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Large Strain Consolidation Stochastic Finite Element Method for Soft-clay Road Embankment Analysis

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Abstract: This paper presents a method for the large strain consolidation analysis of soft-clay road embankment with stochastic parameters to random excitation. Based on the large strain theory of continuum material, Biot consolidation theory and Neumann stochastic finite element method (NSFEM), the large strain consolidation NSFEM (LSC-NSFEM) has been established. A program of LSC-NSFEM is designed to analyze soft-clay road embankment. The residue iteration method is used to deal with the nonlinear fluctuating part of stiffness matrix, and the approximate algorithm of the fluctuating part of stiffness matrix is designed to improve LSC-NSFEM program efficiency.

Keywords: Soft-clay Road Embankment; Large Strain; Consolidation; Neumann Stochastic Finite Element Method

1 Introduction

The deformation analysis is one of the most important research themes in geotechnical engineering, and the consolidation settlement analysis is the key to deal with the deformation analysis for soft clay foundation. Since Terzaghi[15] put forward the one-dimensional consolidation theory, the investigation of soil consolidation settlement theory has been developing quickly. With the expansion of the computer technologies, numerical methods are comprehensively used in settlement analysis, and have improved the investigation of soil consolidation settlement theory.

With the enhancement of consolidation theory research, Biot consolidation method[1] assuming low strain has been found unsuitable for the actual situation. Therefore the issue of large strain consolidation analysis based on continuum mechanics started in 1970's and was researched widely. 2D large strain consolidation model under the Eulerian description was built by Cater et al[3] in 1977, and analyzed

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by finite element method (FEM). The difficulty in this method is due to constitutive equations built by Green strain rate tensor and Jaumann stress rate tensor. Afterwards, Chopra and Dargush[4] derived a FEM for the large strain consolidation problem based on the material description and virtual work principle. The subjective tensor: first Piola-Kirchhoff stress tensor was employed in consolidation equations, and other objective tensors were used in constitutive equations bringing in the complexity. Based on the continuum model of saturated soils and the axiomatized theories in classic continuum mechanics, Ding[6] proposed a mathematical framework of continuum consolidation theory taking variable-mass effect into account during consolidation process. Within the framework of Ding's theory, Truesdell rate-type constitutive relation is introduced into the geometrically-nonlinear consolidation analysis as well as simplified Jaumann stress rate-type and the other cases.

In the last two decades, stochastic finite element methods (SFEM) have a rapid development due to the spectacular improvement of computing power resources[12]. In conjunction with SFEM, the expansion of the inverse of the stochastic global stiffness matrix in a Neumann series had been treated by several researchers, namely Shinozuka, Yamazaki et al[14,16,17], and Ghanem and Spanos[7,8] in the late 1980's. Zhao Lei[18] presented a method of Neumann SFEM for the dynamic analysis of structures with stochastic parameters to random excitation. Excellent agreement between the results of this method and those obtained by the direct Monte Carlo simulation (M.C.S.) indicate that this approach is satisfactory for dynamic analysis of structures with stochastic parameters with respect to accuracy, convergence and computational efficiency.

SFEM has also been applied to the analysis of stochastic geotechnical problems[10], as well as to the analysis of structures with uncertain material, geometric parameters[2,9,11] and uncertainties in time domain such as random loads[5]. Due to the complexity of SFEM formulation, its application to nonlinear and multidimensional consolidation in geotechnical engineering is still very limited. The settlement analysis based on Biot consolidation theory has been initiated by Nishimura[13] using direct M.C.S. Further development is made by Guo and Liu[10] using Taylor SFEM considering the representation of stochastic field.

In this paper, we propose a finite element analysis method based on the large strain theory of continuum material, Biot consolidation theory and Neumann SFEM. Then, the results from Neumann SFEM are compared with those from the direct M.C.S. with respect to accuracy, convergence and efficiency.

2 LSC-NSFEM

2.1 Large strain Biot's consolidation FEM

In the large strain Biot consolidation FEM, fundamental unknown variables are displacements and excess pore pressure. Base on the equilibrium, virtual work, geometry and constitutive equations, the large strain Biot FE equation within a certain time interval has the following form:

$$\tilde{\mathbf{K}}_i \delta\Delta\mathbf{u}_i = \mathbf{R}_i \tag{1}$$

Where i means the i-th time interval, $\tilde{\mathbf{K}}_i$ indicates the large strain consolidation stiffness matrix in this time interval; and $\mathbf{R}_i$ indicates the increment of equivalent stress and flow in the same time interval. $\Delta\mathbf{u}_i$ is the unknown variables increment in this time interval, so the unknown variables $\mathbf{u}_i$ can be generated by:

$$\mathbf{u}_i = \sum_{\mathrm{k}=1}^{i} \delta\Delta\mathbf{u}_k \tag{2}$$

2.1.1 FE equation linearization

Large strain consolidation FE equation is nonlinear because of stiffness matrix $\tilde{\mathbf{K}}_i$ dependence on $\Delta\mathbf{u}_i$. $\tilde{\mathbf{K}}_i$ linearized FE equation presents as

$$\bar{\mathbf{K}}_i \delta\Delta\bar{\mathbf{u}}_i = \mathbf{R}_i \tag{3}$$

Where $\bar{\mathbf{K}}_i$ indicates linearized stiffness matrix of soil skeleton, $\delta\Delta\bar{\mathbf{u}}_i$ is the linearized unknown variables.

2.1.2 The residue iterative method

Based on $\delta\Delta\bar{\mathbf{u}}_i$ got from Eq.(3), the accuracy achieved in $\delta\Delta\mathbf{u}_i$ may be calculated using the modified Newton-Raphson method.. This iterative method is popular in solid and soil mechanics nonlinear total stress analysis method, and the calculation of the out-of-balance force is the key of this method. In consolidation iteration method, it is to calculate not only the out-of-balance force but also the out-of-balance water pressure. Ding[6] brings up the out-of-balance quantity to describe it. In the iterative processes, when the k-th iterative approximate solution $\Delta\mathbf{u}_i^{(k)}$ is given, the out-of-balance quantity $\Phi_i\left(\Delta\mathbf{u}^{(k)}\right)$ can be calculated as follows:

$$\Phi_i\left(\delta\Delta\mathbf{u}^{(k)}\right) = \tilde{\mathbf{K}}_i^{(k)} \delta\Delta\mathbf{u}_i^{(k)} - \mathbf{R}_i^{(k)} \tag{4}$$

Where $\delta\Delta\mathbf{u}_i^{(k)}$ means the k-th iterative approximate solution in LSC problem at the i-th time interval, $\tilde{\mathbf{K}}_i^{(k)}$ indicates nonlinear stiffness matrix of soil skeleton in LSC problem that corresponds to $\delta\Delta\mathbf{u}_i^{(k)}$, and $\mathbf{R}_i^{(k)}$ is the increment of equivalent stress and flow corresponding to $\Delta\mathbf{u}_i^{(k)}$ in i-th time interval. $\Phi_i\left(\delta\Delta\mathbf{u}^{(k)}\right)$ is the out-of-balance vector that weighs $\Delta\mathbf{u}_i^{(k)}$ deviation the balance.

If $\Phi_i\left(\delta\Delta\mathbf{u}^{(k)}\right) \neq \{0\}$, $\Delta\mathbf{u}_i^{(\mathrm{k}+1)}$ can be expanded in the first order Taylor series of $\delta\Delta\mathbf{u}_i^{(k)}$.

$$\delta\Delta\mathbf{u}_i^{(\mathrm{k}+1)}= \Delta\mathbf{u}_i^{(k)}+\phi\left(\delta\Delta\mathbf{u}_i^{(k)}\right) \tag{5}$$

Assumed that $\delta\Delta\mathbf{u}_i^{(\mathrm{k}+1)}$ can meet out-of-balance quantity being zero, then,

$$\Phi_i\left(\delta\Delta\mathbf{u}^{(k+1)}\right) = \Phi_i\left(\Delta\mathbf{u}^{(k)}\right) + \left(\frac{\mathrm{d}\Phi}{\mathrm{d}\left(\delta\Delta\mathbf{u}\right)}\right)^{(k)}\phi_i\left(\Delta\mathbf{u}^{(k)}\right) = 0 \tag{6}$$

Where $\left(\frac{\mathrm{d}\Phi}{\mathrm{d}(\Delta\mathbf{u})}\right)^{(k)}$ means the tangent stiffness matrix, and can be defined as follows:

$$\mathbf{K}_t^{(k)} = \left(\frac{\mathrm{d}\Phi}{\mathrm{d}\left(\delta\Delta\mathbf{u}\right)}\right)^{(k)} \tag{7}$$

So $\phi_i\left(\delta\Delta\mathbf{u}^{(k)}\right)$, the amendment of k-th iterative approximate solution at the i-th time interval, can be calculated as follows:

$$\phi_i\left(\delta\Delta\mathbf{u}^{(k)}\right) = -\left(\mathbf{K}_t^{(k)}\right)^{-1}\Phi_i\left(\Delta\mathbf{u}^{(k)}\right) = -\left(\mathbf{K}_t^{(k)}\right)^{-1}\left(\tilde{\mathbf{K}}_i^{(k)}\Delta\mathbf{u}_i^{(k)} = \mathbf{R}_i\right) \tag{8}$$

Where $\mathbf{K}_t^{(k)}$ indicates the tangent stiffness matrix that corresponds to $\delta\Delta\mathbf{u}^{(k)}$. We use $\mathbf{K}_t^{(k)} = \tilde{\mathbf{K}}_i^{(k)}$ for convenience, where $\tilde{\mathbf{K}}_i^{(k)}$ is nonlinear stiffness matrix that corresponds to $\Delta\mathbf{u}_i^{(k)}$. $\left(\tilde{\mathbf{K}}_i^{(0)}\right)^{-1}$ can be used to replace of $\left(\tilde{\mathbf{K}}_i^{(k)}\right)^{-1}$ in modify Newton-Raphson method. Where $\tilde{\mathbf{K}}_i^{(0)}$ means the nonlinear stiffness matrix that corresponds to $\Delta\mathbf{u}_i^{(0)}$, and $\Delta\mathbf{u}_i^{(0)}= \Delta\bar{\mathbf{u}}_i$ is gained from linearized FE equation.

2.2 Neumann SFEM

Neumann expansion method has developed and is applied to the equation for deriving the statistical solution within the framework of Monte Carlo simulation. In conjunction with stochastic finite elements, the expansion of the inverse of the stochastic global stiffness matrix in a Neumann series, so called as NSFEM, is presented by Shinozuka, Yamazaki et al[14,16,and17] and Ghanem and Spanos[7,8].

The stiffness matrix $\mathbf{K}_1$ is considered with random effect, and the increment vector of equivalent stress and flow $\mathbf{R}$ isn't. For reduced calculation, the stiffness matrix $\mathbf{K}_i$ is decomposed into two matrices:

$$\mathbf{K}_i = \mathbf{K}_{0i} + \delta\mathbf{K}_i \tag{9}$$

Where $\mathbf{K}_{0i}$ represents the part of quasi-static equivalent stiffness matrix in which stochastic variables are replaced by their mean values, and $\mathbf{K}_1$ consists of components representing the fluctuating part of the corresponding components in $\mathbf{K}_i$. Via Neumann expansion, $\mathbf{K}_i^{-1}$ takes the following form:

$$\mathbf{K}_i^{-1} = (\mathbf{K}_{0i} + \delta\mathbf{K}_i)^{-1} = \left(\mathbf{I} = \mathbf{P} + \mathbf{P}^2 - \mathbf{P}^3 + \cdots\right)(\mathbf{K}_{0i})^{-1} \tag{10}$$

Where $\mathbf{I}$ is the identity matrix, and $\mathbf{P}$ is written as

$$\mathbf{P} = (\mathbf{K}_{0i})^{-1}\,\delta\mathbf{K}_i \tag{11}$$

The calculation of $\mathbf{K}_i^{-1}$ cannot be generally performed efficiently in direct Monte Carlo simulation. The Neumann expansion of inverse of the stochastic global stiffness matrix has the advantage that only the non-fluctuating part of the stiffness matrix has to be inverted and this is only once. Due to this, the need of CPU-time can be greatly reduce.

The unknown variation $\delta\Delta\mathbf{u}_i$ may be given by the following form

$$\delta\Delta\mathbf{u}_i = \mathbf{K}_i^{-1}\mathbf{R}_i = \delta\Delta\mathbf{u}_{0i} = \mathbf{P}\delta\Delta\mathbf{u}_{0i} + \mathbf{P}^2\delta\Delta\mathbf{u}_{0i} - \mathbf{P}^3\delta\Delta\mathbf{u}_{0i} + \cdots \tag{12}$$

Once $\delta\Delta\mathbf{u}_{0i} = (\mathbf{K}_{0i})^{-1}\,\mathbf{R}_i$ is obtained, every subsequent $\delta\Delta\mathbf{u}_{ri}$ is determined from

$$\delta\Delta\mathbf{u}_{ri} == (\mathbf{K}_{0i})^{-1}\,\delta\mathbf{K}_i\delta\Delta\mathbf{u}_{r-1} \quad (\mathrm{r} = 1,2,3,\cdots) \tag{13}$$

So $\delta\Delta\mathbf{u}_i$ can be written as

$$\delta\Delta\mathbf{u}_i = \Delta\mathbf{u}_{0i} + \delta\Delta\mathbf{u}_{1i} + \delta\Delta\mathbf{u}_{2i} + \delta\Delta\mathbf{u}_{3i} + \cdots = \sum_{r=0}\delta\Delta\mathbf{u}_{ri} \tag{14}$$

2.3 LSC-NSFEM

We can combine the large strain consolidation FEM with the Neumann expansion to establishing the SFEM, so-called LSC-NSFEM, for analysis soft soil with stochastic parameters.

Since the stiffness matrix $\mathbf{K}_i$ is nonlinear affected by large strain, $\delta\mathbf{K}$ and $\Delta\mathbf{u}_{ri}$ in Eq.(8) becomes nonlinear. Using $\tilde{()}$ to denote nonlinear, Eq.(8) now may be written as

$$\delta\Delta\tilde{\mathbf{u}}_{ri} = \left(\tilde{\mathbf{K}}_0^{-1}\delta\tilde{\mathbf{K}}\right)\left(\delta\Delta\tilde{\mathbf{u}}_{r-1}\right)_i \tag{15}$$

To solve this nonlinear equation, the first step is linearizing Eq.(15).
Introducing $\bar{()}$ denotes linearizing, therefore

$$\delta\bar{\mathbf{K}} = \bar{\mathbf{K}} - \bar{\mathbf{K}}_0 \tag{16}$$

Consequently, the linearized unknown variation $\delta\Delta\bar{\mathbf{u}}_i$ is given by

$$\delta\Delta\bar{\mathbf{u}}_i = \sum_{r=0}\left(\delta\Delta\bar{\mathbf{u}}_r\right)_i \tag{17}$$

Where

$$\left(\delta\Delta\bar{\mathbf{u}}_r\right)_i = \left(\tilde{\mathbf{K}}_0^{-1}\delta\bar{\mathbf{K}}\right)\left(\delta\Delta\bar{\mathbf{u}}_{r-1}\right)_i \tag{18}$$

The second step is using modify Newton-Raphson method to calculate $\delta\Delta\mathbf{u}_i$. The out-of-balance quantity $\Phi\left(\delta\mathbf{u}^{(k)}\right)$ can be computed

$$\Phi\left(\delta\mathbf{u}^{(k)}\right) = \left(\tilde{\mathbf{K}}_0+\delta\tilde{\mathbf{K}}\right)\delta\mathbf{u}^{(k-1)} = \mathbf{R} \tag{19}$$

and the modified vector

$$\phi\left(\delta\mathbf{u}^{(k)}\right) = -\left(\tilde{\mathbf{K}}_0+\delta\tilde{\mathbf{K}}\right)^{-1}\Phi\left(\delta\mathbf{u}^{(k)}\right) \tag{20}$$

The iterative formulas of unknown variations increment can be written as

$$\delta\mathbf{u}^{(k)} = \delta\mathbf{u}^{(k-1)}+\phi\left(\delta\mathbf{u}^{(k)}\right) \tag{21}$$

2.4 The nonlinear part of fluctuating stiffness matrix

In LSD-NSFEM, the nonlinear unknown variations increment $\delta\mathbf{u}^{(k)}$ need to be iterated for each simulation leading to an enormous amount of CPU time.

Approximatively, using the unknown variations increment of deterministic part replaces in each simulated. Partially neglecting the nonlinear part of fluctuating stiffness matrix can obviously improve the efficiency of LSC-NSFEM, it can be named as “partial” LSC-NSFEM., and the method of considering the nonlinear fluctuating part may be called “total” LSC-NSFEM.

Furthermore, absolutely neglecting the nonlinear part of fluctuating stiffness matrix, Eq.(13) can be written as

$$\delta \mathbf{u}_r = \left(\tilde{\mathbf{K}}_0^{-1}\delta\bar{\mathbf{K}}\right)\delta\mathbf{u}_{r-1} \tag{22}$$

It was called as "linear" LSC-NSFEM.

3 Numerical Example

3.1 Finite element model

The "partial" and "linear" LSC-NSFEM both would bring in the error of calculating. To elucidate LSC-NSFEM with respect to the accuracy and efficiency, a 2D soil finite element model (Fig.1) is studied, and the deterministic and stochastic parameters in Gaussian stochastic field are given in table 1.

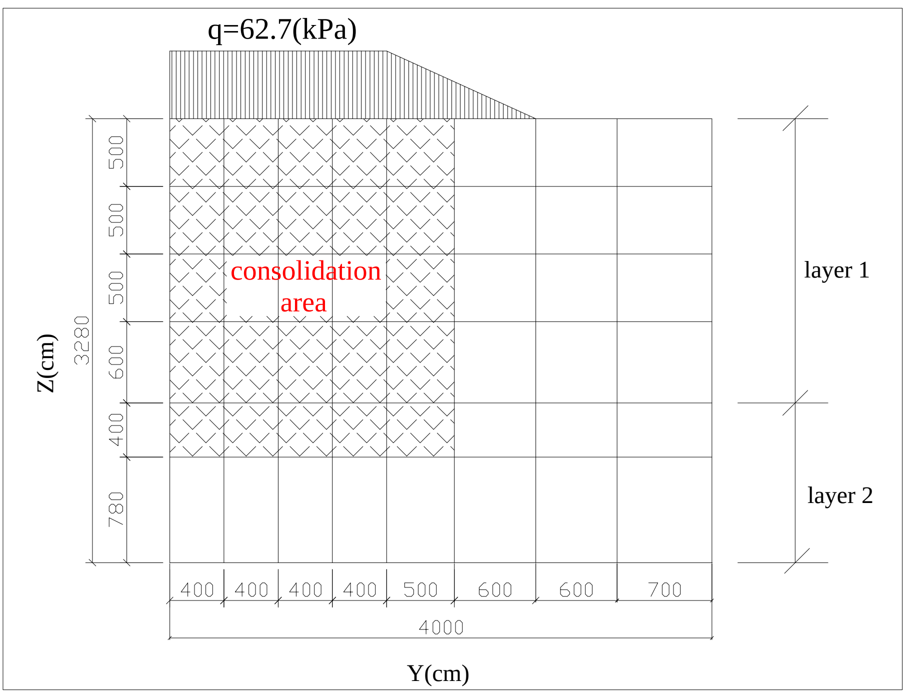

Figure 1: Finite element mesh

Where **E** is strain modulus, μ means the Poisson ratio of the soil, γ_w means weight of dry soil and $\mathbf{k}_{x(y,z)}$ is permeability coefficient.

Table 1: The soil model parameters

Symbol	Layer 1	Layer 2	Variation Coefficient
$\mathbf{E}$	$\mathbf{E}_1 = 948(\text{kPa})$	$\mathbf{E}_2 = 1740(\text{kPa})$	0.3
μ	$\mu_1 = 0.301$	$\mu_2 = 0.301$	0.25
γ_w	$\gamma_{w_1} = 16.3(\text{kN / m}^3)$	$\gamma_{w_2} = 17.4(\text{kN / m}^3)$	0
$\mathbf{k}_x$	$\mathbf{k}_x = 10^{-4}(\text{cm / s})$	$\mathbf{k}_x = 10^{-4}(\text{cm / s})$	0
$\mathbf{k}_y$	$\mathbf{k}_y = 10^{-4}(\text{cm / s})$	$\mathbf{k}_y = 10^{-4}(\text{cm / s})$	0
$\mathbf{k}_z$	$\mathbf{k}_z = 3 \times 10^{-5}(\text{cm / s})$ (consolidation area) $\mathbf{k}_z = 1 \times 10^{-6}(\text{cm / s})$	$\mathbf{k}_z = 3 \times 10^{-5}(\text{cm / s})$ (consolidation area) $\mathbf{k}_z = 1 \times 10^{-6}(\text{cm / s})$	0.25

3.2 Numerical results and analysis

Using 20000 samples, the results of the expected value of maximum surface settlement vs. time computation by four methods (direct Monte-Carlo simulation, "total" LSC-NSFEM, "partial" LSC-NSFEM and "linear" LSC-NSFEM) are compared with the deterministic result shown in Figs 2, and the curve of load vs. time is also shown in Fig.2. It shows that the results of all these methods are very close.

The contours of settlement standard deviation are plotted in Fig.3-6 with the results of four methods. The small difference shows that the same accuracy of the LSC-NSFEM is comparable to the direct M.C.S.

Table 2 compares the CPU time by four methods. The direct M.C.S (direct Monte-Carlo simulation) requires more time than any other method, and the "linear" LSC-NSFEM requires the least amount of CPU time, as expected. However, the "total" and "partial" LSC-NSFEM requires by far greater amount of CPU time than the "linear" method, though there is still much more efficiency than direct M.C.S.

Table 2: Comparison of CPU Time

direct M.C.S.	"total" LSC-NSFEM	"Partial" LSC-NSFEM	"linear" LSC-NSFEM
5:27[a]	4:45	2:49	2:08
[a] Hour: minute			

3.3 Conclusions

The research work was mainly concerned with the six following aspects:

(1) With the impact of large strain consolidation, the fluctuating part of stiffness matrix is included in fundamental unknown quantity. The residual iteration method is

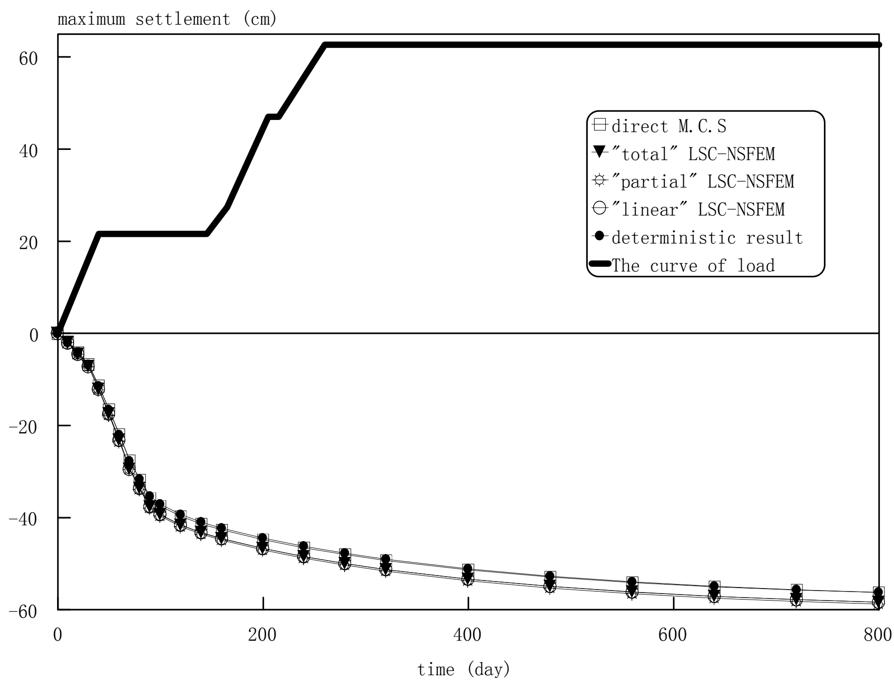

Figure 2: Comparison of maximum surface settlement vs. time computing results

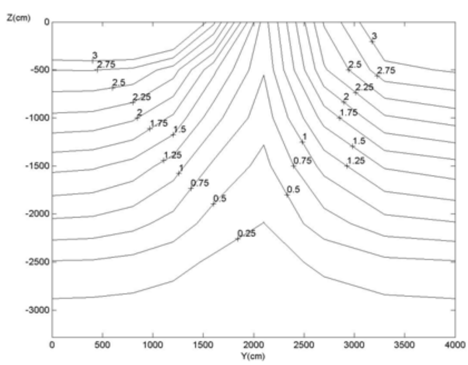

Figure 3: Contours of settlement standard deviation with direct Monte-Carlo simulation

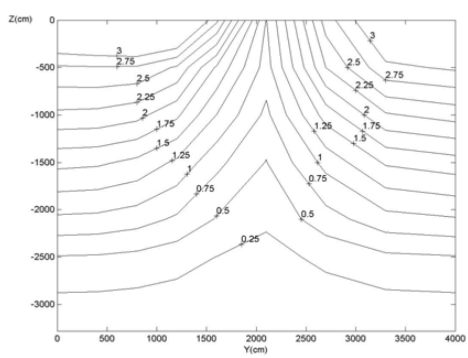

Figure 4: Contours of settlement standard deviation with "total" LSC-NSFEM

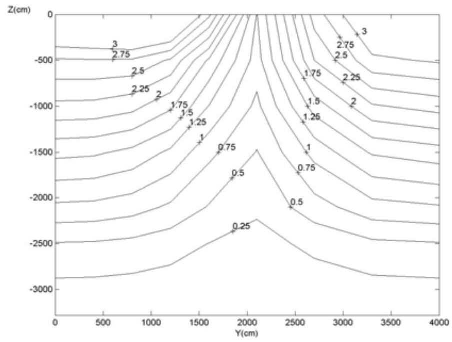

Figure 5: Contours of settlement standard deviation with "partial" LSC-NSFEM

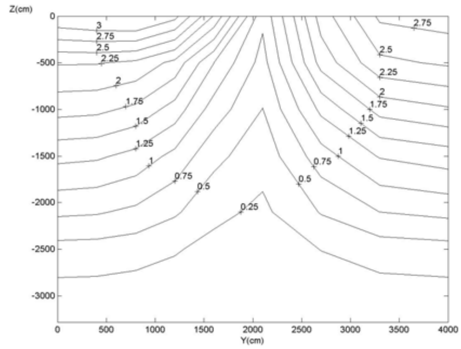

Figure 6: Contours of settlement standard deviation with "linear" LSC-NSFEM

used to deal with the nonlinear fluctuating part of stiffness matrix, and the approximate calculation methods of fluctuating part of stiffness matrix were discussed. The approximate algorithms are chosen to simplify FEM calculation, which improve the efficiency and not damage the accuracy of solution.

(2) The FEM calculation of simplified soft-clay road embankment model was carried out. The laws of time-dependent and distribution in special of mean and variance of settlement were analyzed in detail.

(3) Taking the 2D model for example, the result of LSC-NSFEM was compared with Monte-Carlo direct simulation FEM. All results show that the program presented in this paper is validity and effective.

(4) The settlement standard deviation in consolidation reinforced area is higher than the other place standard deviation in respect of the same depth. It is proved that the settlement in consolidation area is more effected by soil parameters stochastic.

(5) The result of FEM shows that the settlement standard deviation is higher with the increase of depth and minimum in shoulder.

(6) Comparing the CPU time by different methods, the direct M.C.S requires more time than any other method, and the "linear" LSC-NSFEM requires the least amount of CPU time, as expected. However, the "total" and "partial" LSC-NSFEM requires by far greater amount of CPU time than the "linear" method, though there is still much more efficiency than direct M.C.S.

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