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Theoretical Study of Contact Angles of a Linear Guideway

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Abstract: The contact angle affects the life and accuracy of a linear gudieway. In this study, the factors which effect the angle of contact angle of linear guideway includes contact deformation of balls and grooves, downward load, ball diameter, number of load-carrying balls, number of load-carrying rows and conformity. A theoretical approach for finding the contact angle changed of linear gudieway is proposed by using Hertzian theory and Lundeberg/Palmgren approach. The results are useful for modification of loading capability of linear gudieway under load and different preload setting.

Keywords: Linear guideway, contact angle, Hertzian theory, Lundeberg/Palmgren approach.

1 Introduction

Linear guideway transmission mechanisms have been widely used in modern machinery. They are frequently used to support different loading to linear motion. The mechanical durability of the structure should be satisfied in order to assure the reliability against the external loading and long-term usage of devices. In practice, good contact condition between balls and grooves (have good durability and long-term usage) can be assured by the initial contact angle.

In ISO 14728-1 and 14728-2, dynamic load and statics load rating for linear motion rolling bearing, are identified ideally by several parameters (including conformity, groove length, number of load-carrying rows of balls, length of block, number of load-carrying balls, ball diameter and contact angle[ISO (2004)]. These parameters in dynamic load and statics load rating formula of basic load ratings of linear guideway type recirculating linear ball bearings are constant. In practice, the linear rolling ball guideway, to produce adequate preload in order to increase the structural rigidity an oversized rolling balls are usually employed. Contact angle and geometry of block are changed a lot when using high preload setting [AG (2007)]. In a previous experimental study, there is sufficient evidence to show that plastic

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deformation and contact angle changed when variable load applied, as shown in Fig. 1 [Su (2010)]. According to Hertzian theory for elastic deformation, there is a nonlinear relationship between the local deformation at the contact point and the applied load acted on the contact components. For a linear guideway, the deformation of grooves is increased with applied load on the balls. This deformation makes the contact stiffness of the rolling element interface to rise in a nonlinear way [Johnson, (1985)]. In addition to the nonlinear stiffness, the contact angle is also changed. Such a variation in contact angle may affect not only capability of load rating, but also positioning accuracy. Although a number of studies have been made on linear guideway, little was studied about contact angle in theoretical approach. This research is intended to analysis the contact angle by using Hertzian theory and Lundeberg/Palmgren study in elastic deformation range of a linear guideway with rolling balls, angular contact, face-to-face arrangement and with preload setting. Therefore a reliable and reasonable theoretical method is performed for the contact angle of ball and grooves.

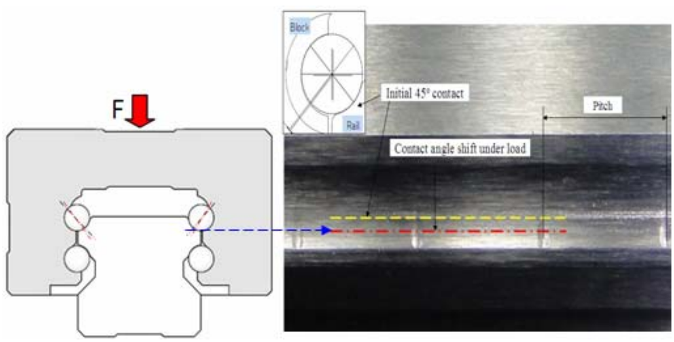

Figure 1: Contact angle changed by load applied

2 2 Theoretical approach

In this study, symmetric arrangement with simplistic model is employed and shown in Fig. 2. There are five assumptions: (1) All of contact deformations are in the elastic range. (2) System employed downward load F_a, preload f_p and initial contact angle α_1 (3) Vertical displacementδ_a, horizontal displacement δ_r and finial contact angle α_2 occurs under downward load applied. (4) Maximum deformation$\delta_{\max}$ occurs at maximum compression force $Q_{\max}$ on ball. (5) Same material of steel is used to make rail, block and ball. First, according to Fig. 2b and maximum deformation at ball can be expressed as follows:

$$\delta_{\max} = \delta_a \sin \alpha_2 - \delta_r \cos \alpha_2 \quad (1)$$

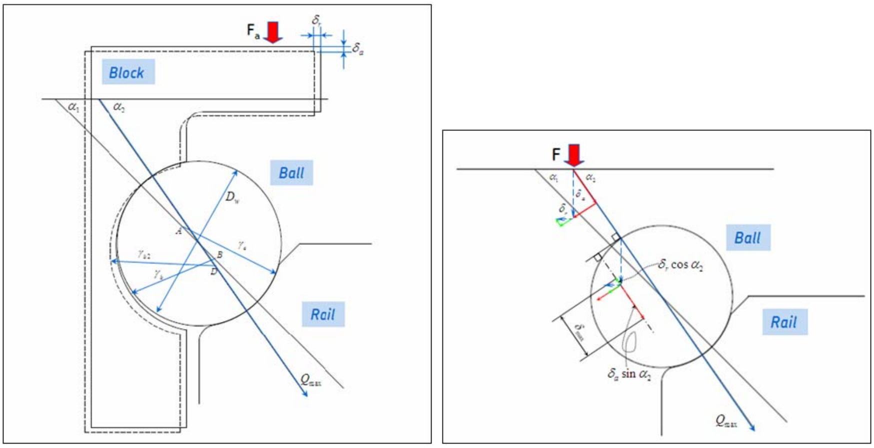

Figure 2: Simplified model of angular-contact ball linear guideway (a) under applied load (b) elastic deformation of ball under maximum normal direction force

According to Eq.1 and according to geometry of Fig. 1 can be rewrited as follows:

$$\delta_a \sin\alpha_2 = \delta_{\max} + \delta_r \cos\alpha_2 = \delta_{\max}\left(1 + \frac{\delta_r \cos\alpha_2}{\delta_{\max}}\right) = \delta_{\max}\left(1 - \frac{1}{\frac{\delta_a \tan\alpha_2}{\delta_r} - 1}\right) \quad (2)$$

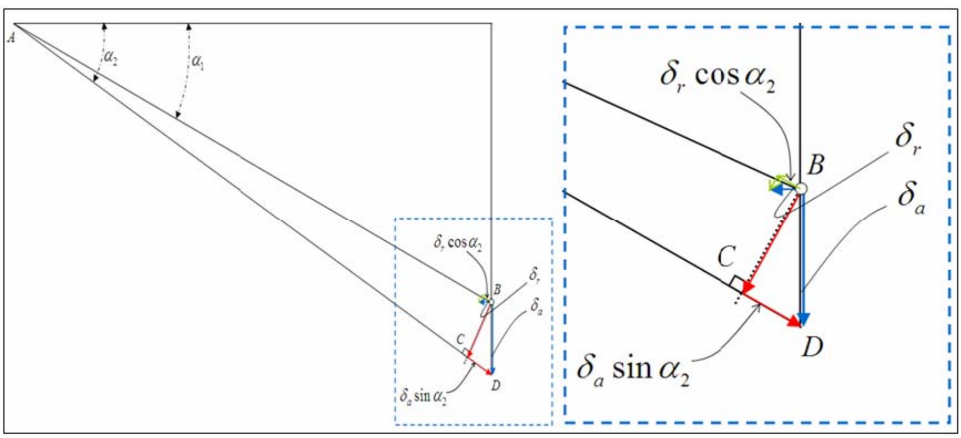

Figure 3: Elastic deformation of ball due to combined horizontal and vertical compression force

By contrast, Fig. 3 demonstrates relation of externally applied load is transmitted and deformation (in blue dash square) is enlarged in right. It is obvious that vertical deformation is a significant parameter than horizontal deformation for considered in analysis. Curvature center of block shifted vertically from point B to point D under downward load applied and contact angle increased from α_1 to α_2. Distance between the centers of curvature increased from $\overline{AB}$ to $\overline{AD}$. Accessed point C in $\overline{AD}$ and assume $\overline{AB} = \overline{AC} = S_g$, the relation can be expressed as follows:

$$\overline{AD} = S_g + \delta_a \sin\alpha_2 \tag{3}$$

$$S_g = r_b + r_e - D_w \tag{4}$$

In above equation, r_b is radius of groove at block and r_e is radius of groove at rail. Assume $\overline{BC}$ and $\overline{AC}$ at right angle, $\overline{AB}\cos\alpha_1 = \overline{AD}\cos\alpha_2$, the relation can be expressed as follows:

$$\frac{\cos\alpha_1}{\cos\alpha_2} = \frac{\overline{AD}}{\overline{AB}} = \frac{S_g + \delta_a \sin\alpha_2}{S_g} = 1 + \frac{\delta_a \sin\alpha_2}{S_g} \tag{5}$$

For the convenience of application in the rolling ball bearing, simplified formulae of the Hertzian theory are used. [Lunberg and Palmgren (1947); Junzo (2002)] Maximum deformation with point contact consisting of the same material and subjected to a maximum compression force $Q_{\max}$ can obtain as follows:

$$\delta_{\max} = e_\delta \sqrt[3]{Q_{\max}^2 \left(\sum\rho\right)} = c_\delta \frac{Q_{\max}^{2/3}}{D_w^{1/3}} = c_\delta D_w \left(\frac{Q_{\max}}{D_w^2}\right)^{2/3} \tag{6}$$

In above equation, e_δ is coefficient of contact and c_δ is coefficient of contact deformation. Table 1 shows the relation among conformity f_m and curvature difference $F(\rho)$. This assumption may allow for the simplification of the geometric relation.

Table 1: Coefficent of contact deformaion of steel ball

f_m	$F(\rho)$	$e_\delta(\times10^{-4})$	$c_\delta(\times10^{-4})$	f_m	$F(\rho)$	$e_\delta(\times10^{-4})$	$c_\delta(\times10^{-4})$
0.51	0.9615	1.510	3.830	0.55	0.8333	2.137	5.543
0.5125	0.9524	1.591	4.041	0.56	0.8065	2.194	5.719
0.515	0.9434	1.660	4.223	0.57	0.7813	2.251	6.021
0.5175	0.9346	1.718	4.377	0.58	0.7576	2.299	6.048
0.52	0.9259	1.770	4.517	0.59	0.7353	2.339	6.180
0.525	0.9091	1.856	4.750	0.60	0.7143	2.374	6.297
0.53	0.8929	1.928	4.948				

The linear guideway, radius of groove at block and rail are the same and can be expressed as follows:

$$r_e = r_b = f_m D_w \tag{7}$$

In ref. [Harris (2001)] indicated the compression force distributed equally among the rolling element under downward load can be expressed as:

$$Q_{\max} = \frac{F_a}{Z.\sin\alpha_2} \tag{8}$$

In above equation, Z is number of load-carrying balls. Considered the existing of preload and face to face arrangement in the linear gudieway, Eq. (8) can be expressed as follows:

$$Q_{\max} = \frac{F_a + f_p}{i.Z.\sin\alpha_2} \tag{9}$$

In above equation, f_p is preload comes from employ oversize balls in the system. i is number of load-carrying row.

After substituting Eq. (2), Eq. (4), Eq. (6), Eq. (7) and Eq. (9) into governing Eq. (5), the equation can be expressed as follows:

$$\frac{\cos\alpha_1}{\cos\alpha_2} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{F_a + f_p}{i.Z.\sin\alpha_2.D_w^2}\right)^{2/3}\left(1 - \frac{1}{\frac{\delta_a}{\delta_r}\tan\alpha_2 - 1}\right) \tag{10}$$

Considered horizontal displacement δ_r is not significant and ignored. Eq. (9) can be written as

$$\frac{\cos\alpha_1}{\cos\alpha_2} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{F_a + f_p}{i.Z.\sin\alpha_2.D_w^2}\right)^{2/3} \tag{11}$$

3 Result and discussion

According to theoretical equation showed in Eq. (11), a series of four diagrams illustrating related cases of loading and preloading to find the contact angle in linear gudieway, as shows in Fig. 4.

Case A: Zero downward load and zero preload employed

Associated boundary condition of $F_a = f_p = 0$ substituted into governing Eq. (11) and $\cos\alpha_1 = \cos\alpha_2$. It showed that ball diameter and conformity are not significant factors affect the contact angle.

Case B: Downward load and zero preload employed

Substituting boundary conditions of $F_a \neq 0$ and $f_p = 0$ into governing Eq. (11) and can be written as

$$\frac{\cos\alpha_1}{\cos\alpha_2} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{F_a}{i.Z.\sin\alpha_2.D_w^2}\right)^{2/3} \tag{12}$$

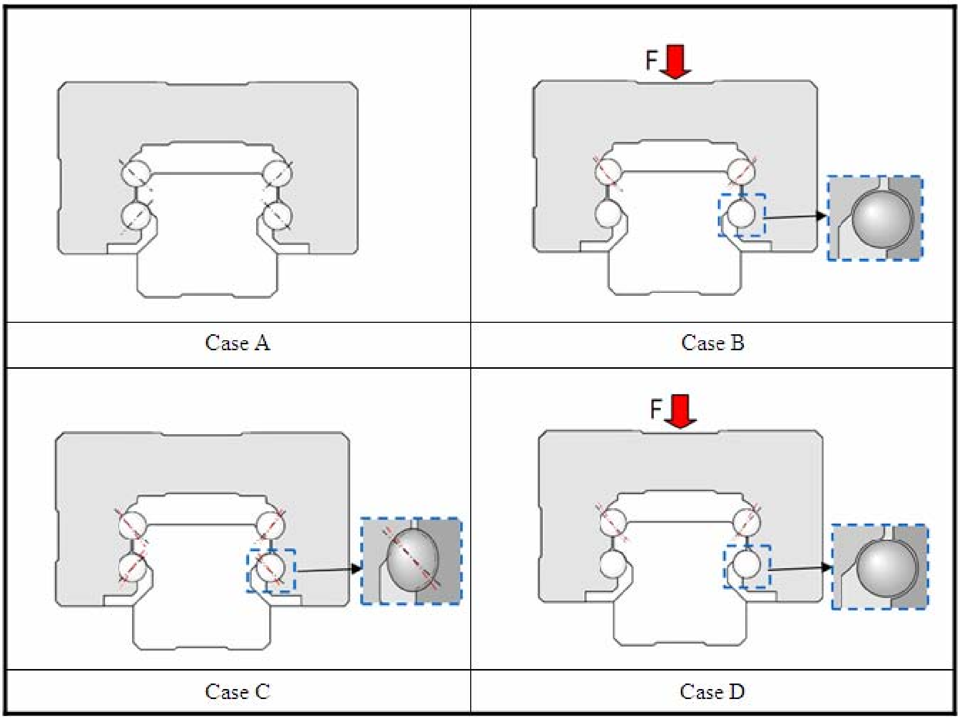

Figure 4: Contact angle changed in different condition

Eq. (12) shows that coefficient of contact deformation, downward load, ball diameter, number of load-carrying balls, number of load-carrying rows and conformity are significant factors affect the contact angle when system established. Compression force supported and contact angle increased at ball of upper two rows when load applied and ball of lower two rows were separated without any contact

Case C: Zero downward load and preload employed

Substituting boundary conditions of $F_a = 0$ and $f_p \neq 0$ into governing Eq. (11) and can be written as

$$\frac{\cos\alpha_1}{\cos\alpha_2} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{f_p}{i.Z.\sin\alpha_2.D_w^2}\right)^{2/3} \tag{13}$$

Eq. (13) shows that coefficient of contact deformation, preload, ball diameter, number of load-carrying balls, number of load-carrying rows and conformity are significant factors affect the contact angle.

Case D: Downward load applied and preload employed

Eq. (11) shows that coefficient of contact deformation, downward load, preload, ball diameter, number of load-carrying balls; number of load-carrying rows of balls and conformity are significant factors affect the contact angle. Compression force supported by upper rows of balls. The contact angle increased at ball of upper two rows when load applied. Contact angle of ball of lower two rows were decreased to α_1 until the ball separated from the groove without any contact when $f_p < 0$.

4 Conclusion

In this study, the contact angle of linear guideway under load and preload effects are developed by theoretical approach. The parameters study had been performed and discussed. Simplified solution of point contact of contact angle can be obtained using for convenience of calculation. It is useful for performance rating and modification such as dynamic load capability, static load capability, life time estimation, positioning accuracy and geometric relation in linear gudieway.

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