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Article

Design of an Observer-Based Controller for T-S Fuzzy Time-Delay Systems under Imperfect Premise Matching

Zejian Zhang[1], Dawei Wang[2,*], Peng Li[1] and Xiao-Zhi Gao[3]

[1]Mechanical Engineering College, Beihua University, Jilin, 132021, China
[2]Mathematic College, Beihua University, Jilin, 132013, China
[3]School of Computing, University of Eastern Finland, Joensuu, 70211, Finland
[*]Corresponding Author: Dawei Wang. Email: wdw9211@126.com

Abstract: In this paper, the stabilization problem of an observer-based T-S fuzzy time-delay system under imperfect premise matching is studied, in which the T-S fuzzy observer model with time-delay and the fuzzy controller do not share the same membership functions. The objective is to design a state observer and un-matching fuzzy controller such that the closed-loop system with time-delay is asymptotically stable. A sufficient condition for the stabilization via observer-based state feedback under imperfect premise matching is presented, and an observer-based state feedback controller under imperfect premise is also constructed. The proposed control scheme is well capable of enhancing the design flexibility, because the membership functions of the observer-based controller can be arbitrarily selected. A few numerical examples are given to illustrate the effectiveness and advantages of our design method.

Keywords: T-S fuzzy system; time-delay; observer-based controller; imperfect premise matching

1 Introduction

Time-delay phenomena intensively exist in the natural science and social life, such as nonlinear and time-delay models. For example, they are in numerous dynamical systems including biology systems, mechanics, economics, chemical systems, network systems, etc. [1,2]. Generally, time-delay can often lead to instability and poor performances. Therefore, it is of great significance to investigate the issue of the stability for time-delay systems. A lot of relevant research work has been reported in [3–5]. However, the results obtained are mainly only based on the linear time-delay systems. It is indeed necessary to generalize them to the nonlinear time-delay systems. As we know that the fuzzy model proposed by Takagi and Sugeno can effectively represent many nonlinear dynamic systems [6]. During the past decade, more and more researchers pay their attention to the issue of the stability for the nonlinear fuzzy systems with time-delay in [7,8]. The stability problem refers to the design of a controller that can guarantee the fuzzy model stability. State feedback plays an important role in handling stabilization in the control systems. However, sometimes the system states cannot be measured directly, and there are limitations of the measurement equipment in use. It is actually impossible to get all the information of the system state variables. Under this circumstance, the physical form of state feedback is difficult to realize. Therefore, state estimation and observation of nonlinear systems are important but challenging problems in modern control theory. The design of observers and controllers design for the T-S fuzzy systems is a demanding topic. For example, the stability analysis and stabilization of the T-S fuzzy models for designing the observers and control laws are addressed in [9,10]. A single-step linear matrix inequality method is developed for the observer-based controller design for discrete-time fuzzy systems in [11], which overcomes the drawbacks induced by the conventional two-step approach, and yields less conservative results. The problem of fuzzy observer-based controller design was investigated for nonlinear networked control systems subject to imperfect communication links and parameter uncertainties in [12]. Some results are generalized to the observer-based adaptive model in [13]. A novel method based on the non-uniform delay partitioning approach was put forward to analyze the stability of the time-delay T-S fuzzy system and design the observer-based feedback controller via the Parallel Distributed Compensation (PDC) scheme in [14]. A fuzzy functional observer method was proposed to design a controller for the observer-based fuzzy model in [15,16]. In particular, some results related to the observer-based controller design problem have been extended to the MIMO time-delay systems in [17].

In the above work, it is always assumed that the observer-based feedback controller is all based on the PDC scheme, which means that the observed-based fuzzy model and the observed-based fuzzy controller share the same membership functions, which leads to the perfect premise matching. As a matter of fact, if the membership functions of the fuzzy controller in the premise of the fuzzy rules are allowed to be designed arbitrarily, a greater design flexibility can be achieved. Thus, some research work on the control problems under imperfect premise matching has been carried out in [18–20], which is different from the observer-based or time-delay approach in [21]. Because the states of the systems are sometimes difficult to be obtained or the costs of measurement are too high in many practical problems, it is critical to design observers of the systems. In addition, nonlinear and time delays are inherent, and not all the states are available in most practical systems. Therefore, the observer-based stabilization control for the time-delay T-S fuzzy systems is an important topic.

In this paper, the observer-based state feedback controllers under imperfect premise matching are investigated for the T-S fuzzy time-delay systems, where the membership functions of the fuzzy observer model are distinctive with those of the observer-based controller. A stabilization approach contains some information of the membership functions for this type of system through fuzzy observer-based controller is proposed. By numerical simulations, it is concluded that the controller we design can guarantee the stability of the observer model, and the design approach is much more flexibility than in previous paper.

The rest of this paper is organized as follows. Section 2 gives the system description and presents some definitions and lemmas. In Section 3, a memory observer-based controller under imperfect premise matching is introduced. Numerical example is further presented to illustrate the effectiveness of the proposed results in Section 4. Finally, some conclusions and remarks are drawn in Section 5.

Notations: In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq) 0$ is used to denote a symmetric positive-definite (positive semi-definite, negative, negative semi-definite, respectively) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand the minimum and maximum eigenvalue of the corresponding matrix respectively. $\|\cdot\|$ denotes the Euclidean norm for vector or the spectral norm of matrix. For convenience, we use w_i, m_i instead of $w_i(z(t)), m_i(z(t))$.

2 System Description

Let r be the number of the fuzzy rules describing the time-delay nonlinear plant. The i th rule can be represented as follows:

IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i, then

$$\begin{aligned} \dot{x}(t) &= A_{1i}x(t) + A_{2i}x(t-\tau_i(t)) + B_i u(t), \\ y(t) &= C_{1i}x(t) + C_{2i}x(t-\tau_i(t)), \end{aligned} \tag{1}$$

where r is the number of the fuzzy rules, $M_a^i \quad a = 1,2,\cdots,p; i = 1,2,\cdots,r$ denotes the fuzzy set. $z_k(t), k = 1,2,\cdots,p$ are the premise variables, which are not necessarily measurable and do not depend on the input variables. $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, and $y(t)$ is the output vector.

$A_{1i}, A_{2i}, B_i, C_{1i}, C_{2i}$ are some constant matrices of compatible dimensions. $\tau_i(t) \leq \tau$, since $i = 1,2,\cdots,r$ is the bounded time-delay in the state.

With the fuzzy inference methods, the final outputs of the fuzzy time-delay model can be formalized as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} w_i(z(t))\left[A_{1i}x(t) + A_{2i}x(t-\tau_i(t)) + B_i u(t)\right], \\ y(t) &= \sum_{i=1}^{r} w_i(z(t))\left[C_{1i}x(t) + C_{2i}x(t-\tau_i(t))\right], \end{aligned} \tag{2}$$

where

$$\sum_{i=1}^{r} w_i(z(t)) = 1,\ w_i(z(t)) \geq 0, i = 1,2,\ldots r \qquad \forall t, \tag{3}$$

$$w_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r}\mu_i(z(t))},\ \mu_i(z(t)) = \prod_{k=1}^{p}\mu_{M_k^i} z_k(t), \tag{4}$$

$z_1(t),\cdots,z_p(t)$ are the premise variable. $\mu_{M_k^i}(z_k(t))$ is the grade of membership of $z_k(t)$ in M_k^i .

In many engineering practices, the state information of system (1) is often not available. Therefore, it is necessary to construct a state observer satisfying the following formal equation to estimate the state of the system.

Observer rule i :

IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i, then

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{1i}\hat{x}(t) + A_{2i}\hat{x}(t-\tau_i(t)) + B_i u(t) + L_i(y(t) - \hat{y}(t)), i = 1,2,\cdots,r, \\ \hat{y}(t) &= C_{1i}\hat{x}(t) + C_{2i}\hat{x}(t-\tau_i(t)), \end{aligned} \tag{5}$$

where L_i is the fuzzy observer gain for the i th subsystem. The overall fuzzy observer is represented by:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} w_i(z(t))\left[A_{1i}\hat{x}(t) + A_{2i}\hat{x}(t-\tau_i(t)) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right], \\ y(t) &= \sum_{i=1}^{r} w_i(z(t))\left[C_{1i}x(t) + C_{2i}x(t-\tau_i(t))\right], \\ \hat{y}(t) &= \sum_{i=1}^{r} w_i(z(t))\left[C_{1i}\hat{x}(t) + C_{2i}\hat{x}(t-\tau_i(t))\right]. \end{aligned} \tag{6}$$

With the above fuzzy observer, the control law under the imperfect premise matching is defined as follows:

IF $z_1(t)$ is N_1^i and … and $z_q(t)$ is N_q^i, then

$$u(t) = F_j\hat{x}(t) \qquad j = 1,2,\ldots,r. \tag{7}$$

The observer-based fuzzy control law is represented by:

$$u(t) = \sum_{j=1}^{r} m_j(z(t)) F_j \hat{x}(t). \tag{8}$$

Combining the fuzzy controller (8) and fuzzy observers (6) and (2), and denoting $e(t) = x(t) - \hat{x}(t)$, we obtain the following systems:

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}w_i m_j\left[\left(A_{1i}+B_iF_j\right)x(t)+A_{2i}x\left(t-\tau_i(t)\right)-B_iF_je(t)\right],\tag{9}$$

$$\dot{e}(t)=\sum_{i=1}^{r}\sum_{k=1}^{r}w_i w_k\left[\left(A_{1i}-L_iC_{1k}\right)e(t)+A_{2i}e\left(t-\tau_i(t)\right)-L_iC_{2k}e\left(t-\tau_k(t)\right)\right].\tag{10}$$

Therefore, the augmented system can be written as

$$\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i m_j w_k\left[G_{ijk}\tilde{x}(t)+\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)\right],\tag{11}$$

where

$$\tilde{x}(t)=\begin{bmatrix}x(t)\\ e(t)\end{bmatrix},\ G_{ijk}=\begin{bmatrix}A_{1i}+B_iF_j & -B_iF_j\\ 0 & A_{1i}-L_iC_{1k}\end{bmatrix},\ \bar{M}_i=\begin{bmatrix}A_{2i} & 0\\ 0 & A_{2i}\end{bmatrix},\ \bar{N}_{ik}=\begin{bmatrix}0 & 0\\ 0 & -L_iC_{2k}\end{bmatrix}.$$

Remark 1. It can be observed from Eq. (8) that the controller we design and the observer-based fuzzy model do not share the same membership functions. Thus, for the observer-based T-S fuzzy time-delay systems, the membership functions of the fuzzy controller can be arbitrarily selected in the design phase, which can enhance the design flexibility. Consequently, it is worthy to investigate the stability conditions of this fuzzy time-delay system.

Lemma 1 [22]. For any vectors $x, y \in R^n$, matrix $0 < S \in R^{n\times n}$, $D \in R^{n\times n_g}$,

$E \in R^{n_g\times n}, G \in R^{n_g\times n_g}$ and any scalar $\varepsilon > 0$, if $G^TG < I$, then

(i) $2x^Ty \le x^TS^{-1}x + y^TSy$

(ii) $DEG + E^TG^TD^T \le \varepsilon^{-1}DD^T + \varepsilon^{-1}E^TE$

Lemma 2 [23]. (Schur complement). The linear matrix inequality $\begin{bmatrix}H & \theta^T\\ \theta & R\end{bmatrix} > 0$ is equivalent to $R > 0, H - \theta^TR^{-1}\theta > 0,$ where $H = H^T, R = R^T$ and θ is a matrix with appropriate dimension.

3 Main Results

Theorem 1. The equilibrium of the closed-loop fuzzy time-delay system (11) with the observer-based control law (8) under imperfect premise matching is asymptotically stable, if the membership functions of the fuzzy model and fuzzy controller satisfy $m_j\left(z(t)\right)-\rho_j w_j\left(z(t)\right)\ge 0$ for all j and $z(t)$, where $0<\rho_j<1$, and there exist matrices $X_1>0, X_2>0, S_{1i}>0, S_{2i}>0, Y_i, \Lambda_{1j}>0, \Lambda_{2j}>0$ and R_i satisfying: $S_{1i}\ge X_1$ $S_{2i}\le X_2$, and Eqs. (23)–(28). The state feedback gain and observer gain can be constructed as $F_i = Y_iX_1^{-1},\ L_i = X_2^{-1}R_i$.

Proof: Select a Lyapunov function as $V\left(\tilde{x}(t)\right)=\tilde{x}^T(t)P\tilde{x}(t)\qquad P>0$.

and we have

$$\dot{V}\left(\tilde{x}(t)\right)=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i m_j w_k\left[2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+\tilde{x}^T(t)\left(G_{ijk}{}^TP+PG_{ijk}\right)\tilde{x}(t)\right].\tag{12}$$

In the following, we will prove the asymptotic stability of the time-delay system (11) based on the Razumikhin theorem [24]. It is evident that there exist $\sigma_1=\lambda_{\min}(P)$ and $\sigma_2=\lambda_{\max}(P)$ such that

$$\sigma_1\left\|x(t)\right\|^2\le V\left(x(t)\right)\le\sigma_2\left\|x(t)\right\|^2.\tag{13}$$

Consider

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i w_k\left(w_j-m_j\right)\Lambda_i=\sum_{i=1}^{r}\sum_{j=1}^{r}w_i w_k\left(\sum_{j=1}^{r}w_j-\sum_{j=1}^{r}m_j\right)\Lambda_i=\sum_{i=1}^{r}\sum_{j=1}^{r}w_i w_k\left(1-1\right)=0,\tag{14}$$

where $\Lambda_i = \Lambda_i^T \in R^{n\times n} > 0, i = 1,2,\ldots,r$ are arbitrary matrices. These terms are introduced to Eq. (12) to alleviate the conservative. From Eq. (12), we have

$$\dot{V}\left(\tilde{x}(t)\right)=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i m_j w_k\left[\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}\right)\tilde{x}(t)+2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)\right]$$

$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i w_k\left(w_j-m_j+\rho_j w_j-\rho_j w_j\right)\tilde{x}^T(t)\Lambda_i\tilde{x}(t)$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i w_k\left(m_j-\rho_j w_j\right)\left[\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i\right)\tilde{x}(t)+2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)\right]$$

$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i w_k w_j\left\{\tilde{x}^T(t)\left[\rho_j\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i\right)+\Lambda_i\right]\tilde{x}(t)+2\tilde{x}^T(t)\rho_j P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)\rho_j P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)\right\}. \quad (15)$$

With $m_j\left(z(t)\right)-\rho_j w_j\left(z(t)\right)\geq 0$ for all j and $z(t)$, let

$$\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i\right)\tilde{x}(t)+2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)<0. \quad (16)$$

It should be noted that by using Lemma 1, and $S^{-1}<P$ and Razumikhin theorem, we assume that there exist a real $\sigma>1$ such that $V\left(x(t-\theta)\right)<\sigma V\left(x(t)\right)$ for all $\theta\in[0,\pi]$. Then Eq. (16) can be represented as follows:

$$\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i\right)\tilde{x}(t)+2\tilde{x}^T(t)P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)$$
$$\leq\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i+P\bar{M}_i S_i\bar{M}_i{}^T P+P\bar{N}_{ik}S_{ik}\bar{N}_{ik}^T P\right)\tilde{x}(t)+\tilde{x}^T\left(t-\tau_i(t)\right)S^{-1}\tilde{x}^T\left(t-\tau_i(t)\right)$$
$$+\tilde{x}^T\left(t-\tau_i(t)\right)S^{-1}\tilde{x}\left(t-\tau_i(t)\right)+\tilde{x}^T\left(t-\tau_k(t)\right)S^{-1}\tilde{x}\left(t-\tau_k(t)\right)$$
$$\leq\tilde{x}^T(t)\left(G_{ijk}{}^T P+PG_{ijk}-\Lambda_i+2\sigma P+P\bar{M}_i S_i\bar{M}_i{}^T P+P\bar{N}_{ik}S_{ik}\bar{N}_{ik}^T P\right)\tilde{x}(t)<0.$$

Let

$$G_{ijk}{}^T P+PG_{ijk}-\Lambda_i+P\bar{M}_i S_i\bar{M}_i^T P+P\bar{N}_{ik}S_k\bar{N}_{ik}^T P+2\sigma P<0., \quad (17)$$

Let

$$G_{ijk}{}^T P+PG_{ijk}-\Lambda_i+P\bar{M}_i S_i\bar{M}_i^T P+P\bar{N}_{ik}S_k\bar{N}_{ik}^T P+2P<0. \quad (18)$$

It is not difficult to find that if Eq. (18) holds, obviously then there exist $\sigma>1$ such that Eq. (17) holds.

Similar to the above proof, by Lemma 1, we have:

$$\dot{V}\left(\tilde{x}(t)\right)\leq\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}w_i w_k w_j\left\{\tilde{x}^T(t)\left[\rho_j\begin{pmatrix}G_{ijk}{}^T P+\\ PG_{ijk}-\Lambda_i\end{pmatrix}+\Lambda_i\right]\tilde{x}(t)+2\tilde{x}^T(t)\rho_j P\bar{M}_i\tilde{x}\left(t-\tau_i(t)\right)+2\tilde{x}^T(t)\rho_j P\bar{N}_{ik}\tilde{x}\left(t-\tau_k(t)\right)\right\}$$

$$=\sum_{i=1}^{r}w_i^3\tilde{x}^T(t)\left\{\rho_i\left[G_{iii}^T P+PG_{iii}-\Lambda_i+P+P\left(\bar{M}_i+\bar{N}_{ii}\right)S_i\left(\bar{M}_i+\bar{N}_{ii}\right)^T P\right]+\Lambda_i\right\}\tilde{x}(t)$$

$$+\sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r}w_i^2 w_j\tilde{x}(t)\left\{\rho_i\left[G_{iij}^T P+PG_{iij}-\Lambda_i+2P+P\bar{M}_i S_i\bar{M}_i^T+P\bar{N}_{ij}S_j\bar{N}_{ij}^T P\right]\right.$$

$$+\rho_j\left[G_{iji}^T P+PG_{iji}-\Lambda_i+P+P\left(\bar{M}_i+\bar{N}_{ii}\right)S_i\left(\bar{M}_i+\bar{N}_{ii}\right)^T P\right]+\Lambda_i+\Lambda_i$$

$$\left.+\rho_i\left[G_{jii}^T P+PG_{jii}-\Lambda_j+2P+P\bar{M}_j S_j\bar{M}_j{}^T P+P\bar{N}_{ji}S_i\bar{N}_{ji}^T P+\Lambda_j\right]\right\}\tilde{x}(t)$$

$$+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{k=j+1}^{r} w_i w_j w_k \tilde{x}^T(t)\Big\{\rho_j\Big[G_{ijk}^T P + PG_{ijk} - \Lambda_i + 2P + P\bar{M}_i S_i \bar{M}_i^T P + P\bar{N}_{ik} S_k \bar{N}_{ik}^T P\Big] + \Lambda_i +$$
$$\rho_k\Big[G_{ikj}^T P + PG_{ikj} - \Lambda_i + 2P + P\bar{M}_i S_i \bar{M}_i^T P + P\bar{N}_{ij} S_j \bar{N}_{ij}^T P\Big] + \Lambda_i +$$
$$\rho_i\Big[G_{jik}^T P + PG_{jik} - \Lambda_j + 2P + P\bar{M}_j S_j \bar{M}_j^T P + P\bar{N}_{jk} S_k \bar{N}_{jk}^T P\Big] + \Lambda_j$$
$$+\rho_k\Big[G_{jki}^T P + PG_{jki} - \Lambda_j + 2P + P\bar{M}_j S_j \bar{M}_j^T P + P\bar{N}_{ji} S_i \bar{N}_{ji}^T P\Big] + \Lambda_j$$
$$+\rho_i\Big[G_{kij}^T P + PG_{kij} - \Lambda_k + 2P + P\bar{M}_k S_k \bar{M}_k^T P + P\bar{N}_{kj} S_j \bar{N}_{kj}^T P\Big] + \Lambda_k$$
$$+\rho_j\Big[G_{kji}^T P + PG_{kji} - \Lambda_k + 2P + P\bar{M}_k S_k \bar{M}_k^T P + P\bar{N}_{ki} S_i \bar{N}_{ki}^T P\Big] + \Lambda_k\Big\}\tilde{x}(t)$$
$$\triangleq \tilde{\boldsymbol{x}}^T(t)\Omega\tilde{\boldsymbol{x}}(t). \tag{19}$$

Let

$$\Theta_i = G_{iii}^T P + PG_{iii} - \Lambda_i + P + P\left(\bar{M}_i + \bar{N}_{ii}\right)S_i\left(\bar{M}_i + \bar{N}_{ii}\right)^T.$$
$$\Theta_{ij} = G_{iji}^T P + PG_{iji} - \Lambda_i + P + P\left(\bar{M}_i + \bar{N}_{ii}\right)S_i\left(\bar{M}_i + \bar{N}_{ii}\right)^T P.$$
$$\Psi_{ij} = G_{iij}^T P + PG_{iij} - \Lambda_i + 4P + P\bar{M}_i S_i \bar{M}_i^T P + G_{jii}^T P + PG_{jii} - \Lambda_j + P\bar{N}_{ij} S_j \bar{N}_{ij}^T P.$$
$$\Psi_{jik} = G_{jik}^T P + PG_{jik} - \Lambda_j + 2P + P\bar{M}_j S_j \bar{M}_j^T P + G_{kij}^T P + PG_{kij} - \Lambda_k + 2P + P\bar{N}_{jk} S_k \bar{N}_{jk}^T P.$$
$$\Psi_{ikj} = G_{ikj}^T P + PG_{ikj} - \Lambda_i + 2P + P\bar{M}_i S_i \bar{M}_i^T P + G_{jki}^T P + PG_{jki} - \Lambda_j + 2P + P\bar{N}_{ji} S_i \bar{N}_{ji}^T P.$$
$$\Psi_{ijk} = G_{ijk}^T P + PG_{ijk} - \Lambda_i + 2P + P\bar{M}_i S_i \bar{M}_i^T P + G_{kji}^T P + PG_{kji} - \Lambda_k + 2P + P\bar{N}_{ki} S_i \bar{N}_{ki}^T P.$$

If

$$\rho_i\Theta_i + \Lambda_i < 0, \tag{20}$$
$$\rho_i\Psi_{ij} + \rho_j\Theta_{ij} + 2\Lambda_i + \Lambda_j \le 0, i, j = 1,2,\cdots,r, j \ne i, \tag{21}$$
$$\rho_j\Psi_{ijk} + \rho_k\Psi_{ikj} + \rho_i\Psi_{jik} + 2\Lambda_j + 2\Lambda_i + 2\Lambda_k \le 0. \quad i = 1,2,\cdots,r-2; j = i+1,\cdots r-1; k = j+1,\cdots,r. \tag{22}$$

and Eq. (18) holds, From Razumikhin theorem, the equilibrium of the closed-loop fuzzy time-delay system is asymptotically on the basis of the observer-based control law.

Let

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2^{-1} \end{bmatrix} = P^{-1},\; S_i = \begin{bmatrix} S_{1i} & 0 \\ 0 & S_{2i}^{-1} \end{bmatrix},\; \Lambda_i = \begin{bmatrix} \Lambda_{1i} & 0 \\ 0 & \Lambda_{2i} \end{bmatrix}.$$

Pre-multiply and post-multiply Eq. (18) and Eqs. (20)–(22) with X, and define $Y_i = F_i X_1, R_i = X_2 L_i$ with the Shur complements, Eq. (18) can be described as Eq. (23) and Eq. (24); and Eqs. (20)–(22) can be described as Eqs. (25)–(28).

$$X_1 A_{1i}^T + A_{1i} X_1 + B_i Y_j + Y_j^T B_i^T + 2X_1 + A_{2i} S_{1i} A_{2i}^T - Z_{1i} < 0. \tag{23}$$

$$\begin{bmatrix} \left(A_{1i}^T X_2 + X_2 A_{1i} - C_{1k}^T R_i^T - R_i C_{1k} + 2X_2 - \Lambda_{2i}\right) & * & * \\ A_{2i}^T X_2 & -S_{2i} & 0 \\ C_{2k}^T R_i^T & 0 & -S_{2k} \end{bmatrix} < 0. \tag{24}$$

$$\rho_i\Big[X_1 A_{1i}^T + A_{1i} X_1 + B_i Y_i + Y_i^T B_i^T + X_1 + A_{2i} S_{1i} A_{2i}^T - Z_{1i}\Big] + Z_{1i} < 0. \tag{25}$$

$$\rho_i\begin{bmatrix} \Pi_i & X_2 A_{2i} - R_i C_{2i} \\ A_{2i}^T X_2 - C_{2i}^T R_i^T & -S_{2i} \end{bmatrix} + \Lambda_{2i} < 0, \tag{26}$$

where $\Pi_i = A_{1i}^T X_2 + X_2 A_{1i} - C_{1i}^T R_i^T - R_i C_{1i} + X_2 - \Lambda_{2i}$.

$$\rho_i\begin{bmatrix}\Pi_{ij} & * & * \\ A_{2i}{}^TX_2+C_{2i}{}^TR_j^T & -S_{2i} & 0 \\ C_{2j}{}^TR_i^T+A_{2j}{}^TX_2 & 0 & -S_{2j}\end{bmatrix}+\rho_j\begin{bmatrix}\Pi_i & * & 0 \\ A_{2i}{}^TX_2-C_{2i}{}^TR_i^T & -S_{2i} & 0 \\ 0 & 0 & 0\end{bmatrix}+2\Lambda_{2i}+\Lambda_{2j}<0, \tag{27}$$

where

$$\Pi_{ij}=A_{1i}{}^TX_2+X_2A_{1i}+A_{1j}{}^TX_2+X_2A_{1j}+4X_2-R_iC_{1j}-C_{1i}{}^TR_j^T-R_jC_{1i}-C_{1j}{}^TR_i^T-\Lambda_{2i}-\Lambda_{2j}.$$

$$\begin{aligned}&\rho_i\begin{bmatrix}\Pi_{jk} & * & * \\ A_{2j}{}^TX_2+C_{2j}{}^TR_k^T & -S_{2j} & 0 \\ C_{2k}{}^TR_j^T+A_{2k}{}^TX_2 & 0 & -S_{2k}\end{bmatrix}+\rho_j\begin{bmatrix}\Pi_{ik} & * & * \\ A_{2i}{}^TX_2+C_{2i}{}^TR_k^T & -S_{2i} & 0 \\ A_{2k}{}^TX_2+C_{2k}{}^TR_i^T & 0 & -S_{2k}\end{bmatrix}+\\ &\rho_k\begin{bmatrix}\Pi_{ij} & * & * \\ A_{2i}{}^TX_2+C_{2i}{}^TR_j^T & -S_{2i} & 0 \\ C_{2j}{}^TR_i^T+A_{2j}{}^TX_2 & 0 & -S_{2j}\end{bmatrix}+2\Lambda_{2i}+2\Lambda_{2j}+2\Lambda_{2k}<0,\end{aligned} \tag{28}$$

where

$$\Pi_{ik}=A_{1i}{}^TX_2+X_2A_{1i}+A_{1k}{}^TX_2+X_2A_{1k}+4X_2-\Lambda_{2i}-C_{1k}{}^TR_i^T-R_iC_{1k}-C_{1i}{}^TR_k^T-R_kC_{1i}-\Lambda_{2k}.$$

$\Pi_{jk}=A_{1j}{}^TX_2+X_2A_{1j}+A_{1k}{}^TX_2+X_2A_{1k}+4X_2-\Lambda_{2j}-C_{1j}{}^TR_k^T-R_kC_{1j}-C_{1k}{}^TR_j^T-R_jC_{1k}-\Lambda_{2k}$. $Z_{1i}=X_1A_{1i}X_1$ and * implies that the blocks are readily inferred by symmetry. If the above equality holds, the equilibrium of the closed-loop fuzzy time-delay system with the observer-based control law is asymptomatic.

Remark 2. From Theorem 1, the observed-based fuzzy controller designed is much more flexible than the former results, because the membership functions can be selected from the model. In addition, the information of the membership functions is utilized in dealing with the stability problem.

4 Numerical Examples

Example 1. Consider the following time-delay T-S fuzzy system:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{3}w_i(t)\left(A_{1i}x(t)+A_{2i}x(t-\tau)+B_iu(t)\right),\\ y(t)&=\sum_{i=1}^{3}w_i(t)\left(C_{1i}x(t)+C_{2i}x(t-\tau)\right),\end{aligned} \tag{29}$$

with the parameters as follows:

$A_{11}=\begin{bmatrix}-1.4274 & 0.0757 \\ -1.4189 & -0.9442\end{bmatrix}$, $A_{21}=\begin{bmatrix}0.25 & 0 \\ 0 & 0.25\end{bmatrix}$, $B_1=\begin{bmatrix}0 \\ 1\end{bmatrix}$, $A_{12}=\begin{bmatrix}-2.0508 & 0.3958 \\ -6.4066 & 1.6168\end{bmatrix}$, $A_{22}=\begin{bmatrix}0.13 & 0 \\ 0 & 0.15\end{bmatrix}$, $B_2=\begin{bmatrix}0 \\ 1\end{bmatrix}$, $A_{13}=\begin{bmatrix}-4.5279 & 0.3167 \\ -26.2228 & 0.9837\end{bmatrix}$, $A_{23}=\begin{bmatrix}0.2 & 0 \\ 0 & 0.45\end{bmatrix}$ $B_3=\begin{bmatrix}0 \\ 1\end{bmatrix}$, $C_{11}=\begin{bmatrix}0 & 1\end{bmatrix}$, $C_{12}=\begin{bmatrix}0 & 1\end{bmatrix}$, $C_{13}=\begin{bmatrix}0 & 1\end{bmatrix}$, $C_{21}=\begin{bmatrix}1 & 0\end{bmatrix}$, $C_{22}=\begin{bmatrix}1 & 0\end{bmatrix}$, $C_{23}=\begin{bmatrix}1 & 0\end{bmatrix}$.

The membership functions are chosen as follows:

$$w_1(x_1(t))=\begin{cases}1, & x_1\le 0.8862\\ 1-\dfrac{x_1-0.8862}{2.7520-0.8862}, & x_1>0.8862 \\ & x_1<2.7520\\ 0, & x_1\ge 2.7520\end{cases},$$

$$w_2(x_1(t))=\begin{cases}1-w_1, & \text{if } x_1\le 2.7520\\ 1-w_3, & \text{if } x_1>2.7520\end{cases},$$

$$w_3(x_1(t)) = \begin{cases} 0, & x_1 \le 2.7520 \\ \dfrac{x_1 - 2.7520}{4.7052 - 2.7520}, & x_1 > 2.7520 \\ & x_1 < 4.7052 \\ 1, & x_1 \ge 4.7052 \end{cases}.$$

From Theorem 1, the observer-based fuzzy control law is represented by:

$$u(t) = \sum_{j=1}^{3} m_j(z(t)) F_j \hat{x}(t). \tag{30}$$

The membership functions of the fuzzy controller can be chosen as follows:

$$m_1(x_1(t)) = \begin{cases} 1, & \text{if} \quad x_1 \le 2 \\ 1 - \dfrac{x_1 - 2}{2.7520 - 2}, & \text{if} \quad 2 < x_1 < 2.7520, \\ 0, & \text{if} \quad x_1 \ge 2.7520 \end{cases}$$

$$m_2(x_1(t)) = \begin{cases} 1 - m_1, & \text{if} \quad x_1 \le 2.7520 \\ 1 - m_3, & \text{if} \quad x_1 > 2.7520 \end{cases},$$

$$m_3(x_1(t)) = \begin{cases} 0, & \text{if} \quad x_1 \le 2.7520 \\ \dfrac{x_1 - 2.7520}{3 - 2.7520}, & \text{if} \quad 2.7520 < x_1 < 3. \\ 1, & \text{if} \quad x_1 \ge 3 \end{cases}$$

For $\tau = 2.0$, we assume that $\rho_1 = 0.45,\ \rho_2 = 0.75,\ \rho_3 = 0.8$ such that $m_j(\theta(t)) - \rho_j w_j(\theta(t)) > 0$ for all j and $\theta(t)$.

Applying the MATLAB LMI toolbox to solve the LMIs in Theorem 1, we have:

$F_1 = [1.3335 \quad -0.5106]$, $F_2 = [1.3558 \quad -0.8027]$, $F_3 = [1.3568 \quad -0.6826]$,

$L_1 = [1.2379 \quad 9.8956]^T$, $L_2 = [1.0855 \quad 9.4773]^T$, $L_3 = [0.5914 \quad 7.8472]^T$.

The best value of t used in MATLAB is –0.012029.

With the initial conditions as $\boldsymbol{x}(0) = [0.8 \quad 0.6]^T$, $\hat{\boldsymbol{x}}(0) = [0 \quad 0]^T$ for $t \in [-1, 0]$, the simulation results under Theorem 1 are shown in Figs. 1–3, which illustrate the effectiveness of the Theorem 1 in this paper.

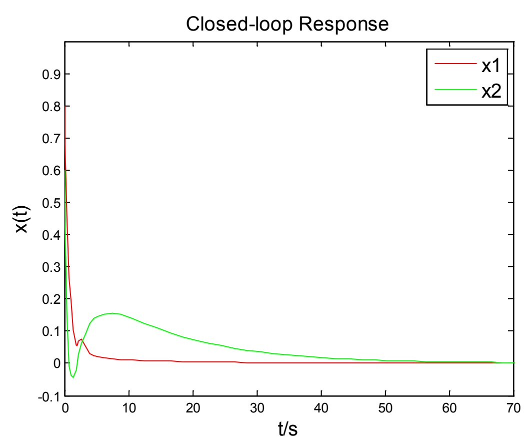

Figure 1: State response of the closed-loop system

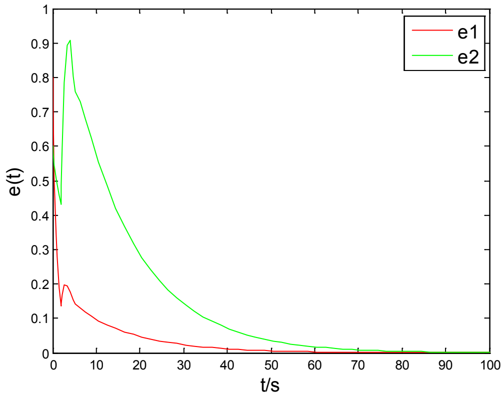

Figure 2: Error state response curve

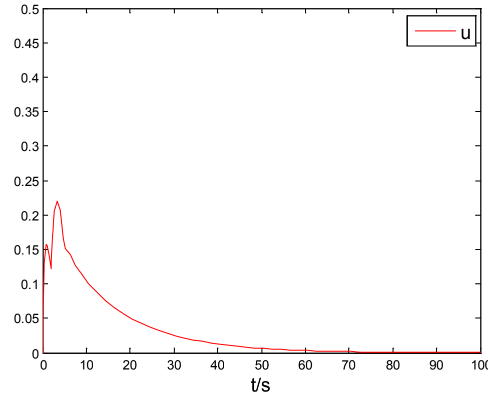

Figure 3: Control input curve

Remark 3. Based on the above example, the membership functions of the observer-based controller $w_i(x(t))$ are different from $m_i(x(t))$, $i=1,2,3$. From Fig. 1 and Fig. 2, we can conclude that the proposed controller ensures the observer model stability. Therefore, the proposed controller design method enhances the design flexibility, since the membership functions of the observer-based fuzzy controllers can be selected arbitrarily.

Example 2. The same fuzzy model with time-delay (29) is also considered. Some advantages of the proposed method are shown in this example. Suppose the membership function $w_i(x(t))$ are chosen as follows:

$$w_1(x_1(t))=\begin{cases}1, & x_1 \le 0.8862\\ 1-\dfrac{e^{x_1-0.8862}+\sin\delta(t)}{2.7520-0.8862}, & x_1>0.8862\\ & x_1<2.7520\\ 0, & x_1\ge 2.7520\end{cases},$$

$$w_2(x_1(t))=\begin{cases}1-w_1, & \text{if } x_1\le 2.7520\\ 1-w_3, & \text{if } x_1>2.7520\end{cases},$$

$$w_3(x_1(t))=\begin{cases}0, & x_1\le 2.7520\\ \dfrac{e^{x_1-2.7520}+\sin(\delta(t))}{4.7052-2.7520}, & x_1>2.7520\\ & x_1<4.7052\\ 1, & x_1\ge 4.7052\end{cases}.$$

where $\delta(t)$ is an uncertain variable.

Compare with $w_j(x(t))$ in Example 1, we can observe that the structure of the membership functions $w_j(x(t))$ are not only complex, but also contain an uncertain variable. Under the PDC controller design framework, the membership functions of the fuzzy controller must be selected as the same as those of the fuzzy model. In addition, $u(t)=\sum_{j=1}^{r}w_j(x(t))F_j\hat{x}(t)$, in which the membership functions of the fuzzy model are required for the implementation of fuzzy controller. This can increase the structure complexity of the fuzzy controller, leading to a higher implementation cost and increasing the difficulty of the fuzzy controller design. In addition, the fuzzy controller may not be realized, because there exists an uncertain variable in $w_j(x(t))$. However, under imperfect premise matching, some simple and specific membership functions $m_j(x(t))$ of the fuzzy controller Eq. (30) can be selected instead of the complex and uncertain functions $w_j(x(t))$, which can also ensure the closed-loop system (1) with time-delay being asymptotically stable.

Therefore, the proposed method can enhance the flexibility of the fuzzy controller as well as retain the robustness property of the T-S fuzzy control systems.

5 Conclusions

In this paper, the observer-based stabilization control for T-S fuzzy time-delay systems under imperfect premise matching is investigated, in which the T-S fuzzy observer model with time-delay and fuzzy controller do not share the same membership functions. The main contribution owes that the information of membership functions of the fuzzy model and controller are considered in the observer design scheme. Furthermore, the design flexibility can be enhanced by arbitrarily selecting simple membership functions for the observer-based controller. The advantages and effectiveness of the proposed method have been illustrated by simulation examples. The case with the unknown delay and fuzzy Lyapunov functions or piecewise Lyapunov functions will be considered in our future work.

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