} from the target node to the common anchor node i and to the reference node. Since the propagation speed of signal is a constant, we will use the distance difference to replace the time difference in next.

The distance difference d_{i0} can be expressed as:

$$d_{i0} = d_i - d_0 + n_i \tag{1}$$

$$= \sqrt{(x-x_i)^2 + (y-y_i)^2} - \sqrt{x^2+y^2} + n_i \tag{2}$$

where d_i and d_0 is the distance from the target node to the common anchor node i and to the reference node 0 respectively, and n_i is the measurement noise.

Based on the criterion of minimizing the sum of squared errors of range difference, the optimal coordinate of the target node can be formulated by Eq. (3):

$$\left(\hat{x}, \hat{y}\right) = \min_{x,y} \sum_{i=1}^{N} \left(d_{i0} - \sqrt{(x-x_i)^2 + (y-y_i)^2} + \sqrt{x^2+y^2}\right)^2 \tag{3}$$

obviously, Eq. (3) is equivalent to the optimal value for solving a nonlinear least squares problem.

For Eq. (2), without considering the measurement error, removing the item $\sqrt{x^2+y^2}$ to the left side and squaring both sides of the equation, after arranging, we can get:

$$2x_i x + 2y_i y + 2d_{i0}d_0 = x_i^2 + y_i^2 - d_{i0}^2 \tag{4}$$

We rewrite it in matrix forms:

$$A\theta = b \tag{5}$$

where $A = \begin{bmatrix} x_1 & y_1 & d_{10} \\ x_2 & y_2 & d_{20} \\ \vdots & \vdots & \vdots \\ x_N & y_N & d_{N0} \end{bmatrix}$, $\theta = \begin{bmatrix} x \\ y \\ d_0 \end{bmatrix}$ and $b = \frac{1}{2}\begin{bmatrix} x_1^2 + y_1^2 - d_{10}^2 \\ x_2^2 + y_2^2 - d_{20}^2 \\ \vdots \\ x_N^2 + y_N^2 - d_{N0}^2 \end{bmatrix}$. Eq. (5) is essentially based on the criterion of minimizing the square error of the square range difference, and a set of linear equations including the coordinates of the target node is obtained. Because of the measurement noise, the linear equations are also incompatible.

3 Localization Algorithm

For Eq. (4), if we can get its global optimal solution, then we can obtain the location of the target node. However, Eq. (4) is a non-linear non-convex optimal problem with regard to the coordinate. Therefore, we will take the global optimal solution of Eq. (5) as the initial point of Eq. (4), and then uses Newton iterations to obtain the location of the target node.

The global optimal solution $\left(x,y\right)$ of Eq. (5) could be achieved by semi-definite programming [12], which is formulated as:

$$\left(x,y\right)=\arg\min_{X,x}\left(Tr(A^{T}AX)-2b^{T}AX+b^{T}b\right)$$
$$s.t.\quad\begin{cases}d_0^2=x^2+y^2\\ \begin{bmatrix}X & x\\ x & 1\end{bmatrix}\geq 0\\ X=\theta\theta^T\end{cases}\tag{6}$$

Next, we take the coordinate value extracted from θ as an initial point, and then use Newton iteration method to search for its local minimum. Since Newton iteration method needs to calculate the first and second derivatives of the objective function, we let $f\left(x,y\right)$ be the objective function in Eq. (3):

$$f\left(x,y\right)=\sum_{i=1}^{N}\left(d_{i0}-\sqrt{(x-x_i)^2+(y-y_i)^2}+\sqrt{x^2+y^2}\right)^2\tag{7}$$

Its first order partial derivatives are:

$$\frac{\partial f\left(x,y\right)}{\partial x}=2\sum_{i=1}^{N}\left\{\left[d_{i0}-\sqrt{(x-x_i)^2+(y-y_i)^2}+\sqrt{x^2+y^2}\right]\times\left[\frac{x}{\sqrt{x^2+y^2}}-\frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}\right]\right\}\tag{8}$$

$$\frac{\partial f\left(x,y\right)}{\partial y}=2\sum_{i=1}^{N}\left\{\left[d_{i0}-\sqrt{(x-x_i)^2+(y-y_i)^2}+\sqrt{x^2+y^2}\right]\times\left[\frac{y}{\sqrt{x^2+y^2}}-\frac{y-y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}\right]\right\}\tag{9}$$

4 Simulations

The simulation environments are as follows: 1) In a two-dimensional plane, there are 10 anchor nodes, one of which is located at the origin of coordinates, the other nine anchor nodes are (16, 42) meters, (34, 52) meters, (58, 30) meters, (78, 18) meters, (66, 48) meters, meters, (30, –12) meters, (22, 12) meters, (53, –3) meters and (12, –28) meters, respectively. The coordinates of the target node are (42, 12) meters; 2) The measurement noise follows the Gauss distribution with the mean value 0 and the variance δ^2.

Fig. 1, Fig. 2 and Fig. 3 show the location errors under different measurement noise variances when the number of anchor nodes is 5, 7 and 9 respectively. It can be seen from the graph that: 1) The positioning error of SDP algorithm and NLLS algorithm in this paper is less than that of CRLB, indicating that both of them belong to biased estimation; 2) When the number of anchor nodes is 5, the performance of SDP algorithm is almost the same as that of NLLS algorithm, but when the number of anchor nodes is 9, especially when the measurement noise is high, the performance of NLLS algorithm will be significantly better than that of SDP algorithm. This means that NLLS algorithm is sensitive to the number of base stations. When the number of base stations is large, NLLS algorithm could achieve better performance.

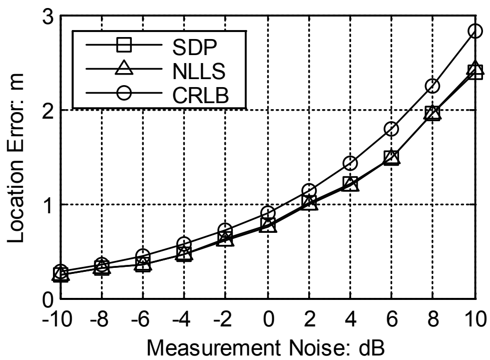

Figure 1: The location error when the number of anchor nodes is 5

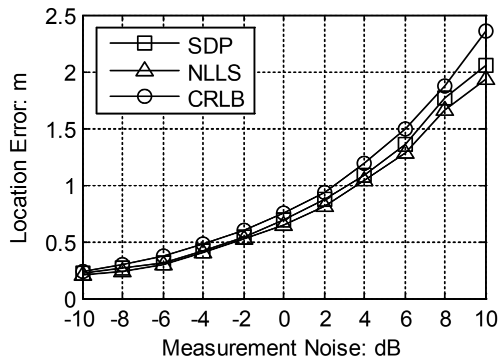

Figure 2: The location error when the number of anchor nodes is 7

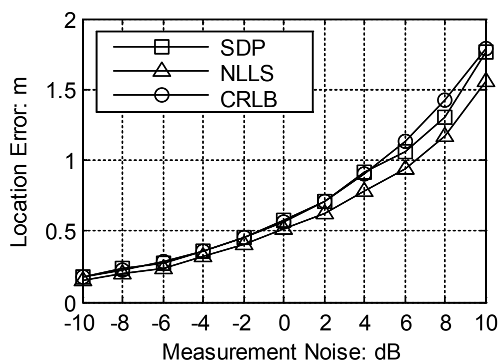

Figure 3: The location error when the number of anchor nodes is 9

In order to further verify the influence of the number of anchor nodes on the performance of the NLLS algorithm. When the measured noise is –5 dB, 0 dB and 5 dB respectively, the location errors under different number of anchor nodes are simulated. The results are shown in Fig. 4, Fig. 5 and Fig. 6. From the three figures, we can see that when the number of anchor nodes is less than 6, the difference between SDP and NLLS is not obvious, but as the number of anchor nodes continues to increase, the performance of the two algorithms shows obvious differences, NLLS algorithm is significantly better than SDP algorithm.

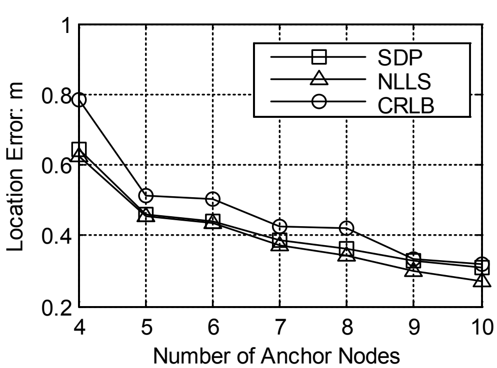

Figure 4: The location error when the measurement noise is –5 dB

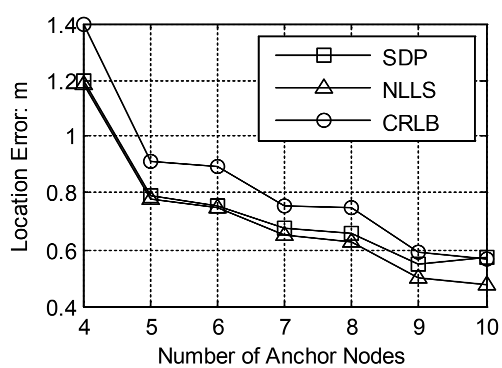

Figure 5: The location error when the measurement noise is 0 dB

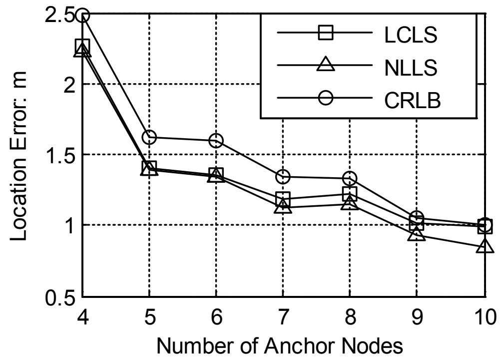

Figure 6: The location error when the measurement noise is 5 dB

5 Conclusions

In this paper, a non-linear least squares localization algorithm based on TDOA measurement is proposed. Firstly, based on the criterion of the minimized sum of square error of time difference of arrival, the location estimation is expressed as an optimal problem of a non-linear programming. Then, an initial point is obtained using the semi-definite programming. And finally, the location is extracted from the local optimal solution acquired by Newton iterations. Simulation results show that when the number of anchor nodes is large, the performance of the proposed algorithm will be significantly better than that of semi-definite programming approach with the increase of measurement noise.

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