Basic concepts and numerical integration issues in the 2D boundary element implementation of strain gradient elasticity problems

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Abstract: The mathematical modeling of microdevices, in which structure and microstructure have approximately the same scale of magnitude, as well as of macrostructures of markedly granular or crystal nature (microcomposites), demands a nonlocal approach for strains and stresses. The present proposition is based on a simplified strain gradient theory laid down by Aifantis, which has also been applied mainly by Beskos and collaborators in the context of the boundary element method. This paper is an extension of a presentation made during the ICCES 2014 Conference in Crete, Greece, now relying on machine-precision evaluation of all singular and hypersingular integrals required in a 2D implementation. The singular fundamental solution needed in a boundary element formulation is rederived from the developments made by Polyzos and co-workers and conceptually assessed. The basic formulation is presented for 2D and 3D problems, but the numerical issues and the displayed examples only concern the 2D case. An important contribution is the evidence that normal displacement gradients along the boundary must be approximated independently from displacements, which seems to better agree with Mindlin's pioneer proposition on the subject of strain gradient elasticity and results that no additional treatment of "corner nodes" is actually required. The terms related to hypersingular integrals are newly assessed in order to cope with the strict requirements of a variationally-based formulation particularly when applied to curved boundaries. The implementation of an accurate numerical quadrature scheme for improper integrals (due to the occurrence of Bessel functions) over a curved boundary is also addressed. The implementation is done for in principle arbitrarily high order 2D elements, with results assessed both conceptually (fulfilment of spectral properties) and in terms of numerical convergence for some academic applications.

Keywords: Boundary elements, strain gradient elasticity, variational methods, hypersingular formulation, numerical integration