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Isogeometric Shape Optimal Design of Elastic Structures under Design-dependent Loads

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Abstract: Using an isogeometric approach, a continuum-based shape optimization method is developed for elasticity problems. To obtain efficient and precise adjoint shape sensitivity, precise normal and curvature information should be taken into account in shape sensitivity expressions, especially for design-dependent problems. In this approach, the basis functions generated from NURBS are directly used to construct a geometrically exact model in response and shape sensitivity analyses. Refinements and design changes are easily implemented within the isogeometric framework. The isogeometric design sensitivity analysis provides more accurate sensitivity of complex geometries including higher order terms such as normal and curvature. Also, it vastly simplifies the design modification without communicating with the CAD geometry during optimization process. Since the NURBS basis functions are used in both isogeometric response and sensitivity analyses, design modifications are easily carried out by the adjustment of control points. We demonstrate some numerical examples, where the accuracy and efficiency of the isogeometric sensitivity are verified by the comparison with finite difference one. Also, some examples of design–dependent shape optimization are demonstrated to verify the applicability and effectiveness of the proposed method.

Keywords: Isogeometric analysis, NURBS, Shape optimization, Design sensitivity analysis, Design-dependent problem

1 Introduction

Typically in finite element (FE) based engineering analysis, designs are embedded in CAD systems and FE meshes are generated from the CAD data. Geometric

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approximation inherent in the mesh may lead to accuracy problems in response analysis and more adversely in design sensitivity analysis. Even though a mesh is constructed, further refinement requires tedious communication with the CAD system during design iterations. The objective of isogeometric analysis is to develop an analysis framework, employing the same basis functions as used in the CAD systems and thus embedding the exact geometry.

A continuum-based shape optimization method using the isogeometric approach is developed for elasticity problems. To obtain efficient and precise adjoint shape sensitivity, correct normal and curvature information should be taken into account in shape sensitivity expressions, especially for design-dependent problems. However, in the conventional FE methods using linear interpolation, the normal and curvature are generally inaccurate or missing due to lack of inter-element continuity of design space. In the isogeometric approach, however, basis functions generated from NURBS are directly used to construct an exact geometric model in the response and shape sensitivity analyses.

The isogeometric design sensitivity analysis provides more accurate sensitivity of complex geometries including higher order effects such as normal and curvature. Also, it vastly simplifies the design modification without communicating with the CAD geometry during optimization. Since the NURBS basis functions are used in the isogeometric response and sensitivity analyses, design modifications are easily obtainable using the adjustment of control points. We present some demonstrative examples for shape optimization, where the accuracy and efficiency of isogeometric sensitivity is compared to finite differencing. The shape optimization of design–dependent structures is demonstrated to verify the applicability and effectiveness of the proposed method. Also, it turns out that the shape optimization method yields physically meaningful results.

2 Isogeometric design sensitivity analysis

In an isogeometric analysis, the shape function used in solution field is adopted directly from the B-spline functions in CAD geometry. The B-spline basis functions are defined, recursively, as

$$N_{i,0}(\xi) = \begin{cases} 1 & if\ \xi_i \leq \xi < \xi_{i+1} \\ 0 & otherwise \end{cases} \tag{1}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \quad p = 1,2,3,\cdots. \tag{2}$$

B-spline surfaces are obtained from the linear combination of B-spline basis and the coefficients or *control points*.

$$S(\blacksquare) = \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} L_i^p(\xi)M_j^q(\eta)N_k^r(\zeta)B_{i,j,k} \equiv \sum_I W_I(\blacksquare)B_I(x_1,x_2,x_3) \tag{3}$$

Using the principle of virtual work, an equilibrium equation for elasticity problems is expressed as

$$\int_\Omega c_{ijkl}z_{i,j}\bar{z}_{k,l}d\Omega = b_i d\Omega + \int_{\Gamma^t} pn_i\bar{z}_i d\Gamma, \forall \bar{\mathbf{z}} \in \bar{Z}, \tag{4}$$

under the body force $\boldsymbol{b}$ and the pressure $p\boldsymbol{n}$. The basis function in Equation (3) can be utilized to discretize the Equation (4) in the isogeometric approach. Taking first order variations of the bilinear strain energy and linear load forms with respect to the shape design parameter τ, the shape design sensitivity equation is derived as [Choi and Kim (2004)]

$$\begin{aligned}
&\int_\Omega c_{ijkl}\dot{z}_{i,j}\bar{z}_{k,l}d\Omega = \\
&\int_\Omega b_{i,m}\bar{z}_i V_m d\Omega + \int_\Omega (b_i\bar{z}_i)V_{j,j}d\Omega + \int_{\Gamma^N} p_{,j}V_j n_i\bar{z}_i + pV_{j,j}n_i\bar{z} - pV_{i,j}n_j\bar{z}d\Gamma \\
&\quad + \int_\Omega c_{ijkl}z_{i,m}V_{m,j}\bar{z}_{k,l}d\Omega + \int_\Omega c_{ijkl}z_{i,j}\bar{z}_{k,m}V_{m,l}d\Omega - \int_\Omega (c_{ijkl}z_{i,j}\bar{z}_{k,l})V_{m,m}d\Omega
\end{aligned} \tag{5}$$

Equations (4)-(5) can be accurately evaluated in the isogeometric approach but cannot in the finite element approach, due to geometric parameters.

In order to verify shape design sensitivity expressions for pressure loading cases, the quarter model of a circular pipe is introduced as shown in Figure 1. Internal pressure of P is applied inside the inner hole, and symmetric boundary condition is given. Due to the symmetry, only the displacement sensitivities in x-direction for control points 1 to 6 are considered. For the sensitivity verification, analytic sensitivities compared with finite difference sensitivities

In order to generate non-normal shape design velocity field, only three control points on the oblique line are perturbed as shown in Figure 2. When pressure loading is applied along the curved boundary, the direction of loading is simultaneously changed under this non-normal perturbation. In Table 1, analytic sensitivities are also compared with finite difference sensitivity. Analytic sensitivities for a pressure loading case show very good agreement compared with finite difference sensitivity as shown in the last column.

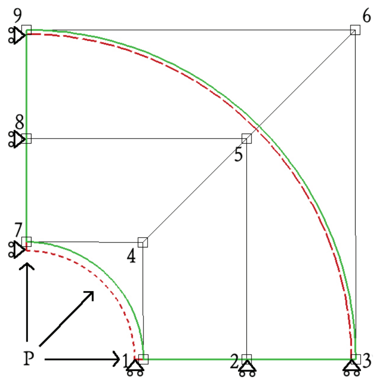


Figure 1: Shape sensitivity model

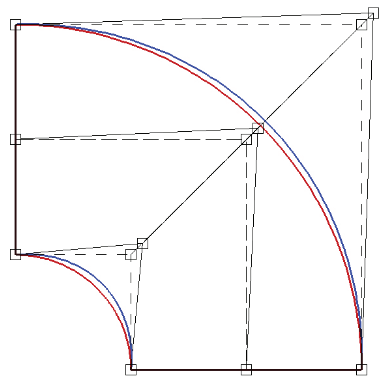


Figure 2: Non-normal design perturbation

Table 1: Shape design sensitivity for non-normal design perturbation

DOF	FDM	Analytical Sensitivity	%
1x	2.935e-5	2.938e-5	99.895
2x	1.283e-5	1.284e-5	99.971
3x	1.140e-5	1.140e-5	99.969
4x	1.866e-5	1.858e-5	99.924
5x	8.162e-6	8.181e-6	99.769
6x	1.409e-5	1.411e-5	99.870

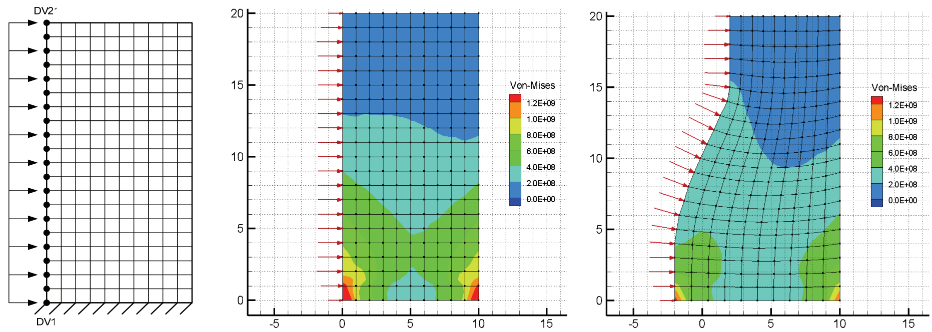
3 Numerical example

Based on the shape design sensitivity of pressure loading in Equation (5), the shape design optimization is performed for the initial rectangular shape in Figure 3. The model consists of 231 control points (262 DOF) and the order of a basis function is quadratic. The plane stress analysis is utilized with a thickness of 1 mm. Material properties are given as E=207.4Gpa and νÍ=0.3. There are 21 design variables along the left-hand side, and the uniform pressure loading is given on this side. For the given rectangular model, a shape design optimization is performed to minimized the compliance, satisfying the requirement that the volume is less than or equal to the initial volume. The optimization problem is stated as

$$\min C = \int_{\Omega} \mathbf{f}^{\mathbf{T}} \mathbf{z} d\Omega, \tag{6}$$

subject to

$$V = \int_{\Omega} d\Omega \leq V_{initial}. \tag{7}$$



(a) Initial mode (b) Initial model (c) Optimal model

Figure 3: Shape optimization model and result

The design sensitivity of compliance computed from Equation (5) is compared with the finite difference sensitivity in Table 2, where accurate sensitivities are observed in the last column.

Table 2: Shape design sensitivity of compliance

DV	FDM	DDM	%
1	8.30441E+02	8.30393E+02	99.99
5	2.95844E+02	2.95821E+02	99.99
10	8.15986E+01	8.15920E+01	99.99
16	5.96266E+00	5.96177E+00	99.99
21	-7.67315E+02	-7.67327E+02	100.00

The shape optimization that is based on the finite element method sometimes gives an unrealistic irregular shape [Braibant and Fluery (1984)]. Meanwhile, the optimal result in Figure 3 (c), obtained by the isogeometric approach shows a smooth boundary along the boundary where the pressure is applied, due to the design parameterization and an exact representation of pressure loading direction.

The optimization history of the given numerical model is presented in Figure 4. As shown, the uniform and fast convergence of optimization process is observed. After 6 sensitivity evaluations, the compliance is decreased from 1.3×10^8 to 7.7×10^7 (reduced by 39%) without volume change.

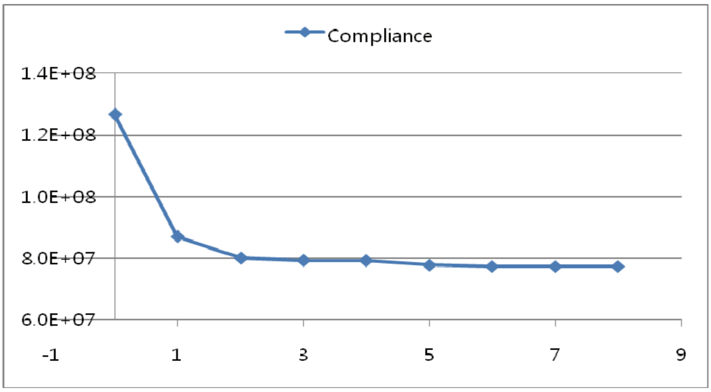


Figure 4: Optimization history

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