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A Unique Constitutive Model for Soils in Landslide Analysis

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Abstract: In this contribution, an original constitutive model is proposed to describe, within a unique framework, the initiation, the propagation and the arrest phases of landslides. The model is built such that it is enable to model in one hand a stable stage with an elasto-plastic behaviour, and in another hand a viscous dominated behaviour. The transition between the two behaviours is performed by means of the second order work stability criterion. This model is applied for an undrained triaxial test, in which the stress invariant consistently falls after the transition.

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1 Landslides Modelling

1.1 Landslides from a mechanical point a view

In a general framework 'landslide' stands for a ground movement due to gravity forces, but it gathers in fact phenomena of very different dynamics and magnitude. One can mention for example a very fast landslide that happened in Sarno and Quindicci (Italy, 1998) with a velocity of about 5 to 20 m/s, and a very slow one which still keeps on going at la Clapière (French Alps), advancing 1 to 10 *cm* a year (Fig. 1). This large velocity range can be explained notably by the soil composition: firstly the saturation degree can vary between 10 and 99% [Coussot and Meunier (1996)], secondly the solid fraction can be constituted of either fine and cohesive particles, frictional grains, or even metric blocks. All these differences justify the lot of denominations found in the literature: mudflows, debris flows, solifluction. . . The common points between these phenomena finally lie in the soil loss of stability and the transition from a static to a dynamic or quasi-static state. This changeover of the intrinsic behaviour can be related to a mechanical failure.

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1.2 Numerical approaches for landslides modelling

The modelling of landslides requires to accurately track the history variables involved in elasto-plasticity (in situ soils), and also to describe large transformations (flowing soils). Most of the numerical methods are well suited to deal with only one of these requirements, but very few are able to take both into account. For instance, methods using a finite element Lagrangian description are more adapted to describe elastoplasticity, whereas those using an Eulerian description are more used to model fluid. The global model proposed here has been developed in a particular numerical method, benefitting from both Lagrangian and Eulerian advantages: the Finite Element Method with Lagrangian Integration Points [Moresi, Dufour, and Mühlhaus (2002, 2003)]. This method is based on one hand, on an Eulerian FE grid where nodal velocities are computed, and on another hand, on a set of Lagrangian particles that carry material history variables. Those particles are also used as integration points when computing the elementary viscous matrix. Furthermore, at the end of a timestep, they are advected by interpolation of nodal velocities to an updated configuration.

Figure 1: Diversity in landslides velocity: examples of La Clapière (a,b) and Sarno (c)

2 A Pre-failure Behaviour: Elasto-plastic Law 'Plasol'

The elasto-plastic law named Plasol reproduces the main characteristic of soils:

- Firstly, it is based on a Van Eekelen plastic criterion close to the Mohr Coulomb one without singularities. It follows [Barnichon (1998)]:

$$F = J_{2\sigma} + m\left(J_{1\sigma} - \frac{3c}{\tan\varphi_c}\right) = 0 \text{ with: } m = a(1+b\sin 3\theta)^n \text{ and: } \sin 3\theta = -\left(\frac{3\sigma 3}{2}\frac{J_{3\sigma}}{J_{2\sigma}^3}\right) \tag{1}$$

$J_{1\sigma}$, $J_{2\sigma}$ and $J_{3\sigma}$ are the three invariants of the Cauchy stress tensor, φ_c is the friction angle in compression, n is a dimensionless number usually taken as -0,229, a and b are functions of the friction angles in compression and extension φ_c and φ_e. Those angles together with the Lode's angle θ modify the trace of the criterion in the deviatoric plane, in order not to have a circle but a shape whose radius is not constant.

- Secondly, it is able to represent hardening. Along the plastic path the plastic parameters (friction and dilatancy angles) vary -according to the equivalent plastic strain- between an initial (index 0) and a final (index f) values which respectively define using eq (1) the elastic limit and the plastic failure criterion.

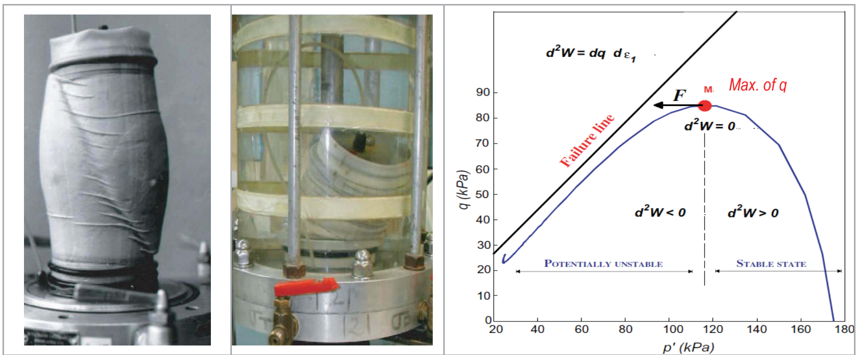
- Finally, Plasol allows to describe a non associated plastic flow, i.e. not perpendicular to the surface defined by F (eq. (1)) but with a dilatancy angle ψ. This specific nonassociativity of soils is notably determinant for the failure detection.

3 A Failure Criterion: The Second-Order Work

3.1 A limitation of the elasto-plasticity theory

Many communities in solid mechanics are focusing on failure and instability issues. Nowadays, the more developed criterion to define the failure state is the plastic limit criterion. It can be formalized this way: let $\underline{\underline{M}}$ be the constitutive matrix linking the stress and strain increments ($\underline{d\sigma} = \underline{\underline{M}}\underline{d\varepsilon}$), then the failure condition is $\det(\underline{\underline{M}}) = 0$, i.e. $\underline{d\sigma}$ is becoming null for a non-zero strain increment $\underline{d\varepsilon}$. In other words, a bounded stress increment does not produce a bounded strain response, as expressed the Lyapounov stability definition (1907), but an undefined response. This failure condition is well fulfilled for metals. However, in non-associated materials, such as soils, not only localized failure modes (Fig. 2a) have been observed but also diffuse ones (Fig. 2b). In this latter case, the loss of stability cannot be predicted by the plasticity theory. Such an example of diffuse failure is classically obtained during an undrained triaxial test for which the material suddenly collapses while getting over a peak of deviatoric stress, although the plastic criterion is not reached (Fig. 2b, 2c).

Thus, elasto-plasticity theory with limit states is not capable to precisely detect instabilities in geomaterials, which is problematic to accurately predict landslides. In some cases with a very small slope angle, elasto-plasticity approach cannot even detect the failure.



a. Desrues and Chambon (2002)	b. Servant et al. (2005)	c. Darve et al, 2004

Figure 2: Localized and diffuse failures in drained/undrained triaxial test (a,b).Stress path for b(c)

3.2 A more general failure criterion

In this framework a more general failure criterion has been developed by Hill (1958): the second order work criterion. The normalized form of the stability condition is written as follows:

$$d^2W_n = \frac{d\sigma_{ij}d\varepsilon_{ij}}{|d\sigma_{ij}||d\varepsilon_{ij}|} > 0 \qquad \forall||d\varepsilon \neq 0|| \tag{2}$$

It has been successfully applied for geomechanical issues [Laouafa and Darve (2002)], with a typical example being the undrained triaxial test for which d^2W_n changes sign just at the invariant stress peak (Fig. 2c). According to Darve, Servant, Laouafa, and Khoa (2004), a system has reached a failure state when it is still subjected to strains, while no more energy is transferred to it. For landslide modelling, this more general failure criterion has been chosen, upon which viscosity appears.

4 A Post Failure Behaviour: The 3D Bingham Viscosity

Once soils are no longer in static equilibrium, they can drastically change their way to behave. If we focus more particularly on mudflows, this material is known to behave as viscous fluid, with a yield stress [Daido (1971)] and a non linear viscosity [Coussot and Boyer (1995)]. In a first approximation mudflows are considered with a mere Bingham's constitutive relation (linear viscosity with a yield stress).

In the context of a global transition model where elasto-plasticity is defined in 3D, it is necessary to write it also in 3D. According to Duvaut and Lions (1971) the 3D expression is:

$$\text{if } J_{2\dot{\varepsilon}} \neq 0: \quad s_{ij} = 2\eta \dot{e}_{ij} + s_0 \frac{\dot{e}_{ij}}{J_{2\dot{e}}}, \quad \text{else: } J_{2\sigma} \leq s_0 \tag{3}$$

$\underset{\approx}{s}$ and $\underset{\approx}{\dot{e}}$ are the deviatoric stress and strain rate tensors, $J_{2\sigma}$ and $J_{2\dot{\varepsilon}}$ are the second invariant of stresses and strain rates, s_0 and η are the yield stress and the viscosity of the viscous material. Inversely, the viscous strain rate is expressed as:

$$\text{if } J_{2\sigma} > s_0: \quad \dot{e}_{ij} = \frac{1}{\eta}\left(s_{ij} - s_0 \frac{s_{ij}}{J_{2\sigma}}\right) = \frac{J_{2\sigma} - s_0}{2\eta} \cdot \frac{s_{ij}}{J_{2\sigma}}, \text{ else: } \dot{e}_{ij} = 0 \tag{4}$$

The direction of the threshold s_0 is now given by the stress direction s_{ij} instead of the strain rate $\dot{e}_{ij}$. The global model considers the sum of elasto-plastic and viscous strain rates (Fig. 3). The latter is null before the failure, it is activated as soon as d^2W_n becomes negative, and it increases according to the difference between the current invariant $J_{2\sigma}$ and the yield stress s_0 (eq. 4). If $J_{2\sigma}$ is below s_0, the viscous flow stops according to the Bingham's relation, and the soil retrieves an elasto-plastic behaviour. This model thus reproduces two different kinds of behaviour, one solid and one fluid. Since the coupled model (elastoplasticity / failure criterion / nonlinear viscosity) presented in this contribution is general, any constitutive law for the solid-like (e.g. Cam-Clay) phase and the fluid-like (e.g. Herschel-Bulkley) phase can be used depending on the intrinsic properties of the material.

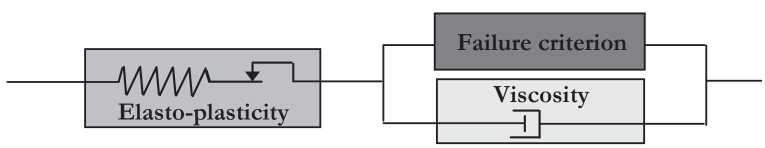


Figure 3: Scheme of the global constitutive relation in 1D for a soil subjected to failure

5 Application of The Model on An Undrained Triaxial Test

As mentioned previously, the undrained triaxial test is typical for failure within the plastic criterion, and for sudden collapse of the soil. A unit square of confined soil (in plane strain condition) is modelled, with velocity boundary conditions such as to apply an isochoric loading (equivalent to undrained conditions) with a strain rate of $0.6s^{-1}$. Taking into account that friction angle in compression and extension are equal, the mechanical parameters of the soil are gathered in Table 1.

Table 1: Constitutive parameters of soil

$\rho(kg/m^3)$	$E(MPa)$	v	$\varphi_0/\varphi_f(°)$	$\psi_0/\psi_f(°)$	$C(kPa)$	$Kv(Pa.s)$	$\eta(Pa.s)$
0	15	0,29	3/28	$-20/+5$	1/10	250	10

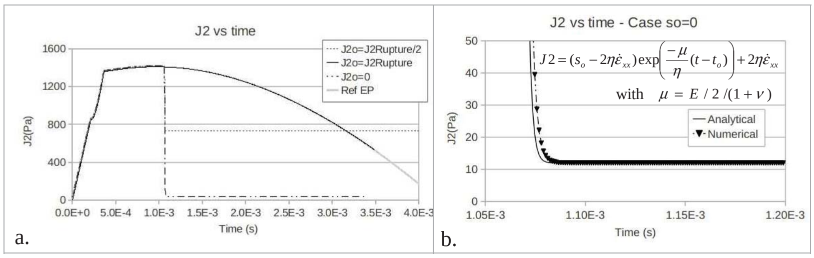


Figure 4: Evolution of J_2, comparison with EP case (a), and analytic case for $s_0 = 0$ (b).

Four tests are performed: a reference one in elasto-plasticity without transition, and 3 tests with a viscous transition at the failure and different yield stresses s_0 null, equal to the second invariant at the failure, J_{2f}, and to half of it. The second order work criterion (and thus the failure) is, as expected, reached at the peak of the evolution of the second stress invariant J_2 with loading (Fig. 4). In the case of $s_0 = J_{2f}$, the sample behaves as elasto-plastic since Bingham's model cannot produce viscous strain rate if there is no stress invariant difference (eq. 4). In the other cases, the stress rapidly decreases, following the analytical exponential resulting from the visco-elastic partial differential equation (Fig. 4).

6 Conclusions and Perspectives

In the context of landslide, this constitutive model has been written to describe in a continuum framework the elasto-plastic behaviour of a in-situ soil and eventually the failure and the transition toward viscous fluid for mud-like devastating flows. The chosen criterion for the transition, i.e. the second order work, is able to detect localized as well as diffuse failure modes, particularly for almost saturated soils. The yield stress 0 s allows to model the arrest of the Bingham fluid. The benchmark of the undrained triaxial test shows that soil turned to follow the analytical solution. The different tests highlight that the yield stress needs to be lower than the failure stress invariant; indeed it is consistent to consider that in the field the mudflow yield stress is lower than the in situ soil stress at failure. The next step is to perform a stress driven benchmark and a heuristic case with a larger scale.

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