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An Optimal Pre-stress Die Design of Cold Backward Extrusion by RSM Method

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Abstract: In this paper, a pre-stress die design method for cold backward extrusion with a non simple hollow cylinder die by using response surface method is proposed. The radius of the interface and the absolute interference in the interface of a prestress die are chosen as the design variables. Both the two design variables are set at four levels, and then 16 combinations of design parameters are constituted totally. A finite element based code is utilized to investigate the elastic deformation characteristic under different design parameters, and the response surface method is then employed to synthesize the data sets obtained from the numerical analysis, thus establishing a maximum die effective stress prediction mode. By using the prediction model, the optimum radius and the optimum absolute interference in the interface under a certain inner pressure can be determined.

Keywords: Response surface method, pre-stress design, optimum die design.

1 Introduction

During cold forging, in order to reduce the stress concentration, die inserts are usually prestressed with stress rings. For die design with prestressed rings, the theory of thick-walled cylinder [Ugral and Fenster(1995)], Lame's equation, is conventionally used. However, Lame's equation is only applicable for die design with a simple hollow cylinder die insert, and is very difficult to predict the local stress of a die insert having a non-simple shape. Therefore, many studies have been conducted on the FE application to the analysis and design of prestressed die [Muramatsu(2008); Lee, Saroosh, Song and Ima(2009)].

In this paper, a pre-stress die design method for cold backward extrusion with a non simple hollow cylinder die by RSM is proposed. Different radius of the interface and the absolute interference in the interface are taken into account as the design parameters in this study. Both the two design variables are set at four levels, and then 16 combinations of design parameters are constituted totally. A finite element

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based code is utilized to investigate the elastic deformation characteristic under different design parameters, and the RSM is then applied to synthesize the data sets obtained from the numerical analysis, then a prediction model of the maximum effective stress of the die is established. From the prediction model, the optimum radius and the absolute interferences in the interface under a certain inner pressures will be found.

2 Method of analysis

2.1 Analysis modeling

A commercial FE code DEFORM-2D [S.F.T.C (2010)] is adopted to analyze the elastic deformation of the shrink-fitting of a backward extrusion die with a non simple hollow cylinder die insert. The schematic diagram of the backward extrusion die set is shown in Figure3. As shown in Figure 1, an inner pressure p_0 uniformly acts toward the inner surface of the die. During the analysis, the die and the prestressed ring are assumed elastic. The Young's modulus *E* of the die insert (Tungsten Carbide) and the prestressed ring (SKD61 die steel) are 4.9x10^5 MPa and 2.05x10^5 MPa, respectively. The poisson's ratio of the die insert and the prestressed ring are 0.24 and 0.30, respectively. The Yield strengths *Y* of the die and the ring are 1650 MPa and 1300 MPa, respectively. The other shrink-fitting parameters used in the FE-simulation on the backward extrusion die design are summarized in Table 1.

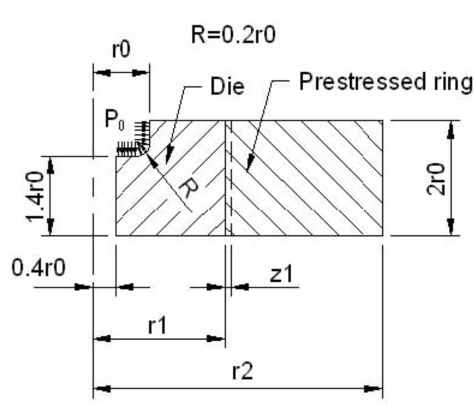

Figure 1: The schematic diagram of the backward extrusion die set with a non simple hollow cylinder die insert.

Table 1: The shrink fitting parameters used in the FE-analysis on the backward extrusion die design

Radius of interface, r_1 (mm)	16.03, 18.32, 22.90, 27.48
Absolute interference in radius, z_1 (mm)	0.0665, 0.0831, 0.0997, 0.1080
Inner pressure p_o (MPa)	840
Inner radius of the die insert, r_0 (mm)	10
Outer radius of the prestressed ring, r_2 (mm)	50

2.2 Response surface method

RSM consist of a group of techniques used in the empirical study of relationships between one or more measured responses and a number of input factors. Response surface models are often in form of low order polynomials. Among these common models, the quadratic polynomials response surface is the most popular one since it is flexible to approximate nonlinear response and the extremum can be achieved in the middle region of the design space. The mathematical model of the quadratic response surface is described as follows.

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{i>i}^{k} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ij} x_i^2 + \varepsilon \tag{1}$$

where x_1, x_2, ..., $x_k$are the input variables, which influence on the response Y; $\beta_i(i = 1,2,...,k)$, $\beta_{ij}(i = 1,2,...,k; j = 1,2,...,k)$ are the unknown parameters, ε is the error and k is the number of variables.

The model accuracy or the goodness of fit of the response surface model is often assessed by four error measures:

(1) Averages absolute error: averages differences between actual or observed and predicted or approximated values.

(2) Maximum error: maximum differences between actual and predicted values.

(3) Root mean square error RSME: the squared difference between actual and predicted values.

(4) R-squared (R^2): coefficient of determination, between 0 and 1, where $R^2 = 1$ means no error between observed and approximated values.

Once a second order model is fit to the response, the next step is to locate the point of maximum or minimum response. The second order model for k factors can be written as:

$$\hat{y}_i = \hat{\beta}_0 + \sum_{i=1}^{k} \hat{\beta}_i x_i + \sum_{i=1}^{k} \hat{\beta}_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{i>i}^{k} \hat{\beta}_{ij} x_i x_j \tag{2}$$

The point for which the response, $\hat{y}_i$, is optimized is the point at which the partial derivatives, $\partial\hat{y}/\partial x_1$, $\partial\hat{y}/\partial x_2$, ..., $\partial\hat{y}/\partial x_k$ are all equal to zero. This point is called the stationary point. The stationary point may be a point of maximum response, minimum response or a saddle point. The general mathematical solution for the location of the stationary point has to be used. Eq. (2) can be written in matrix notation as:

$$\hat{y}_i = \hat{\beta}_0 + x^T b + x^T B x \tag{3}$$

where:

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_k \end{bmatrix}^T$$

$$b = \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 & \dots & \hat{\beta}_k \end{bmatrix}^T$$

and

$$B = \begin{bmatrix} \hat{\beta}_{11} & \hat{\beta}_{12}/2 & . & . & . & \hat{\beta}_{1k}/2 \\ & \hat{\beta}_{22} & . & . & . & \hat{\beta}_{2k}/2 \\ & & . & . & . & . \\ & & & . & . & . \\ & & & & . & . \\ sym. & & & & & \hat{\beta}_{kk} \end{bmatrix}$$

3 Results and discussion

3.1 The optimizing design of the radius and the absolute interference in the interface

In this paper, two design parameters are selected by varying the variable radius of the interface r_1 and the absolute interference z_1 in the range of 16.03 - 27.48 mm and 0.0665 - 0.1080 mm, respectively. Both of the two variables are set at four levels, therefore, 16(4x4) combination of design parameters are constituted totally and are shown in Table 2. Table 2 shows the FEM analysis results of the maximum effective stress of the die insert and prestressed ring for various combinations of die design parameters. From the Table, it reveals that the maximum effective stresses of the prestressed ring in all cases are less than those of the die insert. Therefore, the maximum effective stress of the die insert is chosen as the response variable only.

The application of RSM yielded the following regression equation which is a relationship between the response variable (the maximum effective stress of the die insert) and the test variables in coded units:

$$(\sigma_{eff\_die})_{\max} = 3067.95 - 3266.18x_1 - 15393.7x_2 + 10531.12x_1x_2 + 2179.035x_1^2 + 51645.68x_2^2 \quad (4)$$

Table 3 shows the results of approximation error analysis of the metamodel. It can be seen that the error values that were normalized by the range of actual values for each response are very small (less than 2.5%) and the R-square value is equal to 0.7848. It means that the metamodel is reasonable adequate, and the next step is deployment of the optimization process.

From Eqn. (4) the stationary point is: $x_1 = 0.5166$, $x_2 = 0.0964$. Then, in terms of the actual values, the stationary point can be found as: the optimizing design of the radius and the absolute interference in the interface are $r_1 = 19.36$mm and $z_1 = 0.0964$mm, respectively. To find the nature of the stationary point the eigenvalues of the B matrix can be obtained as follows using the determinant of the matrix $B - \lambda I$. Solving the quadratic equation in λ returns the eigenvalues $\lambda_1 = 0.16$ and $\lambda_2 = 1.48$. Since both the eigenvalues

are positive, it can be concluded that the stationary point is a point of minimum response. The predicted value of the minimum response can be obtained using Eq. (5) as: $(\sigma_{eff\_die})_{\max} = 1490$ MPa. Figure 2 shows the response surface of the maximum effective stress of the die insert with a minimum point. Figure 3 shows the distribution of the effective stress of the die insert under the condition of inner pressure $p_0 = 840$MPa, the optimal radius of interface $r_1 = 19.36$mm and the optimal absolute interference $z_1 = 0.0964$mm. The figure shows that the maximum effective stress of the die insert locates on the inner fillet (marked with solid circle), and the value is 1490 MPa. The factor of safety of the die is approximately 1.11 for the die material.

4 Conclusions

In this paper, a pre-stress die design method for cold backward extrusion with a non simple hollow cylinder die by response surface method has been proposed. The radius of the interface and the absolute interference in the interface are chosen as the design variables. A finite element based code is utilized to investigate the elastic deformation characteristic under different design parameters, and the response surface method is then applied to synthesize the data sets obtained from the numerical

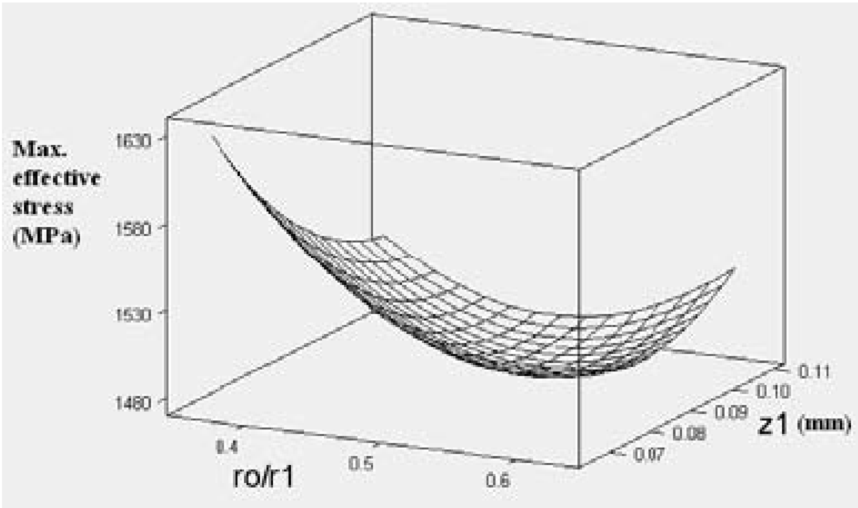

Figure 2: The response surface of the maximum effective stress of the die insert.

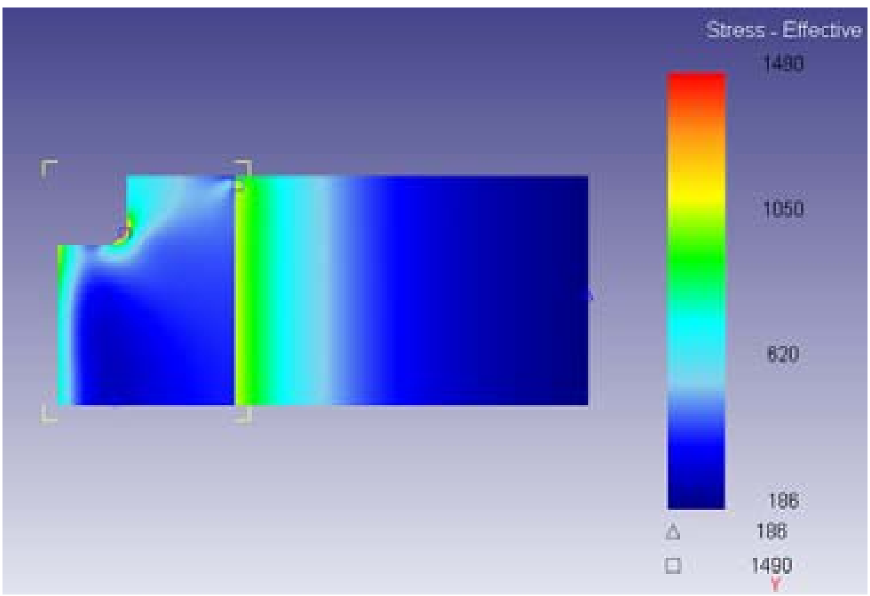

Figure 3: The distribution of the effective stress of the die insert under the condition of inner pressure p_0=840 MPa, the optimal radius of interface r_1=19.36 mm and the optimal absolute interference z_1=0.0964 mm.

Table 2: The combination of the two independent variables in coded and natural values along with the simulated responses under condition of the inner pressure p$_o$ =840 MPa.

Set no.	$x_1 = r_0/r_1$	$x_2 = z_1$(mm)	$(\sigma_{eff\_die})_{\max}$(MPa)	$(\sigma_{eff\_ring})_{\max}$(MPa)
1	0.5459	0.0665	1537.57	818.43
2	0.5459	0.0831	1484.14	990.74
3	0.5459	0.0997	1464.05	1176.13
4	0.4367	0.0997	1503.25	938.67
5	0.4367	0.0665	1580.34	640.98
6	0.4367	0.0831	1536.96	788.71
7	0.3639	0.0831	1565.73	650.58
8	0.3639	0.0997	1535.16	776.00
9	0.3639	0.0665	1603.37	526.29
10	0.6238	0.0665	1512.20	1034.82
11	0.6238	0.0831	1495.28	1221.78
12	0.6238	0.0997	1503.20	1409.11
13	0.5459	0.1080	1469.52	1270.51
14	0.4367	0.1080	1490.18	1013.86
15	0.3639	0.1080	1520.90	838.61
16	0.6238	0.1080	1565.90	1503.00

Table 3: Normalized error analysis results

Error types	Max. effective stress of die insert
Average	0.0085
Maximum	0.0243
RSME	0.0107
R-square	0.7848

analysis, then a maximum die effective stress prediction model is established. From the prediction model, the optimum radius and the optimum absolute interference in the interface under a certain inner pressure can be found.

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