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***Article***

# Blockchain Queuing Model with Non-Preemptive Limited-Priority

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**Abstract:** Blockchain technology has recently obtained widespread attention. And it is being regarded as potentially even more disruptive than the Internet, whose usage includes large areas of applications ranging from crypto currency, financial services, reputation system, Internet of Things, sharing economy to public and social services. The existing works of blockchain primarily are focused on key components and potential applications. However, in the existing blockchain systems, the waiting time of transactions is too long. Furthermore, it may produce serious consequences because many important transactions are not handled timely. To solve the problem, in the paper, the blockchain Queuing model with non-preemptive limited-priority is established, which considers the different transactions having different priority when being mined. There are two classes of transactions in the model, one is high-priority transaction with pay or with prior interest, the other is low-priority transaction without pay or without prior interest. And high-priority transactions can be mined preferentially when mining process is not occupied. If low-priority transaction is being mined, the arriving high-priority transaction will wait for the mining accomplishment. Through the analysis of the model, we compute average waiting time, average staying time and average length of queue. From simulation of the model, we find that transactions with pay or with prior interest and increase service rate of mining are effective for reducing waiting time. Besides, the two factors mutually reinforce to shorten waiting time. Finally, we conclude this paper and point out the direction of future research.

**Keywords:** Blockchain; queuing model; non-preemptive; limited-priority

## 1 Introduction

Blockchain is one of the hottest topics discussed extensively in recent years, which has already changed the lifestyle of people in some real areas for its great influence on finance, business, industry, transportation, healthcare and so forth. Nakamoto first introduced blockchain technology alongside with the crypto-currency bitcoin. Another technology ethereum founded in 2013, brought new features to the blockchain technology, including smart-contracts, which changed the whole game for this technology, allowing it to integrate more services and have more value to many industries and academic fields [1]. Projects about blockchain spring up like mushrooms, such as EOS, Fabric, R3 Corda implement distributed ledger received widespread attention [2].

The existing works of blockchain primarily are focused on key components and potential applications [3]. So one solution for this problem is that different transactions are given different priority and the high-priority transactions are mined preferentially whose waiting time can be shorter. But the existing blockchain queuing model cannot consider that the different transactions have different priority when being mined.

In this paper, the blockchain queuing model with non-preemptive limited-priority is established, which considers the different transactions having different priority when being mined. The waiting time of blockchain transaction is the problem that may confuse many people. Furthermore, it may produce serious consequences because many important transactions are not handled timely. We analyze the problem through queuing model. This is the achievement about blockchain queuing model which considers that transactions waiting for being mined to build block have different priority because of transactions with pay, with prior interest or not. Then, we simulate the model and find an interesting conclusion.

The remainder of this paper is organized as follows: In Section 2, we introduce the related work of this work. In Section 3, we descript the blockchain queuing. In Section 4, we present queuing model with non-preemptive limited-priority. In Section 5, we provide the simulation results. Finally, we give our concluding remarks in Section 6.

## 2 Related Work

The development of information security technology gave birth to blockchain technology. Blockchain combines information security technology, which has formed plenty of application cases. Blockchain technology has recently obtained widespread attention from media, businesses, public sector agencies, and various international organizations. It is being regarded as potentially even more disruptive than the Internet, especially with the progress of security cloud service [4–6]. Blockchain technologies have become widely adopted in many real applications, for example, business and information systems [7]; finance [8]; applications to companies [9]; Internet of Things and shared economy [10]; healthcare [11]; and the others.

So far there have been more papers belong to the literature of blockchain which describes key components, discusses potential applications and establishes the mathematical models to analyze and optimize blockchain systems [12]. Especially, Lin et al. [13] developed queuing theory of blockchain systems and provided system performance evaluation, but they did not consider the different transactions having different priority when being mined because of the transactions with pay, with prior interest or not. In this paper, the blockchain queuing model with non-preemptive limited-priority is established, which considers the different transactions having different priority when being mined.

Kasahara et al. [14] provided an early research (in fact, so far there have been only their two papers in the literature) on applying queuing theory to deal with the transaction–confirmation time for bitcoin, and they gave some interesting idea and useful numerical results to heuristically motivate future promising research. Kawase et al. [15] assumed that the block-generation times follow a general continuous probability distribution function, but this paper concerns mining process.

Queuing theory was first founded by the work of A.K. Erlang to study problems of congestion in telephone service. Nowadays, queuing theory has been applied in a variety of fields, e.g., communication networks [16], computer systems [17], machine plants, etc. And non-preemptive priority queuing model has been used widely, such as communication [18] and medical treatment [19]. It is an innovation that we use non-preemptive priority queuing model to analyze the blockchain transactions being mined.

## 3 Description for Blockchain Queuing

In this section, based on the real scene of blockchain, an interesting blockchain queuing model with no non-preemptive limited-priority is designed, where there are two classes of transactions waiting for blockchain-building through mining process, one is high-priority transaction with pay or with prior interest, the other is low-priority transaction without pay or without prior interest, shown as Fig. 1.

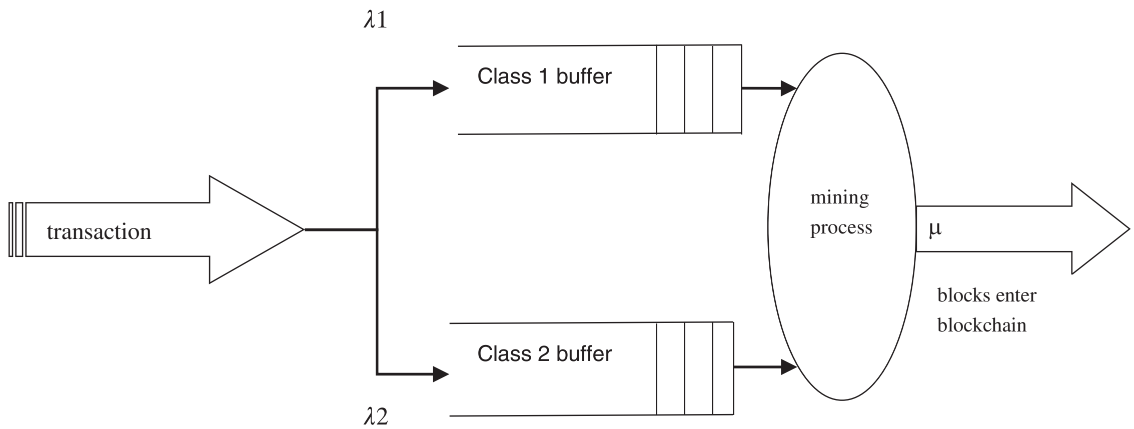


**Figure 1:** Blockchain queuing system

The development of information security technology gave birth of blockchain technology. Blockchain combines information security technology, which has formed plenty of application cases. Blockchain technology has recently obtained widespread attention from media, businesses, public sector agencies, and various international the model is decrypted as follows:

**Arrival process:** Transactions arrive at the blockchain system, which can be divided into 2 classes. The high-priority transactions enter class 1 buffer according to poisson process with arrival rate waiting for mining process, which have priority processing right when mining process is not occupied. If low-priority transaction is being mined, the arriving high-priority transaction will wait for the mining accomplishment. And the low-priority transactions enter class 2 buffer according to poisson process with arrival rate waiting for mining process, which has not priority processing right. Note that the arrival process of transactions is denoted in the left part of Fig. 1.

**Mining process:** Each arrival transaction in 2 classes' buffers waits for being successfully mined as a block; this is regarded as mining process. The miner can compute the transaction to find a hash output for the data in its block that starts with a certain amount of zero's. Then the block is generated after verification. Note that the mining process of transactions is denoted in the middle part of Fig. 1.

**Blocks entering blockchain:** The transaction is mined into a block that is sent to the chain with the service rate. Thus, the process from transaction to blockchain is finished. Note that the blocks entering blockchain is denoted in the right part of Fig. 1.

## 4 Presentation for Blockchain Queuing Model

### *4.1 Hypothesis*

Inspired by the work on the M/M/1 queuing system model, we propose an M/M/1 queuing system model under non-preemptive limited-priority. To begin with, we give the following definition and hypotheses to simplify our analysis:

**Hypothesis 1:** The transactions are divided into two classes, with class 1 enjoying the highest priority and class 2 enjoying the second priority.

**Hypothesis 2:** The transactions reach the system according to the poisson distribution of parameters for that is the average arrival rate of transactions, i = 1, 2.

**Hypothesis 3:** The transaction service time of each level is subject to the negative exponential distribution of the parameter, and the average service time is the average service rate for the transactions of class i, i = 1, 2.

**Hypothesis 4:** Integer α is a limited priority parameters, α and α, which means that when waiting for service, transactions of class 2 will be inserted the queue to receive service if they have waited α transactions. Therefore, when the transactions of class 1 and class 2 are queuing together in the system, the class 2 transactions are spaced with α transactions of class 1 to receive the service is the service time of class i transaction, i = 1, 2. S is serving time for the system.

$\overline{S_i} = ES_i = \frac{1}{\mu}, \rho_i = \frac{\lambda_i}{\mu} = \lambda_i \overline{S_i} \qquad i = 1,2$

The arrival of transactions of class 1 and class 2 is subject to independent poisson streams, so the probability of transaction belong to class i is $\frac{\lambda_i}{\lambda}$. The average service time of the system is $\bar{S}$.

$$\overline{S} = ES = \rho_1 ES_1 + \rho_2 ES_2 = \frac{1}{\mu}$$

The volume of system business is ρ.

$$\rho = \rho_1 + \rho_2 = \frac{\lambda_1 + \lambda_2}{\mu}$$

**Hypothesis 5:** For the total queuing system, no distinction between class 1 and class 2 customers, arrival of transactions is according to the poisson distribution by parameter. The service time of transactions is according to a negative exponential distribution, and the service rate is μ, then the length of queuing is the following.

$$L_S = \frac{\lambda}{\mu - \lambda} \tag{1}$$

The length of average waiting $L_q$ is the following.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{2}$$

### *4.2 The Waiting Time of Transactions*

When the transaction of class 1 arrives, if the mining process is free, the transaction is mined directly, if the mining process is busy, the transaction cannot occupy the service and only wait in front of some transactions of class 2 for the service. When the transaction of class 2 arrives, if the mining process is free, it can be mined directly, if the mining process is busy, or the α transactions of class 1 have been mined, it can wait for servicing after the previous completion, otherwise it can wait in class 2 buffer. The average number of class i is $L_{qi}$, the average waiting time of class i is $W_{qi}$, $i$ = 1, 2. Two scenarios are discussed, one is that the transaction arriving belong to class 1 is A, the other is that the transaction arriving belongs to class 2 is B.

*4.2.1 The First Scenario*

The average waiting time of transaction A can be analyzed from 3 cases (Aj), the waiting number of class i transactions is $L_{qiaj}$, the average time is $W_{qiaj}$, $i = 1, 2$ $j = 1, 2, 3$.

(A1) The transaction of class 2 waiting for being mined interval α. The waiting time of transaction of class 1 is $W_{q1a1}$ including two parts:

The average waiting time of the front transactions being mined is $T_1$.

$$T_1 = \rho_1 W_{q1a1} + (1 - \rho_1) W_{q1a1} = \rho_1 W_{q1a1}$$

The average free time of mining is $T_2$, the residual time of mining is $S_e$. The mean value is $\overline{S_e}$.

$$\overline{S_e} = ES_e = \frac{ES^2}{2ES} = \frac{1}{\mu}$$

$$T_2 = \rho \overline{S_e} = \frac{\rho}{\mu} \tag{3}$$

$$W_{q1a1} = T_1 + T_2 = \rho_1 W_{q1a1} + \frac{\rho}{\mu}$$

(A2) When transaction A is waiting for being mined, the transactions of class 2 are not more than α. The average waiting time of transaction A is $W_{q1a2}$, including three parts:

The average time of transaction awaiting front transactions is $T_1$.

$$T_1 = \rho_1 W_{q1a2}$$

The average free time of mining is $T_2$.

$$T_2 = \rho \overline{S_e} = \frac{\rho}{\mu}$$

The average delay time of transaction A is waiting for being inserted of class 2 transactions are $T_3$. The waiting time of transaction is $W_{q1a2}$, so the average number of class 2 transactions arrival are $W_{q1a2} \lambda_2$, the average time of mining $T_3$.

$$T_3 = W_{q1a2} \lambda_2 ES_2 = \rho W_{q1a2}$$

$$W_{q1a2} = \rho_1 W_{q1a2} + \frac{\rho}{\mu} + \rho_2 W_{q1a2} \tag{4}$$

(A3) There are not enough transactions of class 2 to wait for being mined. The number of class 2 transactions in front of transaction A is $\frac{W_{q1a3}}{\alpha} \lambda_1 - W_{q2a3} \lambda_2$ so the sum of delay time is $T_3$.

$$T_3 = \frac{W_{q1a3}}{\alpha} \rho_1 - W_{q2a3} \rho_2$$

$$W_{q1a3} = T_1 + T_2 + T_3$$

$$= \rho_1 W_{q1a3} + \frac{\rho}{\mu} + \frac{W_{q1a3}}{\alpha} \rho_1 - W_{q2a3} \rho_2 \tag{5}$$

*4.2.2 The Second Scenario*

The arrival transaction is class 2 B. The average waiting time of transaction B can be discussed in 3 cases (Bj). The average waiting number of class i is $L_{qibj}$, the average waiting time is $W_{qibj}$, $i = 1, 2, j = 1, 2, 3$. (B1) There are not enough transactions of class 2 to wait according to interval α, $W_{q2b1}$ includes two parts:

First, the sum servicing time of class 2 transactions and class 1 transactions is $T_1$ . The waiting time of class 2 transactions is $\rho_2 W_{q2b1}$, the waiting time of class 2 transactions is $\alpha\rho_2 W_{q2b1}$. Generally, the transactions of class A before B will not exactly time the number of the B, if the first transaction at that time is not the transaction B. At this point, the number of class 1 transactions will be more than times the number of class 2 transactions, which is between 0 and $\alpha$. The average extra number of class 2 transactions is b. The servicing time of b is $\frac{b}{\mu}$.

$$T_1 = \alpha\rho_2 W_{q1b2} + \rho_2 W_{q1b1} + \frac{\rho}{\mu}$$

There are 3 ways to compute b,

Method of mean value $b$

$$b = \frac{\alpha}{2}$$

Method of poisson probability $b$

$$b = \sum_{i=1}^{\alpha} (i-1).p(i,\lambda_1) + \alpha.p(i,\lambda_2)$$

Method of system probability $b$

$$b = \sum_{i=1}^{\alpha} (i-1).\rho_1^i + \alpha.\rho_2$$

The 3 methods are used to compute b through python; simulation results show that the optimal method is the system probability method.

Second, the average free time of waiting for mining is $T_2$.

$$T_2 = \rho\overline{S_e} = \frac{\rho}{\mu}$$

$$W_{q2b1} = \alpha\rho_2 W_{q2b1} + \rho_2 W_{q2b1} + \frac{b+\rho}{\mu} \tag{6}$$

(B2) There are enough transactions of class 2 to queue at interval. When transactions B waiting for being mined, after the arrival of class 1 transactions, there are enough class 2 transactions to queue at the interval. The average waiting time of class 2 transactions includes 3 parts.

First, the average sum of waiting time of class 1 transactions and class 2 transactions is $T_1$.

$$T_1 = \overline{S_1} L_{q1b2} + \overline{S_2} L_{q2b2} = \rho_1 W_{q1b2} + \rho_2 W_{q2b1}$$

Second, the average free time of mining is $T_2$.

$$T_2 = \rho\overline{S_e} = \frac{\rho}{\mu}$$

Third, the average delay time of transaction B waiting for being inserted of class 1 transaction is $T_3$. At the same time, the average number of class 1 transactions arrival is $W_{q2a2}\ \lambda_1$, the average time of mining is $T_3$.

$$T_3 = W_{q2a2}\lambda_1 ES_1 = \rho_1 W_{q2a2}$$

$$W_{q2a2} = \rho_1 W_{q1a2} + \rho_2 W_{q2a2} + \frac{\rho}{\mu} + \rho_1 W_{q2a2} \tag{7}$$

(B3) There are enough transactions of class 2 to queue at interval. When transactions B waiting for being mined, after the arrival of class 1 transactions, there are not enough class 2 transactions to queue at interval. The average delay time of transaction B waiting for being inserted of class 1 transactions is $T_3$. The transactions includes 2 parts, one is that the number of the arrival of class 1 transactions before transactions B is $\alpha W_{q2b3}\ \lambda_2 - W_{q1b3}\ \lambda_1$, the other is that the average extra number of class 2 transactions is b.

$$T_3 = \alpha\rho_2 W_{q2b3} - \rho_1 W_{q1b3} + b$$

$$W_{q2b3} = T_1 + T_2 + T_3 = \alpha\rho_2 W_{q2b3} + \rho_2 W_{q2b3} + \frac{b+\rho}{\mu} \tag{8}$$

$$W_{q2b3} = W_{q2b1} \tag{9}$$

$$W_{q1a1} = W_{q1b2}, W_{q2a3} = W_{q2b1} \tag{10}$$

From Eqs. (3)~(10), we can conclude:

$$\begin{cases} W_{q1a1} = \dfrac{\rho}{\mu(1-\rho_1)} \\ W_{q1a2} = \dfrac{\rho}{\mu(1-\rho_1-\rho_2)} \\ W_{q1a3} = \dfrac{\rho}{\mu(1-\rho_1-\frac{\mu_1}{\alpha})} - \dfrac{\rho_2(b+\rho)}{\mu(1-\rho_2-\alpha\rho_2)(1-\rho_1-\frac{\mu_1}{\alpha})} \end{cases} \tag{11}$$

$$\begin{cases} W_{q2b1} = \dfrac{b+\rho}{\mu(1-\rho_2-\alpha\rho_2)} \\ W_{q2a2} = \dfrac{\rho_1\rho}{\mu(1-\rho_1)(1-\rho_1-\rho_2)} + \dfrac{\rho}{\mu(1-\rho_1-\rho_2)} \\ W_{q2a3} = \dfrac{b+\rho}{\mu(1-\rho_1-\alpha\rho_2)} \end{cases} \tag{12}$$

The probability of various scenarios is calculated as follows. The number of waiting transactions of class i is $l_{si}$, $i$ = 1, 2, including the transactions of being mined or waiting for being mined.

$$P(l_{s_i} = k) = \rho_i^k(1-\rho_i)$$

The probability of case (b1) is $p_{b1}$,

$$\begin{aligned} P_{b1} &= P(l_{s1} \geq \alpha l_{s2}) \\ &= \sum_{k=0}^{\infty} P(l_{s1} \geq \alpha l_{s2} | l_{s2} = k)P(l_{s2} = k) \\ &= (1-\rho_1)(1-\rho_2)\sum_{k=0}^{\infty}\sum_{m\geq\alpha k}^{\infty} \rho_1^m\rho_2^k \\ &= \frac{1-\rho_2}{1-\rho_2\rho_1^{\alpha}} \end{aligned} \tag{13}$$

The probability of case (a1) is $p_{a1}$ ($l_{s1} < \alpha l_{s2}$).

$$P_{a_1}(l_{s_1} < \alpha l_{s_2}) = 1 - \frac{1-\rho_2}{1-\rho_2\rho_1^{\alpha}} \tag{14}$$

The probability of case (A1) is $p_{a2}$. When transaction awaiting for being mined, the average number of class 2 transactions arrival is

$$W_{q1a2}\lambda_2 = \frac{\rho_2}{\rho_1}L_{q1a2}$$

The number of transactions is $\frac{\rho_2}{\rho_1}l_{s1}$,

$$l_{s1} \geq \alpha(l_{s2}+\frac{\rho_2}{\rho_1}l_{s1}), \quad l_{s1} \geq \frac{\alpha\rho_1}{(\rho_1-\alpha\rho_2)}l_{s2}$$

So, the probability of case (A2)

$$P_{a2} = P(l_{s1} \geq \frac{\alpha\rho_1}{(\rho_1-\alpha\rho_2)}l_{s2})$$

$$= \sum_{k=0}^{\infty} P(l_{s1} \geq \frac{\alpha\rho_1}{(\rho_1-\alpha\rho_2)}l_{s2}|l_{s2}=k)P(l_{s2}=k)$$

$$= (1-\rho_1)(1-\rho_2)\sum_{k=0}^{\infty}\sum_{m\geq\frac{\alpha\rho_1}{\alpha\rho_1-\alpha\rho_2}}^{\infty}\rho_1^m\rho_2^k$$

$$= \frac{1-\rho_2}{1-\rho_1^{\frac{\alpha\rho_1}{\rho_1-\alpha\rho_2}}\rho_2} \tag{15}$$

The probability of case (A3) is $p_{a3.}$

$$P_{a3} = 1 - P_{a1} - P_{a2} = \frac{1-\rho_2}{1-\rho_2\rho_1^{\alpha}} - \frac{1-\rho_2}{1-\rho_1^{\frac{\alpha\rho_1}{(\rho_1-\alpha\rho_2)}}\rho_2} \tag{16}$$

The probability of case (B2), when transaction B waiting for being mined, the average number of class 1 transactions is

$$W_{q2a2}\lambda_1 = \frac{\rho_2}{\rho_1}L_{q2a2}$$

The number of transactions is $\frac{\rho_2}{\rho_1}l_{s2}$.

$$l_{s1} + \frac{\rho_2}{\rho_2}l_{s2} < \alpha l_{s2},$$

$$l_{s1} < (\alpha - \frac{\rho_1}{\rho_2})l_{s2}$$

$$P_{b2} = P(l_{s1} < (\alpha - \frac{\rho_2}{\rho_1})l_{s2})$$
$$= 1 - P(l_{s1} \geq (\alpha - \frac{\rho_2}{\rho_1})l_{s2})$$
$$= 1 - (1 - \rho_1)(1 - \rho_2)\sum_{k=0}^{\infty}\sum_{m \geq \frac{\alpha\rho_1}{(\rho_1 - \alpha\rho_2)}}^{\infty} \rho_1^m \rho_2^k$$
$$= 1 - \frac{1 - \rho_2}{1 - \rho_1^{\alpha - \frac{\rho_2}{\rho_1}} \rho_2} \tag{17}$$

The probability of case (B3) is $p_{b3}$.

$$P_{b3} = 1 - P_{b1} - P_{b2} = \frac{1 - \rho_2}{1 - \rho_1^{\alpha \frac{\rho_2}{\rho_1}} \rho_2} - \frac{1 - \rho_2}{1 - \rho_2\rho_1^{\alpha}} \tag{18}$$

From Eqs. (11)~(18), the waiting time of transactions

$$\begin{cases} W_{q1} = W_{q1a1}P_{a1} + W_{q1a2}P_{a2} + W_{q1a3}P_{a3} \\ W_{q2} = W_{q2a1}P_{a1} + W_{q2a2}P_{a2} + W_{q2a3}P_{a3} \end{cases} \tag{19}$$

### 4.3 Extension of Computer Results

#### 4.3.1 The Stay Time of Transactions

The stay time of transactions include the waiting time and mining time, which is marked as $W_{si}$, $i = 1, 2$.

$$\begin{cases} W_{a1} = W_{q1} + \frac{1}{\mu} \\ W_{a2} = W_{q2} + \frac{1}{\mu} \end{cases} \tag{20}$$

#### 4.3.2 The Length of Different Class Queue

The length of the queue of class 1 and class 2 transactions waiting for being mined is shown as follow:

$$\begin{cases} L_{q1} = \lambda_1 W_{q1} \\ L_{q2} = \lambda_2 W_{q2} \end{cases} \tag{21}$$

## 5 Simulation Results

To verify the theory of blockchain research, python was used to simulate the experiment. $\lambda_1 = 5.7$, $\lambda_2 = 0.3$, $\alpha 3$, $\mu$ takes different values. The result is shown as Tab. 1.

The figure of the relationship between the mining service rate $\mu$ and waiting time of class 1 and class 2 transactions $W_{q1}$, $W_{q2}$ is drew as follow Fig. 2.

**Table 1:** $\lambda_1$ = 5.7, $\lambda_2$ = 0.3, α3, μ takes different values

| Results | $\lambda_1$ | $\lambda_2$ | α | μ | $W_{q1}$ | $W_{q2}$ |
|---|---|---|---|---|---|---|
| Mean value method | 5.7 | 0.3 | 3 | 5 | 0.1500 | 0.2382 |
| Poisson probability method | 5.7 | 0.3 | 3 | 5 | 0.1500 | 0.1989 |
| System probability method | 5.7 | 0.3 | 3 | 5 | 0.1500 | 0.1602 |
| Simulation method | 5.7 | 0.3 | 3 | 5 | 0.1489 | 0.1406 |
| Mean value method | 5.7 | 0.3 | 3 | 6 | 0.0833 | 0.1852 |
| Poisson probability method | 5.7 | 0.3 | 3 | 6 | 0.0833 | 0.1453 |
| System probability method | 5.7 | 0.3 | 3 | 6 | 0.0833 | 0.0940 |
| Simulation method | 5.7 | 0.3 | 3 | 6 | 0.0833 | 0.0947 |
| Mean value method | 5.7 | 0.3 | 3 | 7 | 0.0536 | 0.1507 |
| Poisson probability method | 5.7 | 0.3 | 3 | 7 | 0.0536 | 0.1169 |
| System probability method | 5.7 | 0.3 | 3 | 7 | 0.0536 | 0.0620 |
| Simulation method | 5.7 | 0.3 | 3 | 7 | 0.0536 | 0.0657 |
| Mean value method | 5.7 | 0.3 | 3 | 8 | 0.0400 | 0.1267 |
| Poisson probability method | 5.7 | 0.3 | 3 | 8 | 0.0400 | 0.0975 |
| System probability method | 5.7 | 0.3 | 3 | 8 | 0.0400 | 0.0438 |
| Simulation method | 5.7 | 0.3 | 3 | 8 | 0.0400 | 0.0441 |
| Mean value method | 5.7 | 0.3 | 3 | 20 | 0.0044 | 0.0426 |
| Poisson probability method | 5.7 | 0.3 | 3 | 20 | 0.0044 | 0.0314 |
| System probability method | 5.7 | 0.3 | 3 | 20 | 0.0044 | 0.0051 |
| Simulation method | 5.7 | 0.3 | 3 | 20 | 0.0044 | 0.0954 |
| Mean value method | 5.7 | 0.3 | 3 | 40 | 0.0010 | 0.0200 |
| Poisson probability method | 5.7 | 0.3 | 3 | 40 | 0.0010 | 0.0145 |
| System probability method | 5.7 | 0.3 | 3 | 40 | 0.0010 | 0.0115 |
| System probability method | 5.7 | 0.3 | 3 | 40 | 0.0010 | 0.0115 |

From Fig. 2, we can find the waiting time of transactions becomes shorter as the mining service rate μ increases. Further when the waiting time gets shorter, the service rate increase of class 1 transactions is more significant than class 2 transactions. For the problem of transactions waiting too long time, we find that transactions with pay or with prior interest and increase service rate of mining are effective for reducing waiting time. Besides, the two factors mutually reinforce to shorten waiting time. Simulation results are consistent with the results of theoretical analysis.

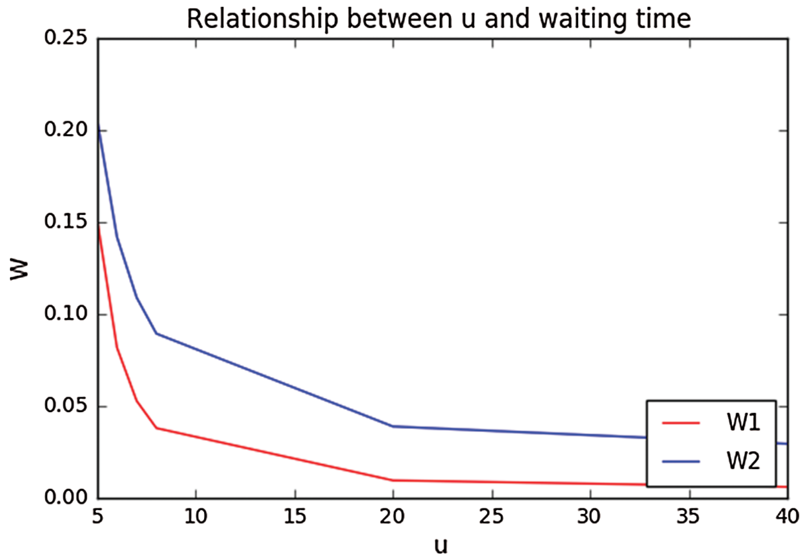


**Figure 2:** A figure of the relationship between the mining service rate and waiting time of class 1 and class 2 transactions

## 6 Conclusion

In order to solute the problem of blockchain transactions waiting too long time for being mined, the blockchain queuing model with non-preemptive limited-priority is established in this paper. From the analysis of the model, we compute average waiting time, average staying time and average length of queue. From simulation of the model, we find that transactions with pay or with prior interest and increase service rate of mining are effective for reducing waiting time. Besides, the two factors mutually reinforce to shorten waiting time.

But the work of this paper is preliminary, and the model which only considers the 2 class of transactions is relatively simple that cannot capture many reality condition of the blockchain transactions. So, it is important that we should found the model more consistent with the actual blockchain system. As future work, we will focus on more general situation, making the model suitable for practical blockchain application.

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