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Investigation of Stress Auto Separating Method in Digital Photo-elasticity and Its Application in Contact Problem

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Abstract: The stress distribution of contact area in engineering structure is complex. Automatic whole-field measurement of photo-elasticity is a feasibility method to solve the contact problem. However, the problem of isoclinic and isochromatic interaction is important in phase-shifting techniques of photo-elasticity. The technique named 8-step phase shifting technique is applied to solve the problem of isoclinic fringe pattern influenced by isochromatic fringe patterns in integer order and half-integer order. A new automatic technique of stress separation improved by an application example of contact problem of root gear, which is discussed in this paper after obtaining the whole-field shearing stress.

Keywords: eight-step phase shifting technique, isoclinic, isochromatic, stress auto separating, contact stress.

1 Introduction

In practical engineering, the contact stress distribution of the multi-body structure is very complicated. Finite element method can not accurately calculate the actual stress distribution of contact area because the boundary conditions are difficult to determine. Therefore, an automatic method to obtain the whole field stress distribution of the contact area accurately is urgently needed. Photo-elascity is a useful experimental method to solve whole field two dimensional and three dimensional stress analysis in engineering. However, the phase-shifting techniques in photoelasticity have a peculiar problem of isoclinic and isochromatic interaction. For solving this problem, numbers of approaches have been proposed to automatically amend it [Patterson and Wang (1991); Sarma et al (1992); Asundi (1993); Ramesh and Ganapathy (1996); Pinit and Umezaki (2007); Ramji et al (2006)]. Six-step phase shifting method suggested by Patterson and Wang (1991), is an effective way to separate the information of isoclinic and isochromatic, but there is a problem of

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isoclinic fringe pattern influenced by isochromatic fringe patterns of integer order and half-integer order. To separate isoclinic angle form the ambiguous fringe pattern, a new technique named eight-step phase shifting technique is suggested by Zhang et al (2009). The Isoclinic angle in the range of [0, π] is determined unambiguously. The true isoclinic angle and phase difference principal value are the key data source of stress separation. The feasibility of this method was proved by the experiments of disk and square under diametral compression. The shearing stress would be obtained as the difference of principle stress and the direction of first principle stress are determined. The shearing stress difference method is selected to separate the stress components of contact area. Automatic technique of stress separation is discussed in this paper after obtaining the whole-field value of the shearing stress. Contact stress of turbine between roots and rims are analyzed by an automatic package of the digital photo-elasticity.

2 Principle of eight-step phase shifting method

As shown in Figure.1, the stress model is arranged in the plane polariscope system. Both 1/4 slides and the analyzer can be rotated.

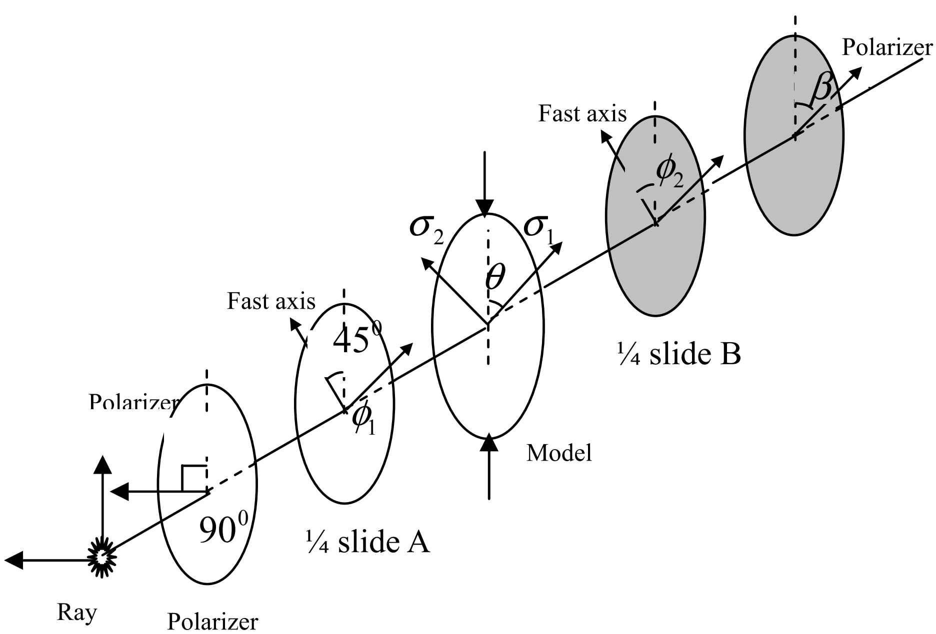


Figure 1: polarizer system of eight-step phase shifting method

Monochromatic ray is use to be a light source. And the emitted light irradiance of the optical system is given by Eq.1.

$$I = E^*E = I_b + 0.5I_a(1 - \cos\delta \cos 2(\beta - \phi_2 + \phi_1) \\ + \sin(\delta + 2(\beta + \phi_2)) \sin 2(\theta + \beta) \sin 2(\beta + \phi_2 - \phi_1)) \quad (1)$$

Where ϕ_1 is the angle of the fast axis of 1/4 slide A and the reference coordinate system x-axis, ϕ_2 is the angle of the fast axis of 1/4 slide B and the reference coordinate system x-axis , β is the angle of the analyzer and the reference coordinate system x-axis, δ is the phase difference in any point of the stress model when ray passed through , θ is the angle of the direction of first principle stress and x-axis. Ib is the background irradiance, Ia is the amplitude of light passed through the Polarizer. $\pi/4$ is the step length of the phase shifting angle of the both 1/4 slides and the analyzer.

According to Eq.2, when δ= 0, π.- π, $\pi/4$, the information of isoclinic angleθ is still included in Eq.2. By adding two phase shifting images, the information of whole field isoclinic can be extracted.

$$\begin{cases} \theta = 0.5\tan^{-1}(\frac{I_3 - I_5}{I_4 - I_6}) & I_4 \neq I_6 \\ \theta = 0.5\tan^{-1}(\frac{I_3 - I_7}{I_8 - I_6}) & I_4 = I_6 and I_6 \neq I_8 \\ \theta = \pi/4 or 3\pi/4 & ; I_4 = I_6 and I_6 = I_8 \end{cases} \quad (2)$$

3 Simulations and automatic stress separation steps

Combining finite element software ANSYS with fitting of function , eight-step phase shifting isochromatic phase patterns of a square plate under diametral compression can be obtained as Figure.2(a)-(h). The first six images of eight-step phase shifting method are totally the same with the six-step method suggested by Patterson and Wang while the last two images are the new adding images.

The whole field isoclinic angle of square plate in the range of (0, $\pi/2$), calculated by eight-step phase method, is shown in Figure.2 (i). The result of eight-step method is almost the same as the analytical solution. The absolute error between eight-step method and analytical solution is shown in Figure.2 (j). It can be seen that the calculation impact of isoclinic angle on isocharomatics is well removed by eight-step method.

The shearing difference method is use to calculate the whole field three stress components automatically. Before the digital photoelastic method, the shearing difference method was relied to manual calculation. There were too many difficult

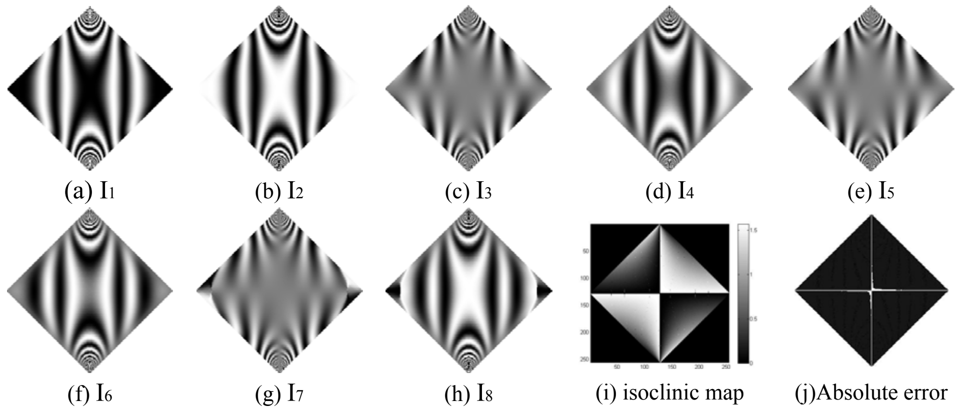


Figure 2: Simulated images of a square plate on eight phase shifting method and isoclinic maps.

problem to apply in engineering, such as great amount of work, more human disturbance factors, more errors, hardly to obtain whole field information, etc. Digital photoelastic makes stress separation automatically. Parts process of this technique is shown in Figure.3. An automatic package, compiled by Matlab and Visual C++ is used to do image processing , fringe pattern calculation and stress separation .

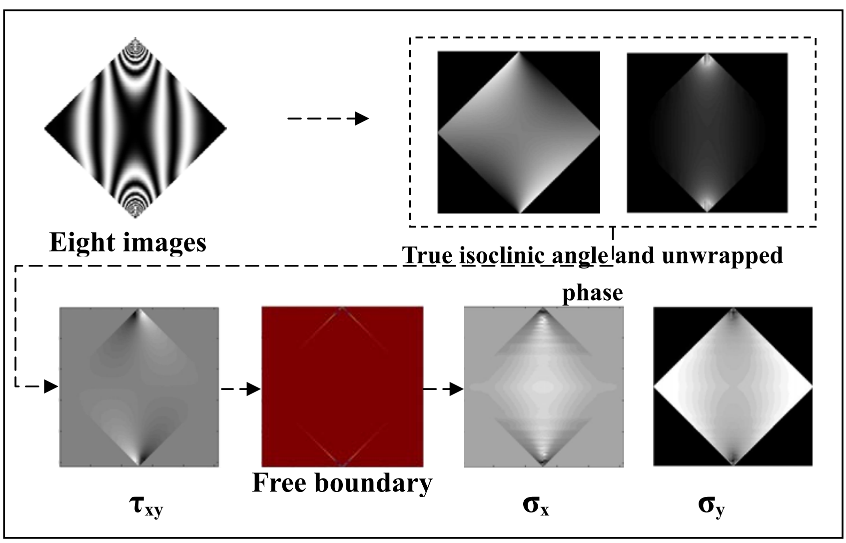
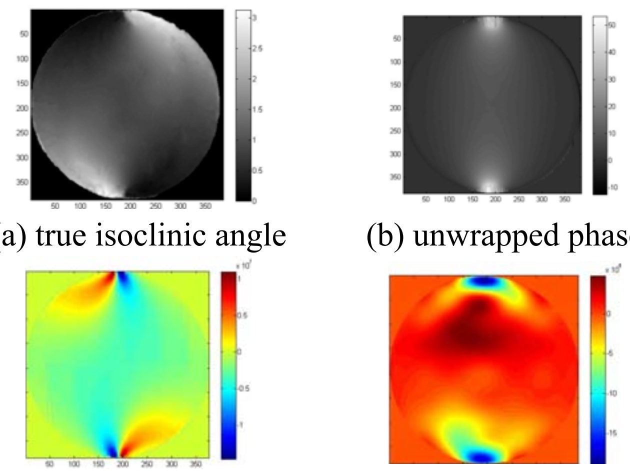


Figure 3: Process of automatic stress separation

4 Experimental setup

The frozen disk is arranged in the plane polarization system. Eight-step images are taken by CCD. The whole field true isoclinic angle calculated by eight-step method is shown in Figure.4 (a), and the unwrapped phase is shown in Figure.4 (b). Fig4. (c) –(d) are components of the disk obtained by automatic package. Obviously, the experimental results are very close to the theoretical results.



(a) true isoclinic angle (b) unwrapped phase

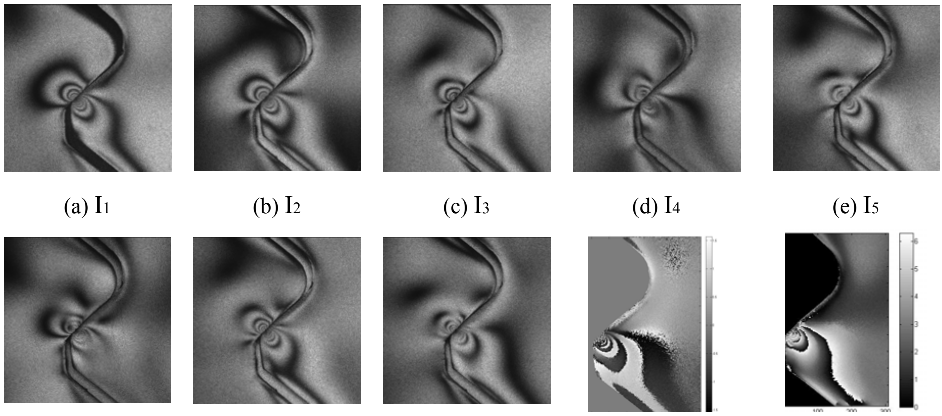
(c) τ_{xy} (d) σ_x

Figure 4: Maps of isoclinic angle based on eight step phase shifting method and components of frozen disk

5 Stress calculation of contact area and application example

As an application example, the whole field isoclinic angle and isocharomatics of turbine between roots and rims are analyzed by the eight-step phase shifting method. As shown in Figure.5 (a)-(h), eight images of the contact area are taken by CCD. Figure.5 (i)-(j) are the isoclinic and phase difference maps obtained by eight-step.

Three stress components and equivalent Von Mises stress of any point in the model are calculated by the shearing stress difference method. Automatic data processing and stress separation are integrated in a package. Efficacy and sensitivity of stress analysis will be greatly increased as the digital photo-elasticity method is applied in contact problem of engineering.

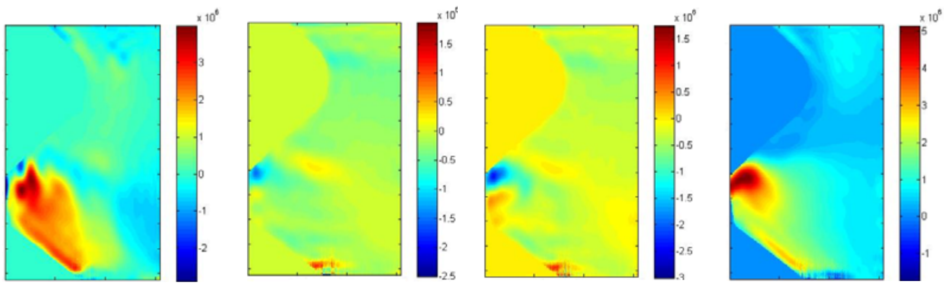


(a) I_1 (b) I_2 (c) I_3 (d) I_4 (e) I_5

(f) I_6 (g) I_7 (h) I_8 (i) isoclinic map (I) phase difference

Figure 5: Images of the contact area based on eight-step phase shifting method and isoclinic angle and isocharomatics of the root gear.

The whole field isoclinic angle and the unwrapping phase are the key data source of stress separation. Based on that, three stress components and equivalent Von Mises stress of any point in the root gear calculated automatically by the package are shown in Figure.6,.



(a) τ_{xy} (b) σ_x (c) σ_y (d) Equivalent Von Mises stress

Figure 6: Three stress components and equivalent Von Mises stress.

The most concern of contact problems is the scope of the contact area. Contact solution is a nonlinear problem. Contact stress analysis is difficult, only the Hertz contact solution has the exact analytical solution. The contact problems in engineering are always complicated. Compared with finite element method which is difficult to determine the real boundary conditions and calculate the stress distribution, the technique above-mentioned is suitable for obtaining the real whole field contact stress distribution.

6 Conclusions

This paper presents a new photoelastic technique to obtain the whole field stress distribution of contact area. This method has some advantages, such as separating the isoclinic angle from ambiguous fringe pattern, calculating the stress distribution of contact area automatically and accurately.

True whole field isoclinic angle and phase difference are the key data to obtain the stress distribution of stress model. To ensure the accuracy of the information obtained from experiments and engineering, the feasibility of eight-step phase shifting method and stress separation method are proved by the simulation of a square plate and the experiments of frozen disk under diametral compression.

A practical contact problem is used to verify the performance of the new technique. Compared to finite element simulation, the results obtained by the new automatical photoelastic technique can be more objectively and accurately reflected the actual engineering situation.

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