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Interpretation of the Entangled States

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Abstract: An interpretation of the entangled states is considered. Two-photon states of photon *A* on path *a* and photon *B* on path *b* with polarizations *H*, *V* are constructed. Two synchronized photons, 1 and 2, can take the paths *a* and *b*, with equal probability 50%. In the bases *a*, *b* and *H*, *V*, the states of the photons form the product states. In the basis 1, 2, the states of the photons form the entangled state. The states of the photons in the bases 1, 2; *a*, *b*; *H*, *V* are inseparable. The correlation of the photons due to the entanglement in the basis 1, 2 can be seen in the bases *a*, *b* and *H*, *V*. The interference term in the approach under consideration and in the standard approach is the same thus providing for the same correlation.

Keywords: Entanglement; polarization state; inseparability; correlation

1 Introduction

As known entangled state is defined as the global state which cannot be written as a product of the individual states. The phenomenon of entanglement had been studied since the seminal paper by Einstein, Podolsky, and Rosen (EPR) [1]. They formulated the problem for the wave functions of position and momentum. Bell [2] considered entanglement of spin states for massive particles (polarization states for photons). The individual states constituting the entangled state are correlated. The correlations of position and momentum of the EPR states had been shown in the experiments on ghost imaging [3]. The correlations of spin (polarization) of the Bell states had been shown in several experiments, e.g., [4] and references therein.

According to the standard interpretation [5], the measurement makes collapse of the entangled state into the individual states. The correlation is recorded through the individual states which, after the collapse of the entangled state, form the product state. We come to the contradiction. The correlation being a feature of the entangled state is recorded through the individual states forming the product state. The way to overcome the contradiction was proposed in [6]. The entangled state is composed of the individual states in one basis, and the correlation is measured through the individual states forming the product state in the other basis. The approach was formulated for the entanglement in position and momentum. In the present paper, we shall consider the problem for the entanglement in polarization.

2 Entanglement in the Basis of Polarization States

Let the source emit two synchronized photons in the states $|\psi_1\rangle$ and $|\psi_2\rangle$ in the opposite directions toward paths *a* and *b*. The photons can take the paths *a* and *b*, with equal probability 50%. The states of the photons can be written as

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\psi_{1a}\rangle + |\psi_{1b}\rangle) \tag{1}$$

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}}(\left|\psi_{2a}\right\rangle + \left|\psi_{2b}\right\rangle) \tag{2}$$

Photon A on path a and photon B on path b form the two-photon state

$$\left|\psi_{AB}\right\rangle = \frac{1}{\sqrt{2}}(\left|\psi_{1a}\right\rangle\left|\psi_{2a}\right\rangle + \left|\psi_{1b}\right\rangle\left|\psi_{2b}\right\rangle) \tag{3}$$

In the basis a, b, the states of the photons are defined either in the space of photon A or in the space of photon B thus forming the product state. In the basis 1, 2, the states of the photons are defined both in the space of photon A and in the space of photon B thus forming the entangled state.

Let detectors D_1 and D_2 register photons A and B respectively. Detector D_1 registers photon A in the state $\left|\psi_A\right\rangle = \left|\psi_{1a}\right\rangle\left|\psi_{2a}\right\rangle$, and detector D_2 registers photon B in the state $\left|\psi_B\right\rangle = \left|\psi_{1b}\right\rangle\left|\psi_{2b}\right\rangle$. The interference term of photons A and B is given by

$$\left|\psi_{1a}\right\rangle\left|\psi_{2a}\right\rangle\left\langle\psi_{1b}\right|\left\langle\psi_{2b}\right|\left|\psi_{1b}\right\rangle\left|\psi_{2b}\right\rangle\left\langle\psi_{1a}\right|\left\langle\psi_{2a}\right|. \tag{4}$$

The interference term provides for the correlation of the outputs of the detectors D_1 and D_2. Detection of photons A and B keeps the entanglement in the basis 1, 2 unchanged. The states of the photons in the bases a, b and 1, 2 are inseparable. Therefore, the correlation of the photons due to the entanglement in the basis 1, 2 can be seen in the basis a, b.

Usually, the entangled state of photons 1 and 2 is written in the form

$$\left|\psi_{12}\right\rangle = \frac{1}{\sqrt{2}}(\left|\psi_{1a}\right\rangle\left|\psi_{2b}\right\rangle + \left|\psi_{1b}\right\rangle\left|\psi_{2a}\right\rangle) \tag{5}$$

Here, the superposition is due to the indistinguishability of the states $\left|\psi_{1a}\right\rangle\left|\psi_{2b}\right\rangle$ and $\left|\psi_{1b}\right\rangle\left|\psi_{2a}\right\rangle$. The superposition $\left|\psi_{12}\right\rangle$ Eq. (5) and the superposition $\left|\psi_{AB}\right\rangle$ Eq. (3) give the same interference term Eq. (4). Detection of photons 1 and 2 is supposed to project the state $\left|\psi_{12}\right\rangle$ onto the product states $\left|\psi_{1a}\right\rangle\left|\psi_{2b}\right\rangle$ or $\left|\psi_{1b}\right\rangle\left|\psi_{2a}\right\rangle$. This is in contrast to the two-photon interference of the state $\left|\psi_{12}\right\rangle$. Detection of photons A and B keeps the state $\left|\psi_{AB}\right\rangle$ thus allowing for the two-photon interference of the state $\left|\psi_{AB}\right\rangle$.

Consider two-photon states in the basis of horizontal H and vertical V polarizations. The Bell states are canonical entangled two-photon states in the basis H, V,

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}(\left|\psi_{1H}\right\rangle\left|\psi_{2V}\right\rangle \pm \left|\psi_{1V}\right\rangle\left|\psi_{2H}\right\rangle). \tag{6}$$

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}(\left|\psi_{1H}\right\rangle\left|\psi_{2H}\right\rangle \pm \left|\psi_{1V}\right\rangle\left|\psi_{2V}\right\rangle). \tag{7}$$

Detection of photons 1 and 2 projects the Bell states Eq. (6) onto the states $\left|\psi_{1H}\right\rangle\left|\psi_{2V}\right\rangle$ or $\left|\psi_{1V}\right\rangle\left|\psi_{2H}\right\rangle$, and the Bell states Eq. (7) onto the states $\left|\psi_{1H}\right\rangle\left|\psi_{2H}\right\rangle$ or $\left|\psi_{1V}\right\rangle\left|\psi_{2V}\right\rangle$. After the detection, photons 1 and 2 form the product state. This is in contrast to the correlation of photons 1 and 2 due to the entanglement of photons 1 and 2.

Consider two-photon states in the basis H, V within the framework of the proposed scheme. Let photons 1 and 2 be unpolarized. The states of photons 1 and 2 can be projected onto polarizations H and V, with equal probability 50%. In view of Eq. (3), the two-photon state of photons A and B in the basis H, V can be written in the form

$$\left|\psi_{AB}\right\rangle=\frac{1}{\sqrt{2}}(\left|\psi_{1a}\right\rangle\left|\psi_{2a}\right\rangle(\left|\psi_{H}\right\rangle \pm\left|\psi_{V}\right\rangle)+\left|\psi_{1b}\right\rangle\left|\psi_{2b}\right\rangle(\left|\psi_{H}\right\rangle+\left|\psi_{V}\right\rangle)). \tag{8}$$

From this one can construct four two-photon states

$$\left|\psi_{AB}\right\rangle=\frac{1}{\sqrt{2}}((\left|\psi_{1aH}\right\rangle\left|\psi_{2aH}\right\rangle+\left|\psi_{1bV}\right\rangle\left|\psi_{2bV}\right\rangle)+(\pm\left|\psi_{1aV}\right\rangle\left|\psi_{2aV}\right\rangle+\left|\psi_{1bH}\right\rangle\left|\psi_{2bH}\right\rangle)). \tag{9}$$

$$\left|\psi_{AB}\right\rangle=\frac{1}{\sqrt{2}}((\left|\psi_{1aH}\right\rangle\left|\psi_{2aH}\right\rangle+\left|\psi_{1bH}\right\rangle\left|\psi_{2bH}\right\rangle)+(\pm\left|\psi_{1aV}\right\rangle\left|\psi_{2aV}\right\rangle+\left|\psi_{1bV}\right\rangle\left|\psi_{2bV}\right\rangle)). \tag{10}$$

The interference term of the photons in the states Eqs. (9,10) is given by respectively

$$\left|\psi_{1aH}\right\rangle\left|\psi_{2aH}\right\rangle\left\langle\psi_{1bV}\right|\left\langle\psi_{2bV}\right|\left|\psi_{1bV}\right\rangle\left|\psi_{2bV}\right\rangle\left\langle\psi_{1aH}\right|\left\langle\psi_{2aH}\right| \pm\left|\psi_{1aV}\right\rangle\left|\psi_{2aV}\right\rangle\left\langle\psi_{1bH}\right|\left\langle\psi_{2bH}\right|\left|\psi_{1bH}\right\rangle\left|\psi_{2bH}\right\rangle\left\langle\psi_{1aV}\right|\left\langle\psi_{2aV}\right|. \tag{11}$$

$$\left|\psi_{1aH}\right\rangle\left|\psi_{2aH}\right\rangle\left\langle\psi_{1bH}\right|\left\langle\psi_{2bH}\right|\left|\psi_{1bH}\right\rangle\left|\psi_{2bH}\right\rangle\left\langle\psi_{1aH}\right|\left\langle\psi_{2aH}\right| \pm\left|\psi_{1aV}\right\rangle\left|\psi_{2aV}\right\rangle\left\langle\psi_{1bV}\right|\left\langle\psi_{2bV}\right|\left|\psi_{1bV}\right\rangle\left|\psi_{2bV}\right\rangle\left\langle\psi_{1aV}\right|\left\langle\psi_{2aV}\right|. \tag{12}$$

The two-photon states of photons *A* and *B* in the basis *H*, *V* Eqs. (9,10) and the Bell states Eqs. (6,7) give the same interference terms Eqs. (11) and (12) respectively.

In the basis *H*, *V*, the states of the photons in Eqs. (9) and (10) are defined either in the space of photon *A* or in the space of photon *B* thus forming the product states. In the basis 1, 2, the states of the photons in Eqs. (9) and (10) are defined both in the space of photon *A* and in the space of photon *B* thus forming the entangled states. The states of the photons in the bases *H*, *V* and 1, 2 are inseparable. Therefore, the correlation of the photons due to the entanglement in the basis 1, 2 can be seen in the basis *H*, *V*.

3 Conclusion

We have considered an interpretation of the entangled states. The aim of the interpretation is to overcome the contradiction of the standard treatment of the correlation of the entangled states. In the standard approach, the correlation is recorded through the individual states which, after the collapse of the entangled state, form the product state.

We have considered two-photon states of photon *A* on path *a* and photon *B* on path *b*. Two synchronized photons 1 and 2, can take the paths *a* and *b*, with equal probability 50%. In the basis *a*, *b*, the states of the photons are defined either in the space of photon *A* or in the space of photon *B* thus forming the product state. In the basis 1, 2, the states of the photons are defined both in the space of photon *A* and in the space of photon *B* thus forming the entangled state. Detection of photon *A* on path *a* and photon *B* on path *b* keeps the entanglement in the basis 1, 2 unchanged. The states of the photons in the bases *a*, *b* and 1, 2 are inseparable. Therefore, the correlation of the photons due to the entanglement in the basis 1, 2 can be seen in the basis *a*, *b*. The interference term in the approach under consideration and in the standard approach is the same thus providing for the same correlation.

We have considered two-photon states of photons *A* and *B* in the basis of horizontal *H* and vertical *V* polarizations. In the basis *H*, *V*, the states of the photons are defined either in the space of photon *A* or in the space of photon *B* thus forming the product states. In the basis 1, 2, the states of the photons are defined both in the space of photon *A* and in the space of photon *B* thus forming the entangled states. The states of the photons in the bases *H*, *V* and 1, 2 are inseparable. Therefore, the correlation of the photons due to the entanglement in the basis 1, 2 can be seen in the basis *H*, *V*. The interference term in the approach under consideration and in the standard approach is the same thus providing for the same correlation.

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