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A New Combined Scheme of Discrete Element Method and Meshless Method for Numerical Simulation of Impact Problems

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Abstract: In the present paper, a combined scheme of discrete element method (DEM) and meshless method for numerical simulation of impact problems is proposed. Based on the basic principle of continuum mechanics, an axisymmetric DEM framework is established for modeling the elastoplastic behavior of solid materials. A failure criterion is introduced to model the transformation from a continuum to a discontinuum. The friction force between contact elements is also considered after the failure appears. So our scheme can calculate not only the behavior of continuum and discontinuum, but also the transformation process from continuum to discontinuum. In addition, a meshless interpolation method is adopted to calculate the strain tensor, which is a non-local data fitting algorithm. Numerical simulations are carried out to validate our scheme. The numerical results agree well with those obtained by the finite element method (FEM) and the corresponding experiment respectively, which proves the feasibility and reliability of our computational scheme for analyzing the impact problems.

Keywords: Discrete element method; Meshless method; Impact problems; Axisymmetric framework; Algorithm of strain tensor

1 Introduction

Various mechanical phenomena can be observed in materials and structures under impact loading. Accurate description and modeling of those phenomena is a very challenging task for traditional mesh relied and continuum-based numerical methods, such as the finite element method (FEM) and the boundary element method (BEM). Some certain inherent drawbacks are well recognized: severe element distortion under large deformation; frequent re-meshing; mass losing while damage

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appears, etc. All the drawbacks are caused by their reliance on meshes and the unsuitability for dealing with discontinuum.

The discrete element method (DEM), which was first proposed by Cundall [Cundall (1971)], has been proved to be a powerful and versatile numerical tool for modeling behaviors of discontinuum. In recent years, Liu et al. [Liu, Gao and Tanimura (2004); Liu and Gao (2003); Liu and Liu (2006); Cheng, Liu and Liu (2009); Shan, Cheng, Liu, Liu and Chen (2009)] established a framework for developing the DEM as a general method.

The basic physical variables calculated in the DEM are internal forces between contact elements and displacements of elements, rather than strain tensor and stress tensor. An interpolation approach is adopted to determine the displacement gradient tensor, and then the strain tensor is calculated from the displacement gradient tensor. Most of the currently used interpolation approaches are linear interpolations, and a higher order interpolation approach is necessary to be introduced into the DEM.

In recent years, many meshless methods are introduced, which have been used in solving many engineering problems, such as elasto-statics and elasto-dynamics problems [Atluri and Zhu (2000); Han and Atluri (2004); Gao, Liu and Liu (2006)]. The meshless interpolation methods used in meshless methods are non-local data fitting algorithms entirely based on nodes, which are higher order interpolation approach suitable to the DEM.

2 Elastoplastic DEM model

As shown in Fig. 1 (B), for the axisymmetric problem, a continuum is separated into assemblages of circular ring elements which have the same section radius. Each element is surrounded by six elements as illustrated in Fig.1 (A), which forms regular hexagon lattice. The neighboring elements are linked by normal and tangential links and each link is combined by a spring and a plastic link as shown in Fig.1 (C).

As illustrated in Fig. 1 (D), taking the neighboring elements i and j into consideration, we can set up two Cartesian coordinate systems, one of which is for the global cylindrical coordinate system (r, z) and the other one is for the local coordinate system (n, s). Similar to the derivation of equations of motion presented in the reference [Cheng, Liu and Liu (2009)], the equations of motion for the axisymmetric problem can be derived.

Based on the basic principles of continuum mechanics, the spring constants k_n, k_s

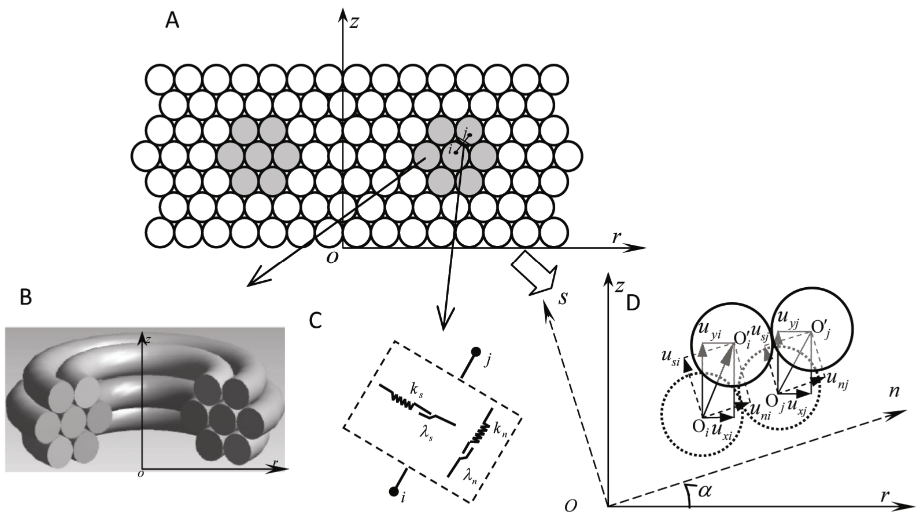


Figure 1: Discrete element model based on circular ring elements. A: discrete element discretization of a continuum. B: an assemblage of circular ring elements. C: normal and tangential links between neighboring elements. D: the global cylindrical coordinate system and the local coordinate system.

and the parameters of plastic links λ_n, λ_s are:

$$\begin{array}{l} k_n = \frac{EV_i}{3R_i^2(1+\nu)(1-2\nu)},\ k_s = \frac{(1-4\nu)EV_i}{3R_i^2(1+\nu)(1-2\nu)} \\ \lambda_n = \frac{1}{3(1-2\nu)},\ \lambda_s = \frac{5-24\nu+24\nu^2}{3(1-6\nu+8\nu^2)} \end{array}, \tag{1}$$

where V_i and R_i are the volume and the section radius of the circular ring element i, respectively, E and ν are the Young's modulus and the Poisson's ratio, respectively. The von Mises yield criterion and the flow rule of our DEM model can be expressed as

$$\begin{array}{l} F(f_n, f_s) = \frac{1}{2}\frac{\lambda_n}{k_n}f_n^2 + \frac{1}{2}\frac{\lambda_s}{k_s}f_s^2 - \frac{V_i\sigma_s^2}{18G} = 0 \\ \mathrm{d}u_n^p = \mathrm{d}\lambda\frac{\partial\varphi}{\partial f_n},\ \mathrm{d}u_s^p = \mathrm{d}\lambda\frac{\partial\varphi}{\partial f_s} \end{array}, \tag{2}$$

where $f_n = k_n u_n$, $f_s = k_s u_s$, σ_s is the yield stress, G is the shear modulus, $\mathrm{d}u_n^p$ and $\mathrm{d}u_s^p$ are the plastic displacement increments along normal and tangential direction, respectively, ϕ is the subsequent yield surface, $\mathrm{d}\lambda$ is a positive proportionality coefficient.

The loading and unloading function can be written as

$$\begin{cases} \varphi(f_n, f_s) < 0 & elasticity \\ \varphi(f_n, f_s) = 0, d\varphi \geq 0 & loading \\ \varphi(f_n, f_s) = 0, d\varphi < 0 & unloading \end{cases} \tag{3}$$

3 Failure criterion and determination of friction force

We introduce the maximum tensile stress criterion and the maximum compressive stress criterion as the failure criteria. For the numerical calculation after failure, the friction force in the tangential direction between contact elements is also considered.

4 A new algorithm of strain based on meshless interpolation method

In our algorithm, we consider the center of the element in the DEM as the node in the meshless method. Since displacements of all nodes have been calculated in the DEM, a meshless interpolation method will be adopted to calculate the strain tensor.

The trial function $u^h(x)$ at the point $x = [x, y]^T$ for the approximation of displacement is

$$\mathbf{u}^h(\mathbf{x}) = \mathbf{p}^\mathbf{T}(\mathbf{x})\mathbf{A}^{-\mathbf{1}}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u}_I, \ \mathbf{x} \in \Omega_\mathbf{x}, \tag{4}$$

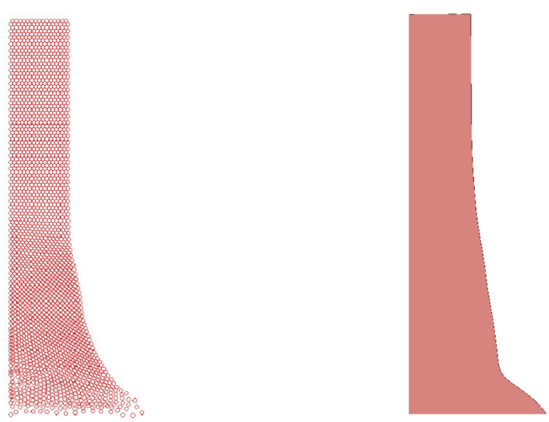
where Ω_x is a sub-domain around the point x, $\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w_I(\mathbf{x})\mathbf{p}(\mathbf{x}_I)\mathbf{p}^\mathbf{T}(\mathbf{x}_I)$, $\mathbf{B}(\mathbf{x}) = [w_1(\mathbf{x})\mathbf{p}(\mathbf{x_1}), w_2(\mathbf{x})\mathbf{p}(\mathbf{x_2}), \cdots, w_n(\mathbf{x})\mathbf{p}(\mathbf{x_n})]$, $p(x) = [p_1(x), p_2(x), \cdots, p_m(x)]^T$ is a complete basis vector, in which m is the number of terms in the basis, $\mathbf{x}_I$ and $\mathbf{u}_I$ are the position vector and the displacement vector of node I, respectively, $w_I(\mathbf{x}) = w(\mathbf{x} - \mathbf{x}_I)$ is the weight function. Then the strain tensor and the stress tensor can be obtained.

5 Numerical examples and discussion

Taylor bar test is used to measure dynamics yield stress, which is firstly designed by Taylor [Taylor (1948)]. We simulate a small cylindrical copper bar against a rigid planar wall by using the elastoplastic DEM model for the axisymmetric problem. The bar has an initial radius 3.2mm and an initial length of 32.4mm. The initial velocity of the bar is 227 m/s and the termination time of the problem is 80μs. In addition, the LS-Dyda software is also used to analyze this example. Tab. 1 lists the maximum radius and the residual length of the bar calculated by our scheme, the LS-Dyna and the corresponding experiment [Taylor (1948)], respectively. The results calculated by the DEM scheme agree well with those calculated by the FEM software and the experiment, which demonstrate that our DEM model has the same reliability with the FEM. Fig. 2 shows the deformed shapes at time $80\mu s$ obtained by the DEM and the LS-Dyna respectively. The numerical results show that the final deformed shape obtained by our scheme is in good agreement with that obtained by the FEM software.

Table 1: The maximum radius and the residual length of the bar calculated by the DEM, the LS-Dyna and the experiment.

	The maximum radius (mm)	The residual length (mm)
DEM	7.11	21.47
LS-Dyna	7.15	21.00
Experiment	7.21-7.24	21.42-21.44



(a) (b)

Figure 2: The deformed shapes at time $80\mu s$. (a) The DEM result. (b) LS-Dyna result.

Post compressive failure fragmentation (PCFF) occurs during the quasi-static compressive tests of brittle material specimens. Fig. 3 shows the explosive fragmentation of alumina cylinder under quasi-static compression obtained by the experiment [Zhou and Wang (2009)]. We use the scheme proposed in the present paper to simulate the PCFF process in the quasi-static compressive test of the alumina cylindrical specimen. The lower end of the cylinder is fixed and the upper end is applied a uniform speed 0.01 mm/s. The explosive fragmentation at various stages of the numerical simulation is shown in Fig. 4. The alumina cylinder explosively bursts into small pieces, which occurs in a very short time. Comparing Fig. 4 with Fig. 3, we can see that the numerical simulation results are in good agreement with the experiment results.

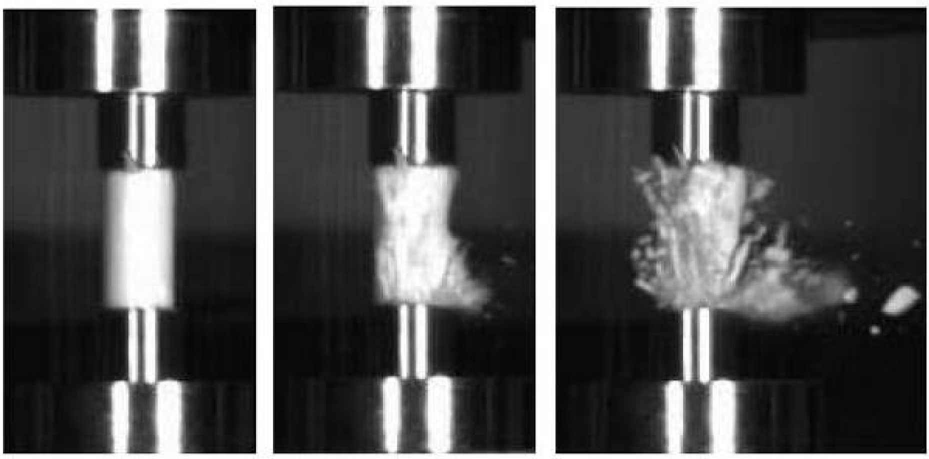


Figure 3: The explosive fragmentation of alumina cylinder under quasi-static compression obtained by the experiment [Zhou and Wang (2009)].

6 Conclusions

Summarizing the studies above, we can believe that the combined scheme of the DEM and the meshless method proposed in this paper is efficient for the numerical simulation of the impact problems. By comparing the numerical results with the corresponding results obtained by the FEM and experiments, the feasibility and reliability of our computational scheme are demonstrated. The numerical scheme proposed in the present paper offers a number of attractive features in simulating elastoplastic materials. Our combined scheme extends the application range of the DEM and also creates a new way for simulating the impact problems. Extending the present model to 3D problems and establishing computer models for viscoplastic materials will be carried out in the future.

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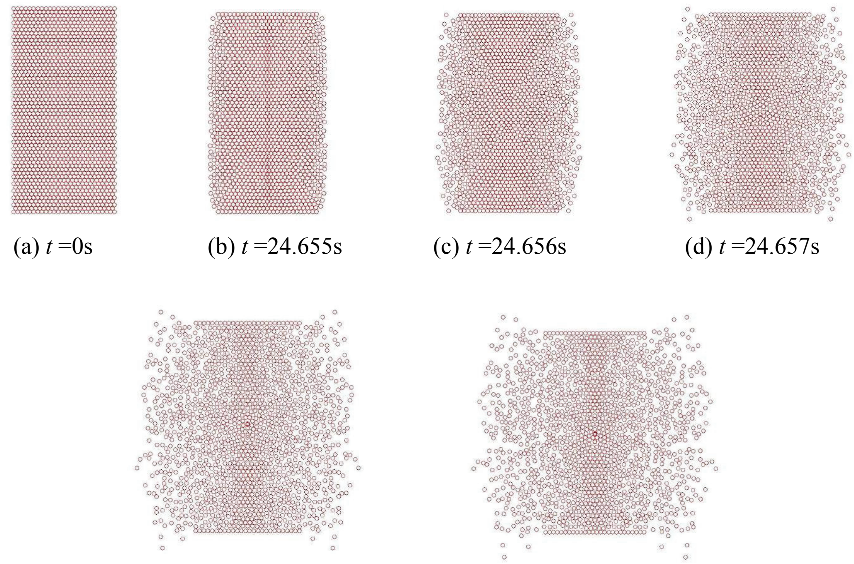

(a) t =0s (b) t =24.655s (c) t =24.656s (d) t =24.657s

(e) t =24.658s (f) t =24.659s

Figure 4: The explosive fragmentation of alumina cylinder under quasi-static compression obtained by the numerical simulation.

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