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Modified Approach for Optimum Position and Sizing of Piezoelectric Actuator for Steering of Parabolic Antenna

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Abstract: Various applications of piezoelectric actuators have been explored over the years. One such application is use of piezoelectric actuators for shape control of structures. In this paper, steering of parabolic antenna by deforming the antenna surface using piezoelectric actuators has been explored. Optimization based on Genetic Algorithm is carried out to find out optimum location, length and applied electric field to the piezoelectric actuators to achieve desired steering of antenna. Constraints are included in the objective function using penalty approach. Shell finite element model is used to determine deformations induced by the actuators. As the wavelength is sufficiently smaller than the aperture dimension, far field radiations are calculated using geometric optics. It is observed that new optimization approach gives better result.

Keywords: Piezoelectric actuators, parabolic antenna, radiation pattern, finite element, genetic algorithm.

1 Introduction

Piezoelectric actuators are used for shape control and active vibration control of structures. Various researchers explored use of piezoelectric actuators for shape control. One such research currently being explored is shape control of antennas. It has been demonstrated that piezoelectric actuators can deform the antenna shell in desired shape which can ultimately result in beam steering and shaping of antenna[1−4]. The antenna deformations influence the radiation pattern by affecting the path length of rays (phase difference). Washington[1] and Yoon and Washington[2] proposed the use of PVDF film and PZT strips respectively for shaping and steering of the cylindrical antennas. Yoon et al[3] used analytical solution based on Reisner's shell theory to demonstrate use of piezoelectric actuators to control the spherical (doubly curved) antenna's coverage area. Gupta et al[4] obtained steering and shap-

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ing of a cylindrical parabolic antenna by generating nearly linear phase variation at aperture plane, using piezoelectric actuators. Chen et al[5] investigated use of PVDF actuators for controlling surface accuracy of membrane reflector.

Optimal shape control involves decision on number of sensors/actuators, their sizes, location and voltage to be applied on piezoelectric actuators etc. Rao et al[6] proposed a genetic algorithm based optimization approach for placement of piezoelectric actuators on a structure. Kudikala et al[7] considered the problem of finding optimal distribution of piezoelectric actuators and corresponding actuation voltages for static shape control of a plate. Gupta et al[8] used genetic algorithm for optimal steering of paraboloid antenna. In this paper a modified approach for the optimization is presented for getting better results. It is proposed to modify objective function to include some constraints on beam steering, side lobe ratio and directivity.

2 Antenna Radiation Pattern

Performance of antenna is measured using its radiation pattern. The radiation pattern of an antenna refers to the spatial distribution of radiated energy. Fig. 1 shows a typical radiation pattern of an antenna. While radiation intensity is a measure of power radiated by the antenna in a given direction, directivity is a measure of the maximum intensity in the direction of the peak of the main lobe, and the beam width gives a measure of the area covered by the antenna. In the next section, calculation of radiation pattern is elaborated.

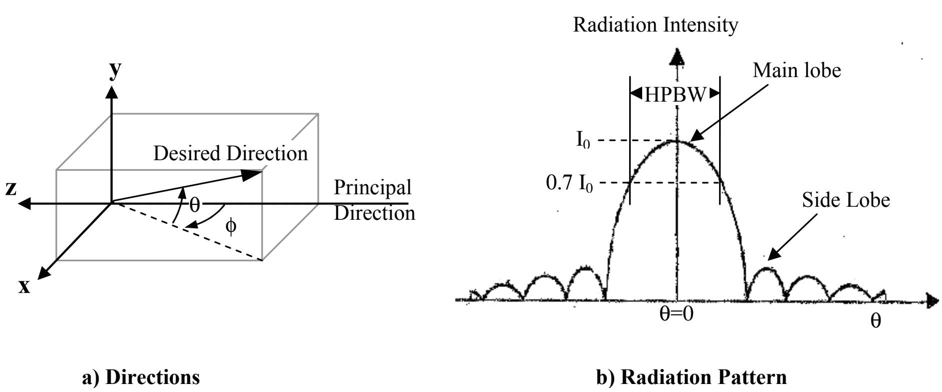

Figure 1: Typical Antenna Radiation Pattern (ϕ=constant)

2.1 Calculation of radiation pattern for parabolic antenna

Two techniques are generally used to determine the radiation pattern of a reflector antenna: *Aperture Distribution Method* (also known as Geometric Theory of Diffraction (GTD)) and *Current Distribution Method*. GTD is applicable only in cases where the aperture is very large when compared to the wavelength (> 40 times). For the antennas considered here where the aperture size of about 350 mm and wave length of 1mm, GTD is used for determining radiation.

A parabolic antenna is shown in Fig. 2(a). A ray from the point source (feed) at focus F meets the antenna at point G. The reflected ray from G goes parallel to principal axis and meets aperture at point P. The aperture in this case is circular in shape. Secondary waves from the aperture plane are radiated into the open space. Assuming uniform intensity and phase distribution over the aperture, the radiation intensity at the infinitely distant point Q is given by Eq. 1.

$$e(\theta,\phi) = \int\int Ae^{-j\frac{2\pi}{\lambda}(x\sin\phi\cos\theta + y\sin\theta)}dydx \qquad (1)$$

where A is the amplitude of secondary sources and the integration is over the circular aperture.

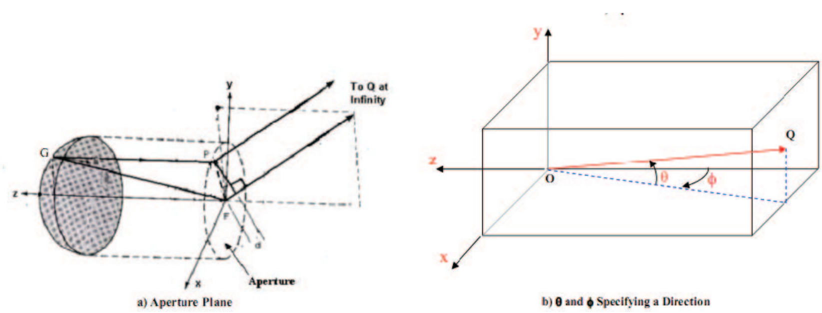

Figure 2: Parabolic Antenna (Balanis[9])

Due to a small deflection δz of the antenna perpendicular to aperture plane at location (x,y), the radiation pattern can be calculated using Eq. 2.

$$e(\theta,\phi) = \int\int Ae^{-\frac{2\pi}{\lambda}p(x,y)}e^{-j\frac{2\pi}{\lambda}(x\sin\phi\cos\theta + y\sin\theta)}dydx \qquad (2)$$

where, p(x,y) is the deflection induced change in length of path of a ray arriving at the point (x,y) on aperture.

It is assumed that the reflected rays travel parallel to axis after reflection (deformations are very small compared to other dimensions). A computer code written in

MATLAB is used for the calculation of the radiation pattern. First, finite element analysis is carried out to calculate deflection of different points on the antenna structure (mesh size of 16 x 64 elements) under piezo-actuation. The component of deflection perpendicular to aperture is taken for the purpose of calculating the radiation pattern. In order to save computation time, θ and ϕ are both varied from -2^o to $+2^o$ in an interval of 0.02^o which is found sufficient to capture the main lobe and sufficient number of side-lobes.

A far-field radiation pattern calculated for an undeformed parabolic shell (circular aperture) is plotted in Fig. 3. Here, the first side lobe has a height which is 13.5% (i.e. side-lobe ratio=7.4) of the main lobe. This is the expected value for circular aperture with uniform phase and uniform intensity distributions [Balanis[9]].

3 Steering of Parabolic Antenna

In this paper steering of antenna using piezoelectric actuators is explored. Steering refers to looking in a direction different from the original direction of antenna as shown in Fig. 4. Lexan®is taken as the material for antenna and PZT-5A is considered as piezoelectric actuators. Properties of these materials are listed in Table 1 and 2. It is assumed that the parabolic antenna has a focal length of 175 mm and a semi-cone angle of 60^o for numerical simulation (Fig. 5). At the apex, a small hole of 5 mm size is taken for fixing the antenna. Piezoelectric actuators are surface mounted on both sides of the antenna surface. In the current study, six numbers of symmetrically mounted actuators are considered.

Table 1: Material properties of antenna

Material	Lexan®9030[10]
Modulus of Elasticity (ISO527)	2.3 x10^9 N/m^2
Thickness	0.2 mm

Table 2: Properties of piezoelectric actuator

Property	**Units**	**PZT**[11]
Modulus of Elasticity	N/m^2	6.6 x 10^{10}
Strain Coefficient (d_{31}=d_{32})	m/V	-190x10^{-12}
Strain Coefficient (d_{33})	m/V	390x10^{-12}
Poisson's Ratiog–	0.178	
Densityg(ρp	Kg/m^3	7500
Max Voltage Before Depoling	V	300

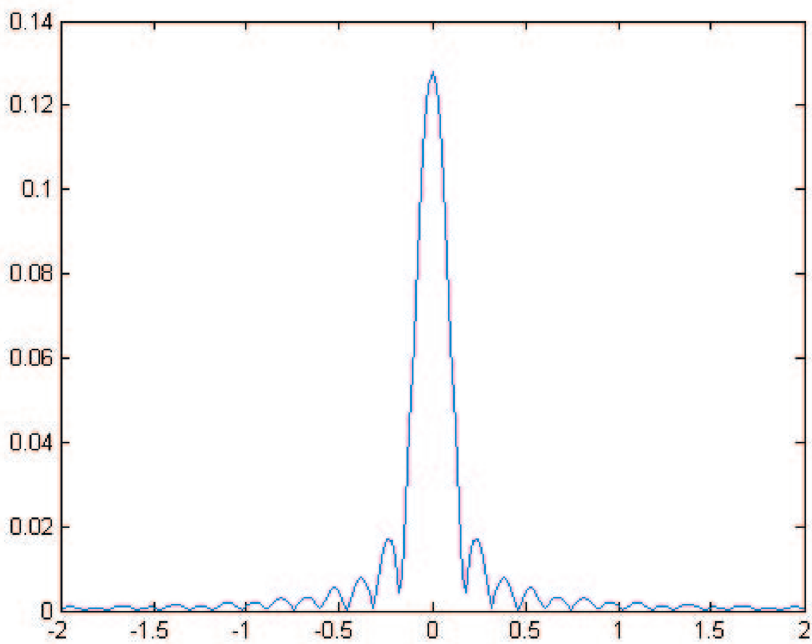

Figure 3: Radiation pattern for Parabolic Shell (along ϕ=0)

Based on a stability study carried out, it is found that a mesh size of 32x8 (i.e 32 elements along periphery and 8 elements along radial direction) gives sufficient accuracy. As this appears too coarse for sizing and locating actuators, a mesh size of 64x16 is considered in this analysis (Fig. 6). The circumferential size is limited to two elements as large sizes creates problem during mounting of actuators on curved surface of antenna. Thickness of the actuators is considered to be equal for all the six actuators. Deformation due to piezoelectric actuation depends on the electric field applied on the actuators. In the current study, it is assumed that the voltage applied on PZT actuator is limited to 300V (limit imposed due to depoling of actuators).

3.1 Finite element modeling of antenna shell

Deformation over the antenna surface is calculated using Finite Element Technique. Reduced shell element as proposed by Ahmed[12] and modified for piezoelectric actuated shell structures by Gupta, et al[13−14] is used. The FE formulation and its experimental validation have been discussed in detail in Gupta et al[13−14].

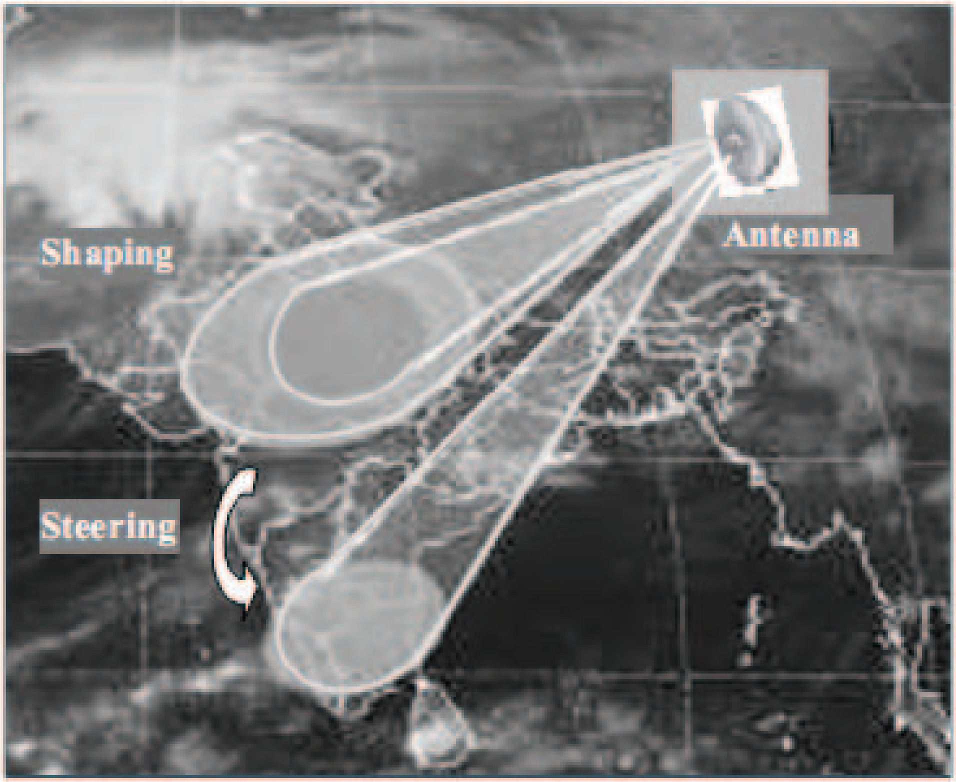

Figure 4: Antenna Steering

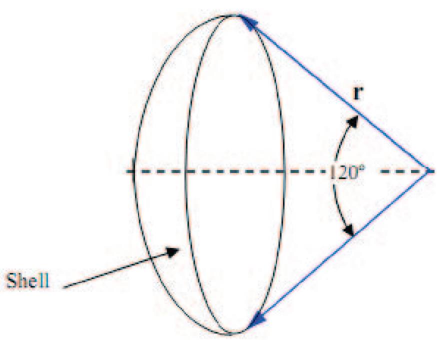

Figure 5: Parabolic Antenna Shell

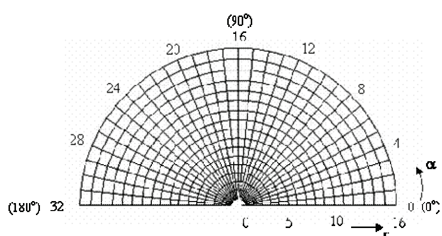

Figure 6: Mesh plot for Half Parabolic Antenna

4 Optimization

For parabolic antenna, steering depends on the actuator sizing, location and applied electric field. Computer based searches are necessary to obtain optimal solutions. An exhaustive search is first carried out to locate a pair of actuators for desired shift. This search reveals that the performance parameters are non-differentiable and multimodal in nature. The design variable set is discrete as it is assumed that actuators will cover full elements. Due to these reasons, gradient based optimization techniques cannot be used for optimization. Genetic Algorithm has been used previously for antenna optimization problems and has been chosen here too.

4.1 Problem Formulation

For the formulation of the optimization problem, two possibilities are:

i) Maximize shift, subject to a minimum quality for the beam.

ii) Maximize quality of beam for a specified shift along ϕ=0 line.

Here, quality of the beam is defined based on maximizing radiation intensity in a particular direction.

Earlier, Gupta, et al[6] optimized size, location and applied voltage for piezoelectric actuators based on the objective function

$$\text{Maximize intensity at } \theta_d(\phi_d = 0) \tag{3}$$

where, θ_d is the desired shift.

It was expected that at the optimal solution, the peak of the main lobe would be in the desired direction and hence the intensity is proportional to directivity. In many cases there was need to store whole data, as the optimum result in a shift away from the desired shift. To improve the optimization process, a new formulation is proposed for optimization -

$$\text{Maximize intensity at } \theta_d(\phi_d = 0) \tag{4}$$

where, θ_d is the desired shift.

Subject to

$$\theta_d = \theta_d \pm 0.02^o \tag{5}$$

$$SideLobeRatio > 2 \tag{6}$$

A constraint on side lobe ratio is applied as it was observed that sometimes, two peaks are observed near to one another with same intensity resulting in erroneous result.

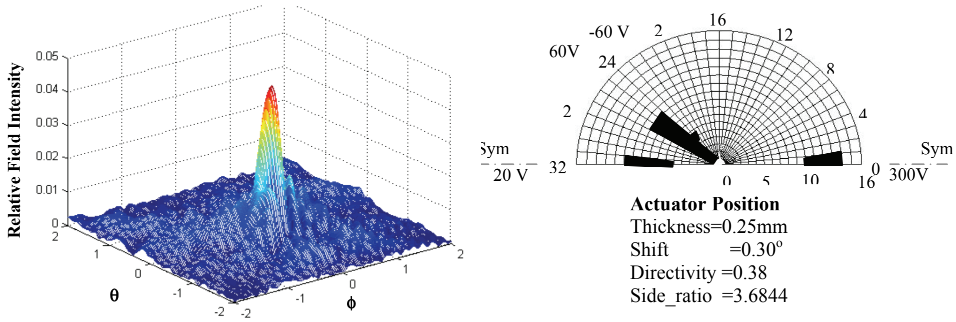

Figure 7: Optimum solution for 0.3^o Shift using PZT Actuators

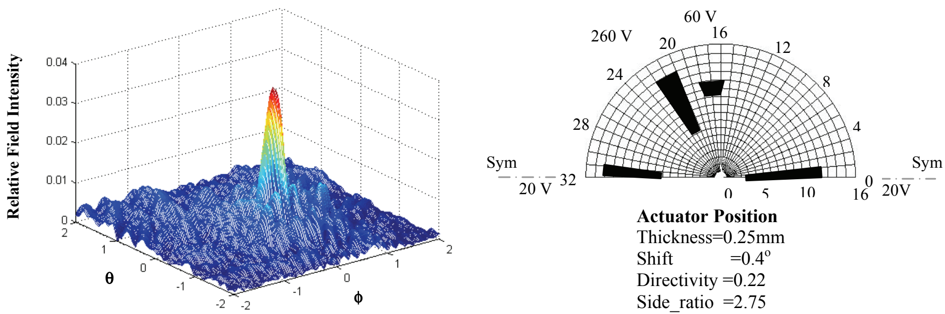

Figure 8: Optimum solution for 0.4^o Shift using PZT Actuators

To implement it in genetic algorithm, constraints were merged with objective function by assigning penalty on violation. The proposed objective function is

$$\text{Maximize } \mathrm{I}\left(\theta_{\mathrm{d}},0\right)*100+\frac{10}{\left(\left||\theta|-\theta_{\mathrm{d}}\right|\right)}+(slr-2)*100 \tag{7}$$

Where θ_d is desired shift

I(θ_d,0) is radiation intensity at θ_d

slr is side lobe ratio

4.2 Design variables chosen for optimization

The variables defining locations and sizes of actuators are: the lower and upper 'r' values and mid-α values (α-width is constant) of the two actuators on one side of α=0^o/180^o line (Fig. 6). The voltages applied to these two actuators form two additional variables. Two more actuators are placed with one on α=0^o radial line and the other at α=180^o radial line. For these actuators, the variables are the lower and upper 'r' values, and the electric field applied. In addition to the above variables, thickness of the actuator is also considered as variable. The total number of variables for the problem is 15. The applied electric field in terms of voltage is restricted to the range [-300V, 300V] and discretized into 256 levels. The 'r' and 'α' values are restricted within the geometry of antenna and discretized as shown in Fig. 6.

4.3 GA search details

Genetic Algorithms has been used for optimization. Parameters for optimization taken were mutation probability of 0.02 and two Cross-over probabilities Xover1 of 0.5 and Xover2 of 0.2. A population size of 500 is found to be effective for this problem. Repeating the search with different starting populations is found to be necessary to reach good solutions. All the intermediate results are stored along with all the performance parameters in a file so that good solutions with other combinations of performance parameters can also be picked up. In the next section optimal solutions obtained for two desired shifts (viz., 0.3^o and 0.4^o) are presented here.

4.4 Results with PZT actuators

Optimization is carried out for obtaining the maximum quality at shifts of 0.3^{o}and 0.4^o. For the desired shift of 0.3^o, the best result obtained has the peak at 0.3^o (Fig. 7) with peak field intensity of 0.38, 1/3rd of the undeformed antenna. For desired shift of 0.4^o, the best result has the peak at 0.4^o with a peak field intensity of 0.22, which is 1/4th of that of undeformed antenna (Fig. 8). Dark patches in the mesh plot correspond to locations of piezoelectric actuators. It can be seen that as the desired shift increases, the directivity and side-lobe ratio decrease. Results are summarized in Table 3.

Table 3: Optimum results for a desired shift using PZT actuator

Desired shiftq	Obtained shiftq	Relative Field Intensity	Widthq (degree)	Widthf (degree)	Side-lobe-ratio
0	0	1	0.20	0.20	7.12
Desired shift of 0.3^o					
0.30	0.30	0.38	0.38	0.20	3.68
Desired shift of 0.4^o					
0.40	0.40	0.22	0.40	0.20	2.75

5 Summary

In this paper an effort has been made to steer antennas using piezoelectric actuators. Constraints are applied on the obtained shift and side lobe ratio. A new objective function is proposed for optimization based on genetic algorithm. A penalty is imposed on objective function on violation of constraint. This gives a better solution to optimization problem. The variables used for optimization are location, size and voltage applied to piezoelectric actuators. It is observed that beam quality decreases as steering increases.

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