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Research on Active Vibration Isolation Based on Chaos Synchronization

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Abstract: Line spectra are the most visible signs of the ships' radiated noise spectrum. The potential of chaotifying vibration isolation systems to reduce line spectra and improve its capability of concealment have been recently reported. Basically, as the existing isolation system design is based on linear theory, it is difficult to produce the nonlinear chaotic motion; and if the vibration isolation system(VIS) is designed directly using the nonlinear theory, it is also difficult to produce the chaotic motion because of the difficulties in accurate calculation of vibration isolation device parameters. In this paper, a controller design method is put forward using the Lyapunov stability theory to synchronize the outputs of a linear system with persistent disturbances with those of a chaotic system. The controller design method is applied in a double layer linear vibration isolation to produce a persistent and steady chaotic motion by tracking the outputs of a chaotic Duffing system, the force transferred to base is effectively reduced, and thus the isolation performance is improved.

Keywords: chaos synchronization, active control, double layer vibration isolation system, Duffing system.

1 Introduction

Since the seminal work of Pecora and Carroll [Pecora and Carroll (1990)] on the synchronization of chaotic systems, synchronization phenomenon has formed a new body of research activities and various synchronization schemes, such as adaptive control [Chen andLu¨(2002)], backstepping design [Tan (2003)], active control [Lei (2005) and Lei (2006)] and nonlinear control [Huang (2004) and Chen (2005)], have been successfully applied to chaos synchronization. Generalized projective synchronization (GPS) is characterized by a scaling factor that defines a proportional relation between the synchronized systems. The performance of projective synchronization can be selected and manipulated by controlling the scaling factor

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[Park (2007) and Li (2007)], where the drive and response systems could be synchronized up to a scaling factor a. It suggests that one can achieve control of this synchronization in general classes of chaotic systems including non-partially-linear systems. Projective synchronization is interesting because of its association with projective synchronization and generalized one.

Linear spectra reduction is of great significance for improving the acoustic stealth of submarines. Lou et al. [Lou (2005)] have studied the application of the chaos in vibration isolation system design, and they discussed how line spectra waterborn noise of the warship can be reduced. It should be mentioned that there exist two aspects of difficulties in the application of the method to practical engineering. The one is how to sustain the persistent chaotic motion of a vibration isolation system; the other is how to preserve the performance of vibration isolation as the system becomes chaotic. Liu et al. [Liu (2008)] presented the feedback control for chaotifying vibration isolation systems based on the calculation of the Lyapunov exponent. Unfortunately, the classical chaos criteria such as calculation of the Lyapunov exponent and Melnikov method are infeasible in real time anti-control of chaos under complex submarine circumstances. Yu et al. [Yu (2007)] proposed the chaos synchronization method to make the chaotic signal to drive a vibration isolation system persistent. The intensity of linear spectra at the primary harmonic frequencies is effectively reduced. However, although the improvement of vibration isolation at a particular case of parameters is displayed by numerical simulation, there is no guarantee in theory for the synchronization scheme to improve the isolation performance of vibration while the chaotic motion is used to reduce line spectra.

In this paper, the controller design method based on the Lyapunov stability theory is put forward and applied in a double layer linear vibration isolation. The simulation results show that the isolation system can generate chaos, effectively reduce the force passed to the base and improve the system's isolation performance by reasonably choosing the output matrix of vibration isolation system. The rest of the paper is organized as follows. Section 2 the controller is designed for chaos synchronization between a linear system and a chaotic system. The double-layer vibration isolation system model is introduced in section 3. The drive chaotic system is introduced in section 4. Numerical simulations are carried out to confirm the validity and stability of this method in Section 5.

2 Controller design for chaotic synchronization

The state equation of Linear perturbed system:

$$\begin{cases} \dot{X}_r = AX_r + BU + EW \\ Y_r = C_r X_r \end{cases} \tag{1}$$

Where $X_r \in R^n$ is the state vector of linear system,$U \in R^m$ is a controller to be designed. $Y_r \in R^m$ is the output of linear system, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $E \in R^{n\times l}$, $C_r \in R^{m\times n}$. $W \in R^l$ is the outside disturbance, W is continuously differentiable, and its dynamic characteristics is determined by the following external systems (Exosystem) [**Zheng D.Z.**(2002)]

$$\begin{cases} \dot{\xi}(t) = G\xi(t), t > 0 \\ W = H\xi(t) \end{cases} \tag{2}$$

Chaotic drive system:

$$\begin{cases} \dot{X}_d = f_d(X_d(t), t) \\ Y_d = C_d X_d \end{cases} \tag{3}$$

where $X_d \in R^p$ is the chaotic drive system state vector, f_d is the nonlinear system function, $Y_d \in R^m$ is the output of nonlinear chaotic system, $C_d \in R^{m\times p}$.
Tracking error:

$$e(t) = Y_r - Y_d \tag{4}$$

The error dynamic system:

$$\dot{e}(t) = \dot{Y}_r - \dot{Y}_d \tag{5}$$

If:

$$\lim_{t\to\infty} |e(t)| = \lim_{t\to\infty} \|Y_r - Y_d\| = 0 \tag{6}$$

the system is called output synchronization.
Theorem 1: System (1) and system (3) realize output synchronization, if
1) Y_r and Y_d are continuously differentiable in the X_r and $X_d$state vector set;
2) $[A \; C_r]$ is observable. C_rB is reversible. B is the linear transform of C_r^T.

3) Controller satisfy $\dot{U}+KU=-\phi$,

$$U=e^{-K(t-t_0)}U(t_0)+e^{-K(t)}\int_{t_0}^{t}e^{K(\tau)}[-\phi(\tau)]d\tau$$

Where: $K=(C_rB)^{-1}(C_rAB+C_rB)$,

$$\phi=(C_rB)^{-1}\left[\left(C_r+C_rA+C_rA^2\right)X+(C_rEH+C_rAEH+C_rEHG)\xi-C_d\left(\ddot{X}_d+\dot{X}_d+X_d\right)\right]$$

Proof: choose Lyapunov function

$$V(e(t),\dot{e}(t))=e^2(t)+\dot{e}^2(t).$$

Then

$$\begin{aligned}\dot{V}&=2e\dot{e}+2\dot{e}\ddot{e}\\&=2\left(\dot{Y}_r-\dot{Y}_d\right)\left(Y_r-Y_d+\ddot{Y}_r-\ddot{Y}_d\right)\\&=2(C_r\dot{X}_r-C_d\dot{X}_d)(C_rX_r-C_dX_d+C_rA^2X_r+C_rABU+C_rB\dot{U}\\&\quad+(C_rAEH+C_rEHG)\xi-C_d\ddot{X}_d)\end{aligned} \tag{7}$$

Set

$$\begin{aligned}&C_rX_r-C_dX_d+C_rA^2X_r+C_rABU+C_rB\dot{U}+(C_rAEH+C_rEHG)\xi-C_d\ddot{X}_d\\&=-C_r\dot{X}_r+C_d\dot{X}_d\\&=-C_rAX_r-C_rBU-C_rEW+C_d\dot{X}_d\end{aligned} \tag{8}$$

Transform the formula (8):

$$\begin{aligned}-C_rB\dot{U}-(C_rAB+C_rB)U=\left(C_r+C_rA+C_rA^2\right)X_r+(C_rEH+C_rAEH+C_rEHG)\xi\\-C_d\left(\ddot{X}_d+\dot{X}_d+X_d\right)\end{aligned} \tag{9}$$

Multiply $-(C_rB)^{-1}$ Equation (8) both sides then

$$\dot{U}+KU=-\phi \tag{10}$$

Where

$$K=(C_rB)^{-1}(C_rAB+C_rB).$$

$$\phi = (C_r B)^{-1} \left[\left(C_r + C_r A + C_r A^2 \right) X + (C_r EH + C_r AEH + C_r EHG) \xi - C_d \left(\ddot{X}_d + \dot{X}_d + X_d \right) \right]$$

equation (10) is the Nonhomogeneous linear differential equation, its solution is:

$$U = e^{-K(t-t_0)} U(t_0) + e^{-K(t)} \int_{t_0}^{t} e^{K(\tau)} \left[-\phi(\tau) \right] d\tau$$

Set $t_0 = 0, U(0) = 0$, then

$$U = e^{-K(t)} \int_{t_0}^{t} e^{K(\tau)} \left[-\phi(\tau) \right] d\tau \tag{11}$$

Substitute equation(11) to equation (7)

$$\dot{V} = -2(C_r \dot{X}_r - C_d \dot{X}_d)^2 < 0 \tag{12}$$

Proof end.

Different synchronization can be achieved by selecting drive system and response system output matrix $C_d$andC_r, such as anti-chaos synchronization, chaos synchronization projection.

3 Double-layer Vibration Isolation System

Double stage vibration isolation system is shown in Fig.1:

$$\begin{cases} m_1 \ddot{x}_1 + c_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) = F \\ m_2 \ddot{x}_2 + c_1 (\dot{x}_2 - \dot{x}_1) + k_1 (x_2 - x_1) + c_2 \dot{x}_2 + k_2 x_2 = F_u \end{cases} \tag{13}$$

Set $\mu = \frac{m_1}{m_2}$. $\omega_1 = \sqrt{\frac{k_1}{m_1}}$. $\xi_1 = \frac{c_1}{2\sqrt{m_1 k_1}}$. $\omega_2 = \sqrt{\frac{k_2}{m_2}}$. $\xi_2 = \frac{c_2}{2\sqrt{m_2 k_2}}$. $f = \frac{F}{k_1}$. $f_u = \frac{F_u}{k_1}$. The system equations change as following:

$$\begin{cases} \ddot{x}_1 + 2\xi_1 \omega_1 (\dot{x}_1 - \dot{x}_2) + \omega_1^2 (x_1 - x_2) = \omega_1^2 f \\ \ddot{x}_2 + 2\mu \omega_1 \xi_1 (\dot{x}_2 - \dot{x}_1) + \mu \omega_1^2 (x_2 - x_1) + 2\xi_2 \omega_2 \dot{x}_2 + \omega_2^2 x_2 = \mu \omega_1^2 f_u \end{cases} \tag{14}$$

set $X = \left[x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2 \right]^T$. Equations (1) can be written in matrix form:

$$\dot{X} = AX + BU + Ef \tag{15}$$

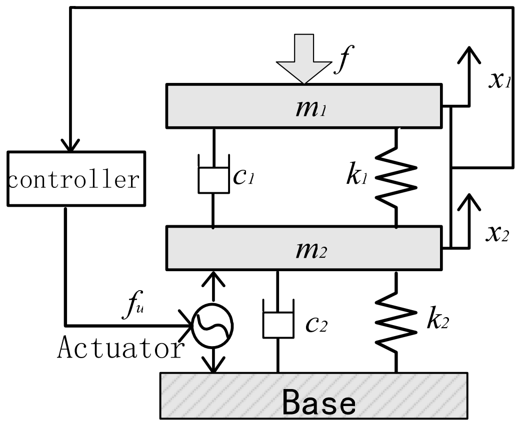

Figure 1: Double-layer Vibration Isolation System

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -\omega_1^2 & -2\xi_1\omega_1 & \omega_1^2 & 2\xi_1\omega_1\\ 0 & 0 & 0 & 1\\ \mu\omega_1^2 & 2\mu\xi_1\omega_1 & -\left(\mu\omega_1^2+\omega_2^2\right) & -\left(2\mu\xi_1\omega_1+2\xi_2\omega_2\right)\end{bmatrix},$$

$$B=\begin{bmatrix}0\\0\\0\\\mu\omega_1^2\end{bmatrix},\quad E=\begin{bmatrix}0\\\omega_1^2\\0\\0\end{bmatrix}.$$

Choose the system output as following:

$$Y_r=C_rX \tag{16}$$

From the demand of Theorem 1 we can get, the selection of C_r should make C_rB reversible and $\begin{bmatrix}A & C_r\end{bmatrix}$ observable, B is the linear transform of C_r^T,choose $C_r=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$ satisfy the above conditions, then $Y_r=\dot{x}_2$.

4 Chaotic drive system

Duffing system is a typical nonlinear dynamic system, the following Duffing equation is selected:

$$\begin{cases}\dot{x}_{d1}=x_{d2}\\ \dot{x}_{d2}=-ax_{d2}-bx_{d1}-x_{d1}^3+q\cos(\omega t)\end{cases} \tag{17}$$

When the parameters are set as $a = 0.4$. $b = -1.1$. $\omega = 1.8$. $q = 1.498$, the system become chaotic.

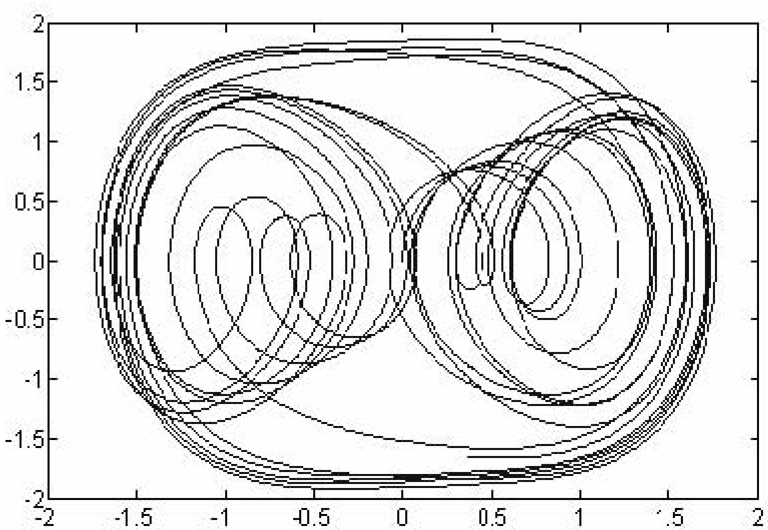

Figure 2: Duffing system $x_{d1} - x_{d2}$ phase portrait

The driving system output is chosen as: $Y_d = C_d X_d$
where $C_d = \begin{bmatrix} c_d & 0 \end{bmatrix}$. $X_d = \begin{bmatrix} x_{d1} & x_{d2} \end{bmatrix}^T$. then $Y_d = c_d x_{d1}$.

5 Simulation

The parameters of the vibration isolation system are $\mu = 2$. $\omega_1 = 3.2113$. $\xi_1 = 4.9038$. $\omega_2 = 10.7529$. $\xi_2 = 4.8978$. $f = 0.9091\cos(6\mathrm{t})$. $C_d = \begin{bmatrix} 10^{-3} & 0 \end{bmatrix}$.

The controller is designed according to Theorem 1. when the vibration isolation system is uncontrolled, the $\dot{x}_2$ is in periodic motion as shown in Figure 3; when the control is applied to the vibration isolation system, synchronization is achieved quickly as shown in Figure 4 in which the time histories of the errors is defined as $e = Y_r - Y_d$. The time history of $\dot{x}_2$ is shown in Figure 5, which indicates that $\dot{x}_2$ is not in periodic motion after control. As is known from the Theorem 1, the value of c_d directly affects the magnitude of $\dot{x}_2$. The magnitude of $\dot{x}_2$ deceases when c_d gets smaller, however the robustness to the initial disturbance of the system will deteriorate when$c_d$gets too small, so the value of c_d is not necessarily the smaller, the better.

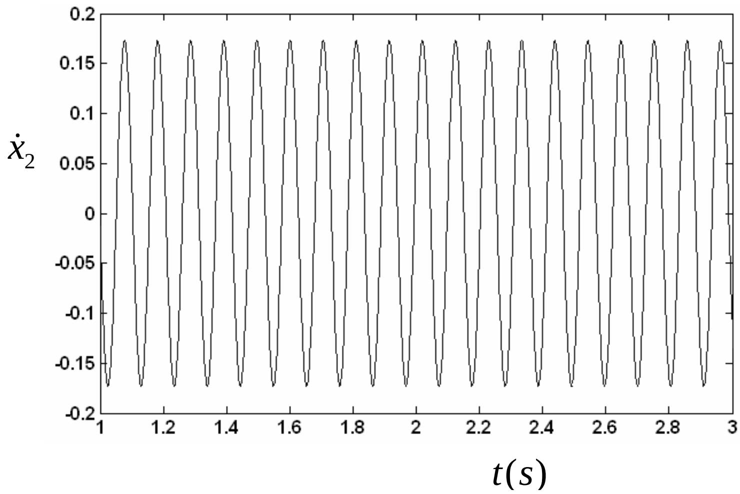

Figure 3: The time histories of $\dot{x}_2$ uncontrolled

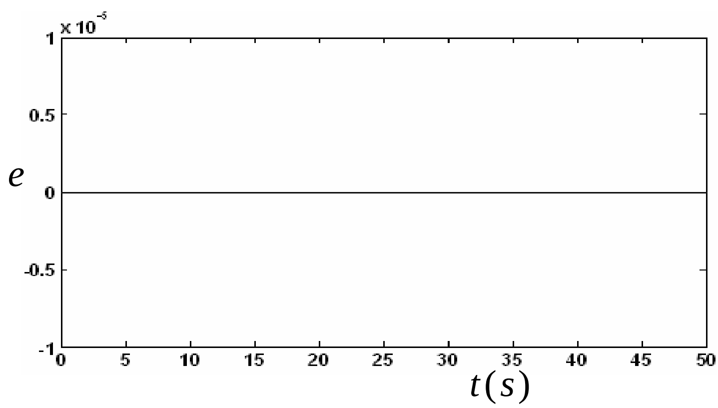

Figure 4: The time histories of the errors

6 Analysis the effect of vibration isolation

Since the external driving forces for passive isolation and active isolation are the same, so the force size passed to the base system can directly reflect the effect of vibration isolation. The force passed to the base by passive vibration isolation is

$$F_{base} = k_2 x_2 + c_2 \dot{x}_2 \tag{18}$$

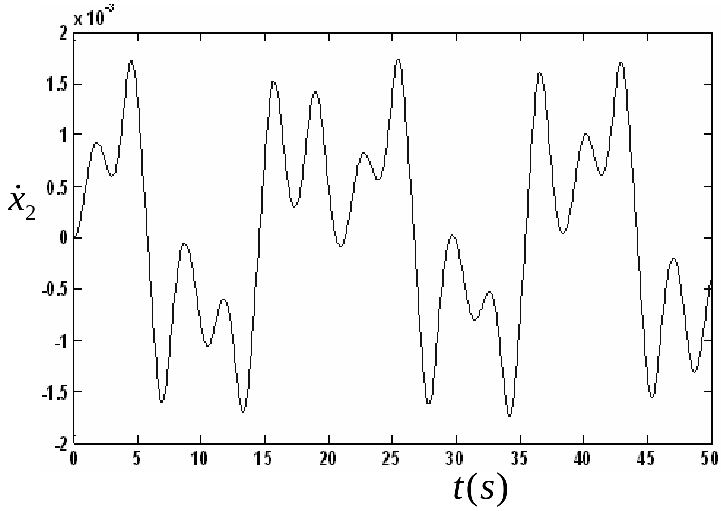

Figure 5: The time histories of $\dot{x}_2$ controlled

Dimensionless transformation of equation (18) is as following:

$$f_b = F_{base}/m_2 = \frac{1}{m_2}(k_2x_2 + c_2\dot{x}_{21}) = \omega_2^2 x_2 + 2\xi_2\omega_2\dot{x}_2 \tag{19}$$

In Fig.6 the force passed to the base is periodic force when the system is uncontrolled. The force transmitted to the base caused by the spring and damping of system can be expressed by equation (18) when the synchronization realized. The time histories and the power spectrum off_b is shown in Fig.7 and Fig.8, which shows that f_b is chaotic, and the line spectrum does not appear in the power spectrum.

Because the actuator is installed between the m2 and the base, the force transmitted to the base should consider the reaction force when active control is applied, which can be written as:

$$F_{base} = k_2x_2 + c_2\dot{x}_2 + F_u \tag{20}$$

Where, F_u is the output of the actuator, dimensionless transformation of equation (18) is as following:

$$f_z = F_{base}/m_2 = \frac{1}{m_2}(k_2x_2 + c_2\dot{x}_2 + f_u k_1) = \omega_2^2 x_2 + 2\xi_2\omega_2\dot{x}_2 + \mu\omega_1^2 f_u$$

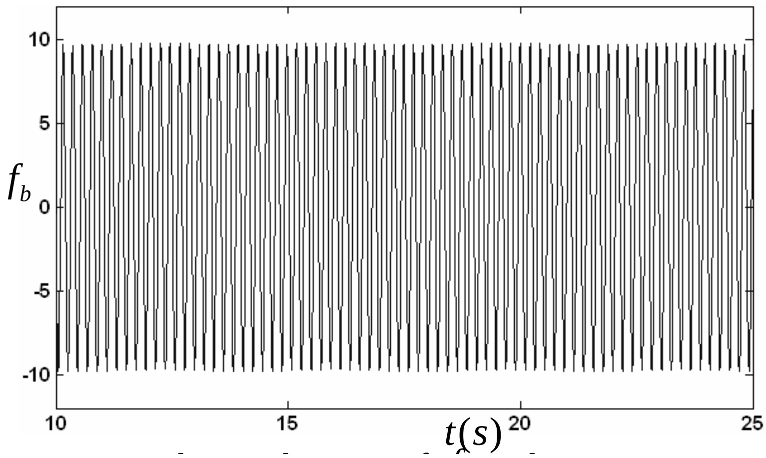

Figure 6: The time histories of f_b with passive system

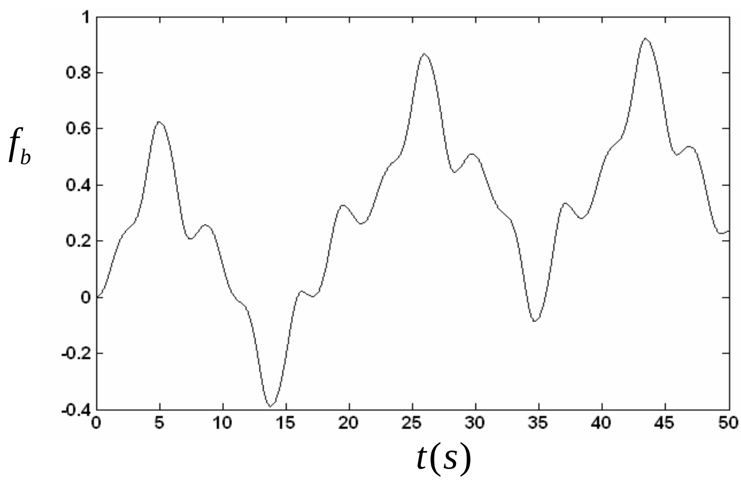

Figure 7: The time histories of f_b with chaos synchronization

From Figure 6 and Figure 9 the amplitude of the force passed to base is smaller when chaos synchronization is applied. And Figure10 shows the amplitude of the actuator output force is less than the amplitude of the external incentive system, it means that the performance of vibration isolation system can be improved with less energy. The power spectrum force transformed to base is shown in Figure 11, after imposing active control synchronization, the force transformed to base is greatly reduced, the power spectrum at the excitation frequency decreases by 25dB, but the

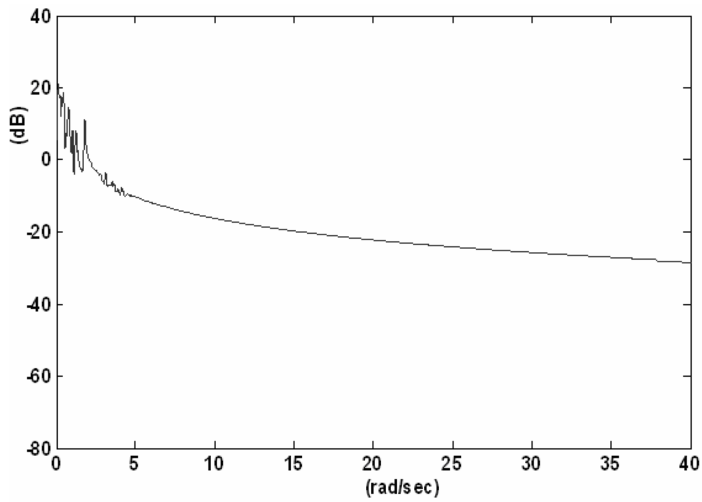

Figure 8: The power spectrum of f_b with chaos synchronization

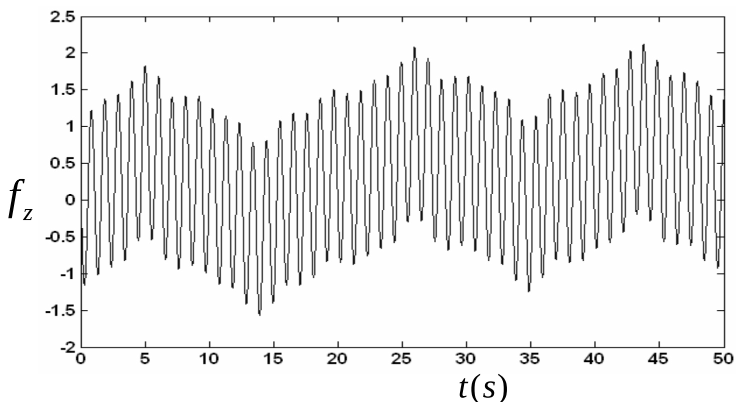

Figure 9: The time histories of force passed to base with chaos synchronization

active vibration isolation system does not eliminate line spectrum characteristics. The main reason is that the actuator, which is installed between the lower quality and the base, generates the line spectrum.

7 Conclusion

Based on Lyapunov stability theory, a controller design method is put forward to synchronize the outputs of a linear system with persistent disturbances with those

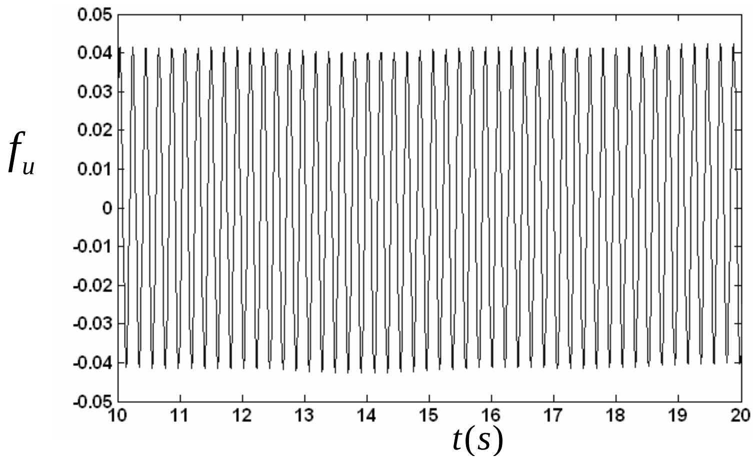

Figure 10: The output curve of actuator

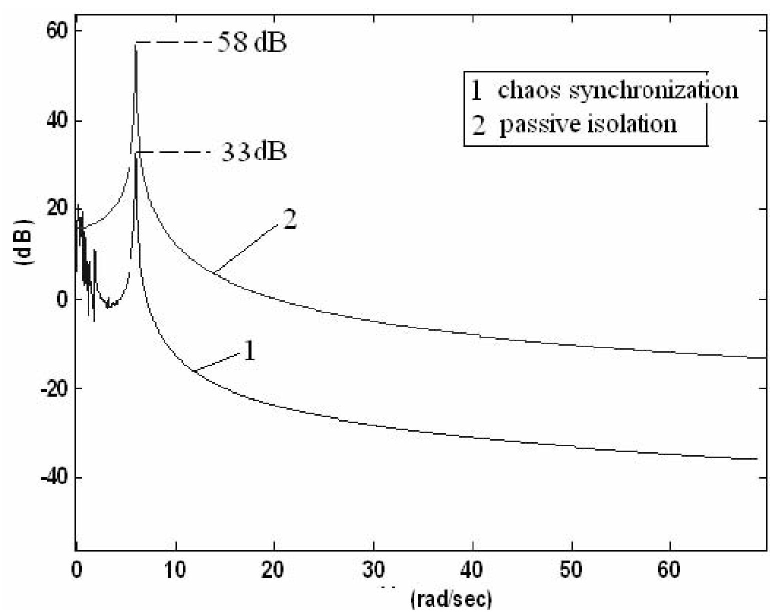

Figure 11: The power spectrum of fore passed to base

of a chaotic system. The controller design method is applied in a double layer linear vibration isolation to produce a persistent and steady chaotic motion by tracking the outputs of a chaotic Duffing system.

1) The magnitude of $\dot{x}_2$ deceases when c_d gets smaller, however the robustness to

the initial disturbance of the system will deteriorate when c_d gets too small, so the size of C_r should be selected considerably to ensure vibration isolation system has both good vibration isolation performance and robustness.

2) It is verified that the power spectrum is a continuous spectrum and the characteristics of line spectrum is eliminated when the double isolation system is in chaotic motion. And additional line spectrum component is found in the power spectrum of the force transmitted to the base, which produced by the actuator installed between the lower mass and the base. However, the isolation performance is significantly improved comparing with the passive isolation system.

3) As the theorem proposed in the paper indicates, the drive chaotic system for vibration isolation system is not limited to Duffing system, other chaotic systems can also be used as the drive system.

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