

# An Experimental Study on Crack Bifurcation by Optical Methods

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**Abstract:** Two kinds of experiments are described on rapid crack bifurcation. One is static experiments with stationary bifurcated cracks. The static experiments demonstrate *COD* method that measures crack opening displacement, *COD*, to obtain stress intensity factor of bifurcated cracks. Moirè interferometry is used to measure *COD*. The other is dynamic experiments on bifurcation of fast propagating cracks. High-speed holographic microscopy is applied to take photographs of rapidly bifurcating cracks and to measure *COD* of the cracks. The measured *COD*s give the energy release rate of bifurcating cracks through the *COD* method developed in the static experiments. The experimental results say that energy release rate increases gradually and continuously across the bifurcation point.

**Keywords:** Fracture mechanics, crack bifurcation, fast propagating cracks, crack opening displacement, stress intensity factor, energy release rate.

## 1 Introduction

When brittle materials break under external forces, fast propagating cracks often appear and propagate at a speed more than several hundred meters per second. When crack speed is high enough, a fast propagating crack bifurcates into two cracks suddenly (Yoffe (1951), Ravi-Chandar and Knauss (1984), Sharon and Fineberg (1999)). Bifurcation is one of the characteristic features of fast propagating cracks, however, its mechanism has not been fully understood yet.

One of some interesting questions is on energy release rate of bifurcating cracks. One crack is propagating before bifurcation, and two cracks are propagating simultaneously after bifurcation. A question is whether the energy release rate of the crack becomes twice at the bifurcation point discontinuously or not.

The present paper describes two kinds of experiments on rapid crack bifurcation to answer the question above.

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One is static experiments with stationary bifurcated cracks in plate specimens (Suzuki and Miyashita (2008), Suzuki, Miyashita, Kimura and Nishikita (2009)). Through the static experiments, a new method is developed to know the energy release rate of bifurcated cracks. The method is called *COD* method since it measures crack opening displacement, *COD*, to obtain energy release rate. Moirè interferometry is used to measure *COD*.

The other is dynamic experiments on bifurcation of fast propagating cracks (Suzuki and Sakaue (2004), Suzuki, Sakaue and Iwanaga (2007)). High-speed holographic microscopy is applied to take photographs of rapidly bifurcating cracks and to measure *COD* of the cracks. The measured *CODs* give the energy release rate of the rapidly bifurcating cracks through the *COD* method developed in the static experiments.

## 2 Theory of COD method

### 2.1 Energy release rate before bifurcation

Figure 1(a) illustrates a single crack. The theory of dynamic fracture mechanics says that the *COD* of a single crack is proportional to the square root of distance  $r$  from the crack tip,

$$COD = K_I(v)L(v)\sqrt{r} \quad (1)$$

$$G(v) = A_I(v)K_I(v)^2 \quad (2)$$

where  $K_I(v)$  is dynamic stress intensity factor,  $G(v)$  is energy release rate,  $v$  is crack speed, and  $L(v)$  and  $A_I(v)$  are functions of the crack speed (Freund (1990)). One can know the dynamic stress intensity factor  $K_I(v)$  of a single crack through Eq.(1) by measuring *COD* along the crack at various position of  $r$ . Energy release rate  $G(v)$  is obtained through Eq.(2). This is the method to measure energy release rate before bifurcation. Substituting  $v=0$  into the above equations, one can obtain the expressions for stationary cracks.

### 2.2 Energy release rate after bifurcation

Figure 1(b) illustrates a bifurcated crack. If bifurcation angle  $\phi$  tends to zero, the bifurcated crack becomes a single crack. In the limit of  $\phi \rightarrow 0$ , the *COD* of the mother crack must be proportional to  $\sqrt{r}$ . It can thus be expected that *CODs* of mother cracks are approximately proportional to  $\sqrt{r}$ , if bifurcation angle  $\phi$  is small enough and if the lengths of the two branch cracks are much shorter than the whole length of the crack. Consequently, one can obtain energy release rate  $G(v)$  by measuring the *COD* of the mother crack, if the measured *COD* is approximately

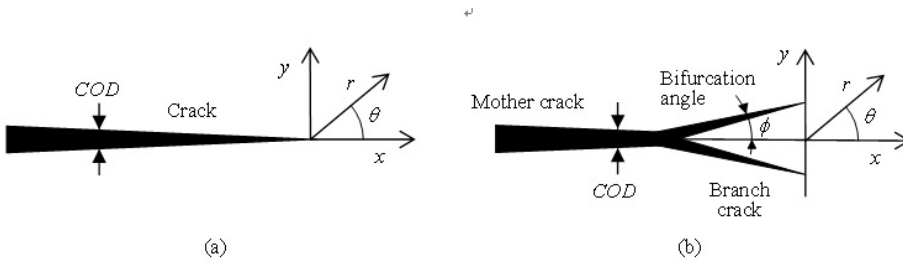


Figure 1: (a) Single crack and its  $COD$ . (b) Symmetrically bifurcated crack. The angle  $\phi$  is the bifurcation angle of a branch crack.

proportional to  $\sqrt{r}$  under the above condition. This is the method to obtain the energy release rate after bifurcation.

### 3 Experimental methods

#### 3.1 Static experiments with Moirè interferometry

The present study uses PMMA plate specimens for static experiment, which is 250mm in length, 250mm in width and 3mm in thickness. Bifurcated notches are used instead of cracks. The length of the bifurcated notches is 125mm. Moirè interferometry (Post (1987)) is used to measure the opening displacement of the bifurcated notch. The opening displacement of the notch is also called  $COD$  in the present study.

#### 3.2 Dynamic experiments with high-speed holographic microscopy

High-speed holographic microscopy is applied to measure the crack opening displacement of rapidly bifurcating cracks. When a crack is rapidly bifurcating in a specimen, three holograms of the crack are recorded successively on a photographic plate with three pulsed lasers. After development, illuminated with a reconstruction beam, every hologram reconstructs their crack images separately. The reconstructed images are photographed through a conventional microscope. Crack opening displacement  $COD$  of the crack can be measured on the microscopic photographs.

## 4 Results

### 4.1 Static experiments with stationary bifurcated crack

Figure 2 shows an example of Moiré interference fringes around notches. The length of two branch cracks is 6mm, and the applied tensile force to the specimen is 304N. The interference fringes are the equi-displacement lines in y-direction (see Fig.1). The displacement between each fringe and the next is  $1.02\mu\text{m}$ . The  $COD$  of the mother notch is given by measuring the difference of fringe orders along the notch.

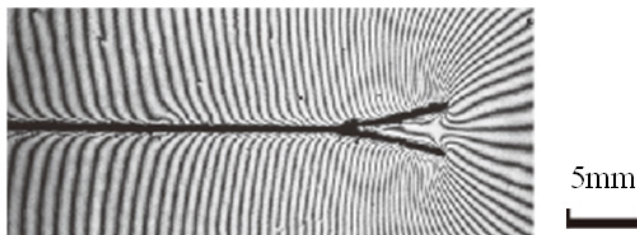


Figure 2: Moiré interference fringes around a bifurcated notch. The branch length is 6mm

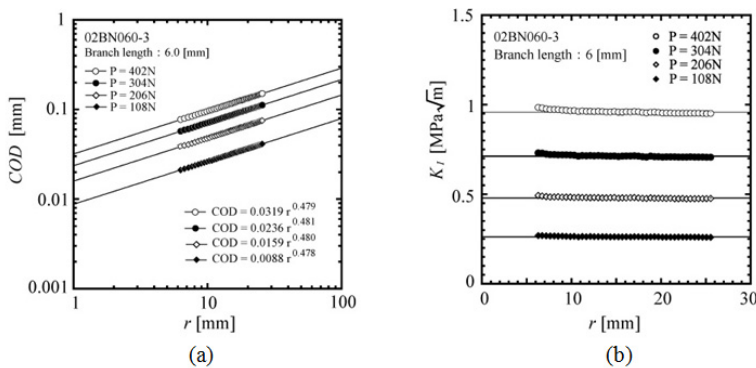


Figure 3: (a)  $COD_M$  vs.  $r$ , (b)  $K_I$  vs  $r$ , of a bifurcated notch.

Figure 3(a) shows opening displacement  $COD_M$  of the mother notch of the bifurcated notch in Fig.5 as a function of the distance  $r$ . Figure 3(a) demonstrates that  $COD_M$  is proportional to 0.48th power of  $r$ , which exponent of 0.48 is very close to 0.5. It can thus be said that  $COD_M$  is approximately proportional to  $\sqrt{r}$ .

Figure 3(b) is the stress intensity factor  $K_I$  obtained from the  $COD_M$  in Fig.3(a) with assuming that  $COD_M$  is proportional to  $\sqrt{r}$ . The  $K_I$  values are almost kept constant even though the distance  $r$  varies. This fact clearly says that the  $COD$  method described in section 2.2 is correct, and energy release rate can be obtained by the measurement of  $COD_M$  of the bifurcated cracks.

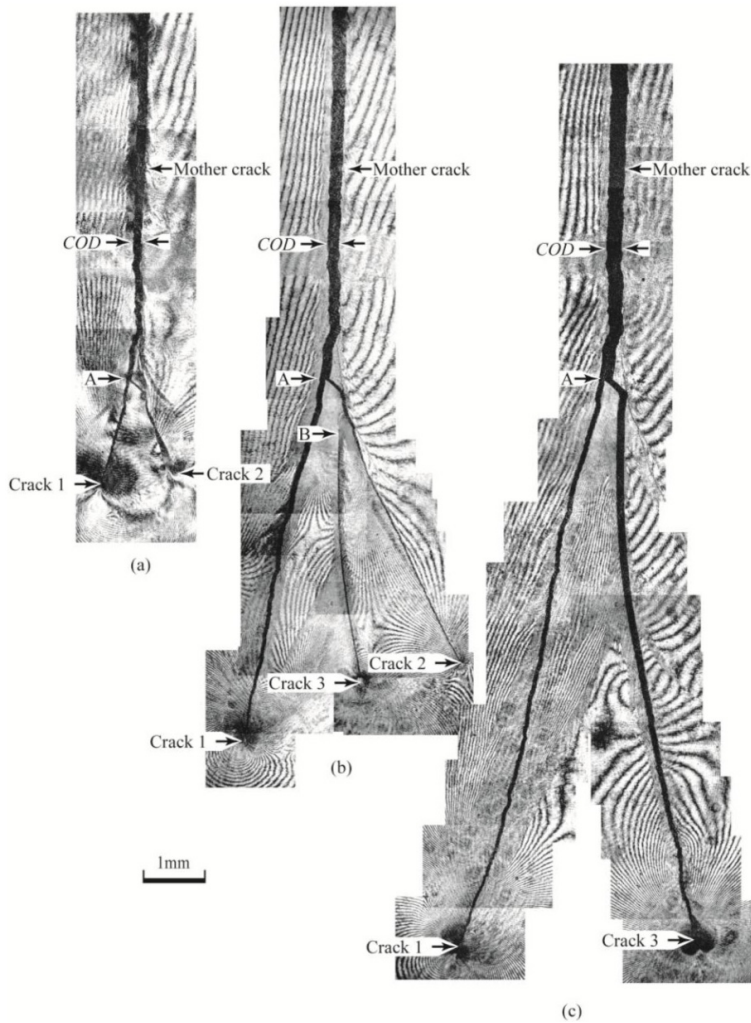


Figure 4: Rapidly bifurcating crack in Homalite 100. Crack speed is 543m/s and frame interval is about  $7\mu\text{s}$ .

#### 4.2 Dynamic experiments on rapidly bifurcating cracks

Figure 4 shows an example of three successive microscopic photographs of rapidly bifurcating cracks in Homalite 100 taken by high-speed holographic microscopy. The crack speed can be measured from the photographs and is 545m/s. The crack bifurcated into two cracks at point A. The first, the second and the third frame were photographed at  $3.0\mu\text{s}$ ,  $10.2\mu\text{s}$  and  $16.8\mu\text{s}$  after bifurcation respectively.

The first frame shows the crack bifurcated into crack 1 and crack 2 at point A. The second frame indicates that crack 1 and crack 2 propagated downward and crack 3 appeared between them after the first frame. Crack 3 seems to have appeared at point B and have propagated downward. Crack 3 is not connected to any other cracks, accordingly, crack 3 may be a branch crack that bifurcated from mother crack on the opposite surface of the specimen. In the third frame crack 2 has stopped propagation and crack 1 and crack 3 are propagating. Crack 1 and crack 3 kept propagation and broke the specimen into three pieces. The photographs in Fig.4 say that the rapid crack bifurcation is not simple two dimensional phenomena but complex three dimensional ones.

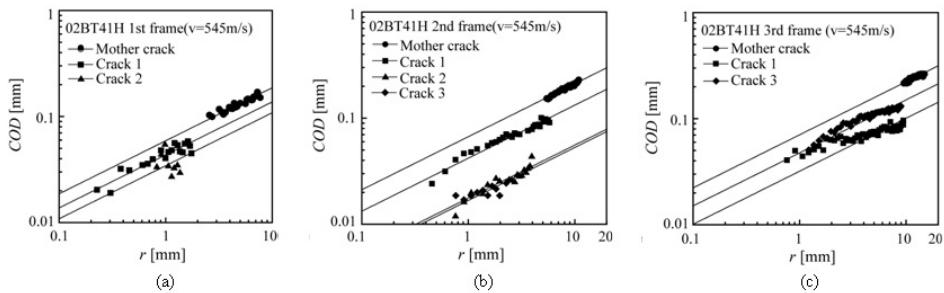


Figure 5: Crack opening displacement  $COD$  as a function of distance  $r$  from crack tip.

Figure 5(a), (b) and (c) indicate crack opening displacement of the crack shown in Fig.4(a), (b) and (c). The dark circles denote the  $COD$ s of the mother crack. The measurement results demonstrate that the  $COD$  of the mother crack is approximately proportional to the square root of

distance  $r$  from the crack tip. Consequently the dynamic energy release rate  $G$  can be obtained from the  $COD$  of the mother crack through Eq. (1) and (2). The  $COD$ s of branch cracks are not always proportional to the square root of distance  $r$  from each tip of branch cracks.

Figure 6 shows the energy release rate of bifurcating cracks. The horizontal axis

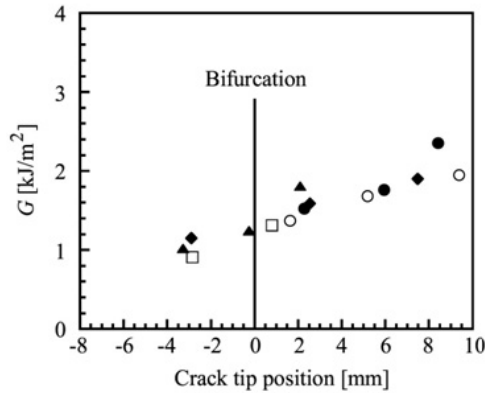


Figure 6: Energy release rate vs. crack tip position (Suzuki, Sakaue and Iwanaga (2007)).

is the crack tip position with respect to the bifurcation point. Energy release rate increases gradually across the bifurcation point, and there is no discontinuity at the bifurcation point. This fact can be explained by the three-dimensional structure of bifurcation process shown in the photographs in Fig.4.

## 5 Conclusions

1. The opening displacement  $COD_M$  of the mother crack of a stationary bifurcated crack is approximately proportional to the square root of the distance  $r$  from the crack tip.
2. The opening displacement  $COD_M$  of mother crack of a rapidly bifurcating crack is also proportional to the square root of the distance  $r$  from the crack tip approximately.
3. Measured  $COD_M$  gives energy release rate  $G$  of the bifurcated crack.
4. The energy release rate increases gradually and continuously across the bifurcation point and there is no discontinuous jump of it.

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