 SL, vol.8, no.1, pp.1-6, 2012

Design of auxetic microstructures using topology optimization

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Abstract: Microstructures can lead to homogeneous materials with negative Poisson's ratio, the so-called auxetics. An automatic way to create such microstructures is provided by topology optimization for compliant mechanisms. Nonconvexity is addressed by a suitable hybrid algorithm, based on differential evolution. This technique is demonstrated in the present paper with numerical examples.

Keywords: Auxetic materials, homogenization, topology optimization

1 Microstructure, homogenized behaviour and auxetics

The microstructure influences the homogenized overal behaviour of a material. The link is provided and studied by the theory of homogenization. In elasticity, Poisson's ratio measures the change of length (deformation) of an elastic material in the perpendicular to the loading direction. Materials with negative Poisson's ratio are characterized as auxetic materials, from the greek word 'afxetos', meaning the one that increases its shape or size Alderson and Alderson (2007). A number of mechanisms have been proposed for the construction of auxetic models Theocaris, Stavroulakis, and Panagiotopoulos (1997); Stavroulakis (2005). The automatic design of microstructures that lead to a specific homogeneour behaviour can be addressed by means of topology optimization, as it is shown here for the case of auxetic materials (see Kaminakis and Stavroulakis (2012) as well).

2 Topology optimization of structures and compliant mechanisms

A quite general structural optimization problem was formulated as an optimal material distribution problem inside a given design domain by, among the first, in Bendsoeand Kikuchi (1988). This way the form and the structural system of the resulting optimal structure does not depend on the initial choise, i.e. the experience

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and the imagination of the design engineer. This flexibility is of great importance for the design of innovative micro- and nano-structures. Topology optimization can be used for the design of compliant mechanisms that deliver motion in a predefined way Rahmatalla and Swan (2005). The deformation form of a periodicity cell used in numerical homogenization can be expressed by using this formulation.

The topology optimization problem is a "0-1" discrete optimization problem. First, the design domain is discretized into finite elements. Each finite element is equiped a design variable which is the density of the material. The problem can be relaxed by substituting the integer variables of density with continuous variables powered, for example, in penalty value $p \geq 3$ leading the design variables near to the ideally desired descrete values 0 and 1. The method is called SIMP (Solid Isotropic Material with Penalization).

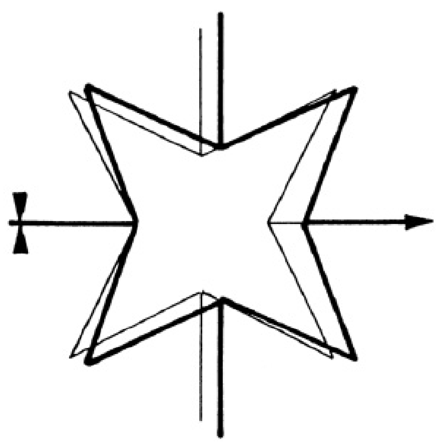


Figure 1: Star shaped, two dimensional representative cell made of beams and deformed shape indication an auxetic homogeneous.

3 Hybrid algorithm for the topology optimization problem

Topology optimization problems have many design variables and therefore are not suitable for general purpose numerical optimization algorithms. For certain cases (mainly flexibility optimization without additional constraints) the construction of local, element-wise update techniques can be very effective, following the lines of the old fully stressed design and optimality criteria methods.

Using the iterative local search method for the solution of the topology optimization problem we found out that the algorithm is very sensitive to the starting point.

In fact, starting from random initial material distributions, the algorithm ends at different final material distributions. This is due to the nonconvex nature of the underlying optimization problem, which may have several local minima. A global optimization algorithm is needed. Genuine global optimization techniques or evolutionary algorithms are not suitable for large scale problems.

For this purpose a hybrid algorithm is used here, which is based on Differential Evolution, for the global optimization part, and classical local iteration steps within each evaluation of the evolution algorithm (see Kaminakis and Stavroulakis (2012)).

4 Results using truss and plane elasticity discretizations

The conceptual design of a microstructure that macroscopically works as an auxetic material is presented. The microstructucture can be simulated by a compliant mechanism in such a way that when axial tensile forces are applied along onte direction, it expands in the perpendicular direction. The objective is to maximize the Poisson's ration that can be expressed as the fraction of the output displacement over the input displacement. The design domain is discretized by means of truss or plane elasticity finite elements. The input force is applied at teh lower left corner of a one forth of a rectangular periodicity cell with direction to the left. The design should produce and upward movement at the upper right corner. The design variables are the cross section areas of each element or the density of each twodimensional finite element. The goal is to determine the design variables in order to maximize the output displacement, subjected to the constraint that the volume of all material used is a fraction of the totally available volume.

For the iterative local search the maximum number of iterations was set equal to 200 and the penalty parameter was set equal to 3. For the DE, the maximum number of generations was 100, and the population size was 16, the mutation factor 1,5 and the recombination factor 0.9, respectivelly (see Kaminakis and Stavroulakis (2012) for technical details).

The implementation has been done with Matlab R2009b on a 8-core, 64bit workstation running Windows Server Enterprice, with the sparse matrix option and the parallel computing offered by Matlab.

4.1 Design domain definition

The design domain is defined as a square ABCD. Input forced is placed on point "D". The output of the mechanism is placed on point "B" on the vertical axis. The domain is illustrated in figure 2.

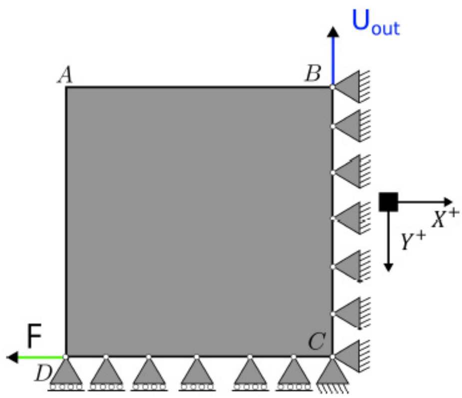
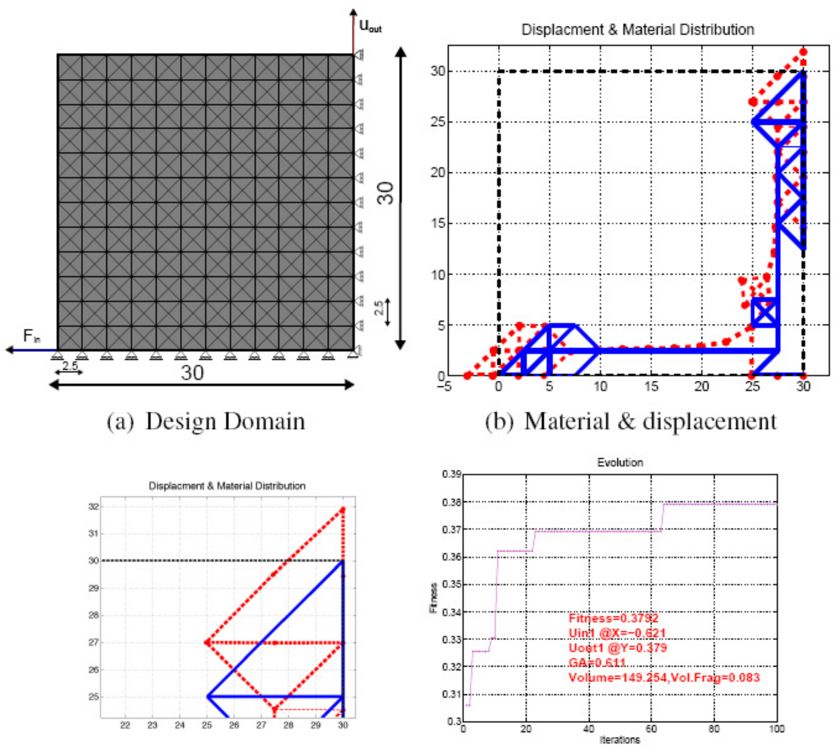


Figure 2: Design Domain



(a) Design Domain (b) Material & displacement

(c) Material & displacement detailed (d) Evolution

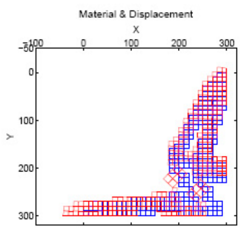
Figure 3: Results: 12x12 subcells, Volume fraction: 8.3% (3*x*3 : 30%)

4.2 A 12x12 cell using truss elements and volume fraction equal to 8.3 per cent

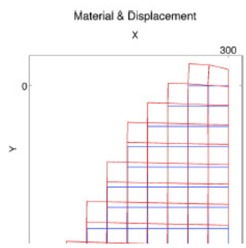
In figure 3 are shown the results for the 12x12 mesh with volume fraction 8.3%. Each subcell has length and width equal to 2.5 unit. The Poisson's ratio is equal to -0.611.

4.3 A 30x30 cell using plane stress elements and volume fraction equal to 30%

Figure 4 demostrates the results for a 30x30 plane finite elements design domain. The goal in this case is filling the 30% of the whole domain with material.

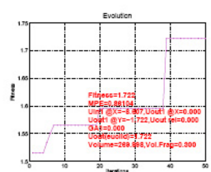


(a) Material & displacement



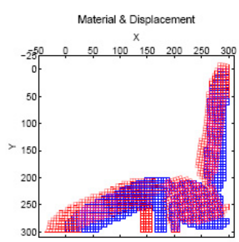
(b) Material & displacement detailed

(c) Final Topology

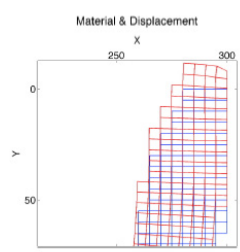


(d) Evolution

Figure 4: A resulted topology for a 30x30 design domain, with Volume goal at 30%



(a) Material & displacement



(b) Material & displacement detailed

(c) Final Topology

Figure 5: A resulted topology for a 60x60 design domain, with Volume goal at 30%

4.4 A 60x60 cell using plane stress elements and volume fraction equal to 30%

Figure 4 demostrates the results for a 60x60 plane finite elements design domain. The goal in this case is filling the 30% of the whole domain with material.

5 Conclusions

Topology optimization for compliant structures, taking into account nonconvexity issues, has been used for the design of microstructures leading to auxetic behaviour. Mesh refinement lead to much richer microstructures. Systematic convergence to a given microstructure would require additional precautions and will be studied in the future. Extension to other, nonlinear or multiphysics problems is also possible.

Acknowledgement

This research has been co-financed by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Heracleitus II. Investing in knowledge society through the European Social Fund.

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