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Solution of Nonlinear Seepage Model for Well Group in Fractured Low-permeability Reservoirs

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Abstract: On the basis of describing the nonlinear seepage characteristics in low-permeability reservoirs and the conductivity varying law of hydraulic fracture, a nonlinear mathematics model which couples low-permeability reservoirs and hydraulic fracture is established. The model takes nonlinear and quasi-linear flow stages in the reservoirs system, and emerging Darcy and non-Darcy flow in the fracture system into account. Finite difference equation set is derived with Taylor expansion method, and stability condition of the scheme is presented; On that basis the computer model is formed. The impact of starting pressure gradient, varying conductivity of the hydraulic fracture on the injection and production well group production performance is analyzed with the computer model. Result shows that adopting the nonlinear model could compute the distribution characteristics of pressure and saturation in the formation more precisely, and describe the production dynamic change law of injection and production well group precisely. The model could be used as a computing tool for injection and production well group hydraulic fracturing optimal design in the low-permeability reservoirs.

Keywords: low-permeability reservoirs, nonlinear seepage, starting pressure gradient, varying conductivity, numerical simulation.

1 Introduction

The fluid flow in low permeability reservoirs no longer meets classic Darcy law, only when the pressure gradient is greater than the minimum start-up pressure gradient, the fluid can flow. With the increase of the pressure gradient, the flow of the fluid will experience not flow stage, nonlinear flow stage and quasi linear flow phase section, but the seepage rules of the latter two flow stages obey different equation of motion. At present, for most scholars, when they study the low permeability reservoir, in most cases they consider the nonlinear seepage stage and quasi linear seepage stage as the quasi linear segment and ignore the influence of

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the flow caused by nonlinear seepage stage [Song and Liu (1999); Cheng (1998); Zhou (2002)] .Based on the concept of dynamic permeability, Yin Zhi-lin [Yin, Sun and Yao (2011)] used unified form of motion to describe seepage rules of nonlinear segment and quasi linear segment, they think that the change of the pressure calculated by using nonlinear seepage rule is gentle compared to quasi linear seepage. Comprehensive consideration of the current research achievements, this paper puts forward an approach to section describe the seepage characteristics of nonlinear and quasi linear flow phase and take the fracturing fracture flow rule into consideration at the same time. The development of low permeability reservoir generally needs to adopt hydraulic fracturing, many scholars have studied fractured well [Lv, Ju and Luan (1998); Su, Wang, Li Tao, et al. (2006); He, Sun, Xu, et al. (2010)], they generally look crack and formation as a same system, this kind of treatment will make simulation not accurate. In addition, the fluid flows fast in fracturing fracture, it may appear high-speed non-Darcy seepage flow, we need to judge flow patterns according to the Reynolds number [Belhaj, Agha and Nouri (2003)]. In addition, fracture conductivity is not fixed, it varies along the fracture length, as well as with cracks in the pressure change [Soliman (1986)]; Wen, Zhang, Wang, et al (2005)]. Based on this, this paper established the low permeability three-dimensional oil-water two phase, reservoir system covers nonlinear segment and quasi linear seepage, fracture system covers Darcy and high-speed non-Darcy seepage coupling mathematical model, then it solved the mathematical model by using finite difference, finally, the calculation results are analyzed.

2 Established fractured injection and production well group coupled mathematical model

2.1 Model hypothesis

(1) Formation rock and fluid is slightly compressible;

(2) Low permeability reservoir fracturing well group has 3 d oil-water two phase flow, establish the reservoir system and fracture system respectively;

(3) Consider nonlinear seepage and quasi linear seepage in reservoir system, oil-water phase start-up pressure gradient remains unchanged;

(4) Consider Darcy and high-speed non-Darcy flow in fracture system, and consider the changing of fracture conductivity characteristics;

(5) Reservoir flow is isothermal flow, and reservoir outer boundary is closed;

(6) Consider the impact of gravity and capillary forces.

2.2 Seepage equation of the reservoir system

The relationship between the velocity and pressure gradient is that when the pressure gradient is between the minimum start-up pressure gradient G_A and the maximum start-up pressure gradient G_C , there is concave up nonlinear section on the flow rate - pressure gradient curve, using quadratic function to characterize the nonlinear segment, as formula (1),

$$v = a(\frac{dp}{dx})^2 + b\frac{dp}{dx} + c \tag{1}$$

When the pressure gradient is greater than the maximum start-up pressure gradient G_C , fluid flow rate - pressure gradient curve becomes a straight line, it can be expressed as formula (2),

$$v = -\frac{k}{\mu}(\frac{dp}{dx} - G_B) \tag{2}$$

Where

G_A is minimum start-up pressure gradient,*MPa/m*; G_B is quasi start-up pressure gradient, *MPa/m*; G_C is maximum start-up pressure gradient, *MPa/m*; v is fluid velocity, m/s; a,b,c is coefficients of nonlinear equations of motion.

Consider the three-dimensional oil-water two-phase flow in low permeability reservoir and there is an interaction between the reservoir and the crack flow items, according to formula (1) and formula (2),we can respectively get reservoir system seepage equation (3) and (4),

When $|\nabla p_l| \geq G_{Cl}$,

$$\begin{aligned}&\frac{\partial}{\partial x}(\frac{\rho_l k_x k_{rl}}{\mu_l}(\frac{\partial p_l}{\partial x} - G_{Bl})) + \frac{\partial}{\partial y}(\frac{\rho_l k_y k_{rl}}{\mu_l}(\frac{\partial p_l}{\partial y} - G_{Bl}))\\ &+ \frac{\partial}{\partial z}(\frac{\rho_l k_z k_{rl}}{\mu_l}(\frac{\partial p_l}{\partial z} + \rho_l g\frac{\partial D}{\partial z} - G_{Bl})) - \tau_{lmf} + q_{lm} = \frac{\partial(\rho_l \varphi S_l)}{\partial t}\end{aligned} \tag{3}$$

When $G_{Cl} > |\nabla p_l| \geq G_{Al}$,

$$\begin{aligned}&-\frac{\partial}{\partial x}[\rho_l(a_l(\frac{\partial p_l}{\partial x})^2 + b_l\frac{\partial p_l}{\partial x} + c_l)] - \frac{\partial}{\partial y}[\rho_l(a_l(\frac{\partial p_l}{\partial y})^2 + b_l\frac{\partial p_l}{\partial y} + c_l)]\\ &-\frac{\partial}{\partial z}[\rho_l(a_l(\frac{\partial p_l}{\partial z} + \rho_l g\frac{\partial D}{\partial z})^2 + b_l(\frac{\partial p_l}{\partial z} + \rho_l g\frac{\partial D}{\partial z}) + c_l)] - \tau_{lmf} + q_{lm} = \frac{\partial(\rho_l \varphi S_l)}{\partial t}\end{aligned} \tag{4}$$

Where, $l = o,w$ means oil-water two phase; m means reservoir system; f means fracture system; $|\nabla p|$ means pressure gradient, *MPa/m*; *D* means height, *m*; *k* means permeability, *Darcy*; k_r means relative permeability; ρ means fluid density, *g/cm*^{3}; μ means fluid viscosity, *mPa·s*; φ means reservoir porosity; S means saturation; τ_{mf} means interaction flow item between reservoir and fracture system *g/(cm*^{3}*·s)*; q_{lm} means production item in unit time and unit volume, *g/(cm*^{3}*·s)*.

2.3 Seepage equation of the fracture system

Crack width is small, so we can ignore fluid flow along the width direction and establish coordinate system$(x^{'},z^{'})$, $x^{'}$ is along the crack extension direction, $z^{'}$ is the same as reservoir coordinate system, in case of confusion with the coordinate system of the reservoir system, crack coordinate system is still recorded as (x,z), the flow form in the fracture is judged by Kaldirafe Reynolds number formula (5), then we get the corresponding motion equation (6), put the motion equation into the continuity equation(8), we can get seepage equation of fracture system,

$$Re_l = \frac{v_{lf}\sqrt{k_f}\rho_l}{17.50\mu_l\phi_f^{3/2}} \tag{5}$$

Equation of motion of the fracture system

$$\begin{cases} \vec{v_l} = -\dfrac{k_f k_{rl}}{\mu_l}\nabla p_l \; Re_l \leq 0.3 \\ -\nabla p_l = \dfrac{\mu_l}{k_l}V_l + \beta\rho_l V_l^2 \; Re_l > 0.3 \end{cases} \tag{6}$$

β means non-Darcy factor, it is determined by medium parameters and shown by formula(7):

$$\begin{cases} \beta = \beta(\varphi, k) \\ \beta = \dfrac{c}{k^a\varphi^b} \end{cases} \tag{7}$$

Continuity equation of the fracture system

$$-\nabla\cdot(w_f\rho_l\vec{v_l}) + w_f\tau_{lmf} + w_f q_{lf} = w_f\frac{\partial}{\partial t}(\rho_l\phi_f S_l) \tag{8}$$

τ_{lmf} means interaction flow item between reservoir and fracture system, shown as formula (9);

$$\tau_{lmf} = \sigma\frac{k_m k_{rl}}{\mu_l}(p_{l,m} - p_{l,f}) \tag{9}$$

σ is substrate block shape factor, it is determined by substrate block shape dimension and its characteristic length

$$\sigma = \frac{4d(d+2)}{L^2} \tag{10}$$

According to the double medium about shape factor calculation method, use Darcy formula to

derivate the interaction flow item between reservoir and fracture system and it can be shown as:

$$\tau_{lmf} = \frac{4D_f}{D_x D_y}\left(\frac{1}{D_x} + \frac{1}{D_y}\right)\frac{k_m k_{rl}}{\mu_l}(p_{l,m} - p_{l,f}) \tag{11}$$

Where R_e means fluid Reynolds number; ϕ_e means fracture porosity; $w_e$means crack width, m; D_e means the length of the crack through the reservoir grid, m; D_X means the step length along the reservoir grid X direction, m; D_y means the step length along the reservoir grid Y direction, m.

According to the research of Soliman [Soliman (1986)], the decreasing way of fracture conductivity covers linear attenuation and index attenuation, in addition, hydraulic fracture crack opens and closes with the change of the pressure in the fracture, conductivity changes accordingly, so fracture conductivity is a function of the crack length and pressure , as formula(12):

$$k_f(i) = kf(i_0) \times f(i, p_{f,i}) \tag{12}$$

Function form varies with the change of fracture properties.

2.4 Auxiliary equation

Considering oil-water two phase flow, it must meet the following auxiliary equation:

For reservoir system:

$$p_{m,cow} = p_{m,o} - p_{m,w} \tag{13}$$

$$S_{m,o} + S_{m,w} = 1 \tag{14}$$

For fracture system:

$$p_{f,cow} = p_{f,o} - p_{f,w} \tag{15}$$

$$S_{f,o} + S_{f,w} = 1 \tag{16}$$

2.5 Initial conditions

For reservoir system:

$$p_{m,o}(x,y,z,0) = p_{oi}(x,y,z) \tag{17}$$

$$S_{m,w}(x,y,z,0) = S_{wi}(x,y,z) \tag{18}$$

For fracture system:

$$p_{f,o}(x,z,0) = p_{oi}(x,z) \tag{19}$$

$$S_{f,w}(x,z,0) = S_{wi}(x,z) \tag{20}$$

2.6 Boundary conditions

(1)Outer boundary conditions

Outer boundary of the reservoir system:

$$\frac{\partial p_{m,o}}{\partial n}|_{\partial\Gamma} = 0 \quad \text{(closed)} \tag{21}$$

$$p_{m,o}|_{\partial\Gamma} = p_{oi} \quad \text{(Constant pressure)} \tag{22}$$

Outer boundary of the fracture system:

$$\frac{\partial p_{f,o}}{\partial n}|_{\partial\Gamma'} = 0 \quad \text{(closed)} \tag{23}$$

(2)Wellbore conditions

Constant liquid quantity:

$$q_m + q_f = \text{cons tan t} \tag{24}$$

Bottom-hole flowing pressure is constant, based on the low speed non-Darcy flow and quasi steady state radial flow we can get the production and water injection rate of the reservoir system, while based on Darcy flow and quasi steady state radial flow, we can get the production and water injection rate of the fracture system.

$$q_{l,mij} = PI_{l,m}[p_{l,mij} - p_{wf} - G_{Bl}(R_e - R_w)] \tag{25}$$

$$q_{l,fij} = PI_{l,f}(p_{l,fij} - p_{wf}) \tag{26}$$

$$q_{mij} = WI_m[p_{iwf} - p_{mij} - G_{Bl}(R_e - R_w)] \tag{27}$$

$$q_{fij} = WI_f(p_{iwf} - p_{fij}) \tag{28}$$

Where $PI_{l,m}$ means production index of the reservoir system; $PI_{l,f}$ means production index of the fracture system; WI_m means injection index of the reservoir system; WI_f means injection index of the fracture system; $q_{l,mij}$ means the l phase production generated by the flow from Reservoir to the well, $g/(cm^3 \cdot s)$; $q_{l,fij}$ means the l phase production generated by the flow from fracture to the well, $g/(cm^3 \cdot s)$; q_{mij} means water injection from water injection well to reservoir system, $g/(cm^3 \cdot s)$; q_{fij} means water injection from water injection well to fracture system, $g/(cm^3 \cdot s)$;

3 Solving the mathematical model

This paper uses finite difference and the method of IMPES to calculate the pressure and saturation distribution in the reservoir and fracture system, for nonlinear segment, by using nonlinear process to get linear difference equations, we should avoid using iterative method to solve the nonlinear difference equations, in this way, we can reduce the computation and improve the calculation speed,

and it gives the convergence conditions of the form.

3.1 *Difference format of the reservoir system*

(1)when $|\nabla p_l| \geq G_{Cl}$, Reservoir system is in quasi linear segment, difference format is as formula (29)

$$\begin{aligned}
&\frac{\lambda_{lxi+\frac{1}{2}}(\frac{p^{n+1}_{li+1jk}-p^{n+1}_{lijk}}{\Delta x}-G_{Bl})-\lambda_{lxi-\frac{1}{2}}(\frac{p^{n+1}_{lijk}-p^{n+1}_{li-1jk}}{\Delta x}-G_{Bl})}{\Delta x}+\\
&\frac{\lambda_{lyj+\frac{1}{2}}(\frac{p^{n+1}_{lij+1k}-p^{n+1}_{lijk}}{\Delta y}-G_{Bl})-\lambda_{lyj-\frac{1}{2}}(\frac{p^{n+1}_{lijk}-p^{n+1}_{lij-1k}}{\Delta y}-G_{Bl})}{\Delta y}+\\
&\frac{\lambda_{lzk+\frac{1}{2}}(\frac{p^{n+1}_{lijk+1}-p^{n+1}_{lijk}}{\Delta z}-\rho g-G_{Bl})-\lambda_{lzk-\frac{1}{2}}(\frac{p^{n+1}_{lijk}-p^{n+1}_{lijk-1}}{\Delta z}-\rho g-G_{Bl})}{\Delta z}\\
&-(\tau_{lmf})_{ijk}+(q_{lm})_{ijk}=(\beta_l)_{ijk}\frac{p^{n+1}_{lijk}-p^{n}_{lijk}}{\Delta t}+(\phi\rho_l)_{ijk}\frac{S^{n+1}_{lijk}-S^{n}_{lijk}}{\Delta t}
\end{aligned} \tag{29}$$

Where

$$\lambda_l=\frac{\rho_l k k_{rl}}{\mu_l}\beta_l=\rho_l\phi S_l(C_p+C_l).$$

(2)when $G_{Cl} > |\nabla p_l| \geq G_{Al}$, reservoir system is in nonlinear segment, the key of finite difference format scheme is to deal with left item, This paper uses the way of Taylor expansion for linear processing and difference solution

$$\frac{\partial}{\partial x}[\rho a(\frac{\partial p}{\partial x})^2]=\frac{(\rho a)_{i+\frac{1}{2}}(\frac{\partial p}{\partial x})^2_{i+\frac{1}{2}}-(\rho a)_{i-\frac{1}{2}}(\frac{\partial p}{\partial x})^2_{i-\frac{1}{2}}}{\Delta x}$$

Differential treatment on the first item:

$$
\begin{aligned}
[(\frac{\partial p}{\partial x})^2]^{n+1}_{i+\frac{1}{2}} &\approx [(\frac{\partial p}{\partial x})^2]^{n}_{i+\frac{1}{2}} + 2(\frac{\partial p}{\partial x})^{n}_{i+\frac{1}{2}}[(\frac{\partial p}{\partial x})^{n+1}_{i+\frac{1}{2}} - (\frac{\partial p}{\partial x})^{n}_{i+\frac{1}{2}}] \\
&= 2(\frac{\partial p}{\partial x})^{n}_{i+\frac{1}{2}}(\frac{\partial p}{\partial x})^{n+1}_{i+\frac{1}{2}} - [(\frac{\partial p}{\partial x})^2]^{n}_{i+\frac{1}{2}} \\
&= 2\frac{p^n_{i+1}-p^n_i}{\Delta x}\cdot\frac{p^{n+1}_{i+1}-p^{n+1}_i}{\Delta x} - (\frac{p^n_{i+1}-p^n_i}{\Delta x})^2
\end{aligned}
$$

Differential treatment on the second item:

$$
[(\frac{\partial p}{\partial x})^2]^{n+1}_{i-\frac{1}{2}} = 2\frac{p^n_i-p^n_{i-1}}{\Delta x}\cdot\frac{p^{n+1}_i-p^{n+1}_{i-1}}{\Delta x} - (\frac{p^n_i-p^n_{i-1}}{\Delta x})^2
$$

The last two difference format of the seepage equation:

$$
\frac{\partial}{\partial x}[\rho b(\frac{\partial p}{\partial x}) + \rho c] = \frac{(\rho b)_{i+\frac{1}{2}}\cdot\frac{p_{i+1}-p_i}{\Delta x} - (\rho b)_{i-\frac{1}{2}}\cdot\frac{p_i-p_{i-1}}{\Delta x} + (\rho c)_{i+\frac{1}{2}} - (\rho c)_{i-\frac{1}{2}}}{\Delta x}
$$

Get the difference format of x direction, then multiply $V = \Delta x\Delta y\Delta z$, getting:

$$
\begin{aligned}
&[\frac{2(\rho a)_{i+\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_{i+1}-p^n_i) + \frac{(\rho b)_{i+\frac{1}{2}}\Delta y\Delta z}{\Delta x}]p^{n+1}_{i+1} + \\
&[\frac{2(\rho a)_{i-\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_i-p^n_{i-1}) + \frac{(\rho b)_{i-\frac{1}{2}}\Delta y\Delta z}{\Delta x}]p^{n+1}_{i-1} + \\
&[\frac{2(\rho a)_{i+\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_{i+1}-p^n_i) + \frac{2(\rho a)_{i-\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_i-p^n_{i-1}) \\
&+ \frac{(\rho b)_{i+\frac{1}{2}}\Delta y\Delta z}{\Delta x} + \frac{(\rho b)_{i-\frac{1}{2}}\Delta y\Delta z}{\Delta x}]p^{n+1}_i \\
&+ \frac{(\rho a)_{i-\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_i-p^n_{i-1})^2 - \frac{(\rho a)_{i+\frac{1}{2}}\Delta y\Delta z}{\Delta x^2}(p^n_{i+1}-p^n_i)^2 \\
&+ [(\rho c)_{i+\frac{1}{2}} - (\rho c)_{i-\frac{1}{2}}]\Delta y\Delta z
\end{aligned}
\tag{30}
$$

the results of Y and z direction can be gotten in the same way, but the pressure gradient of Z direction should be unity to $\partial \mathrm{p}/\partial \mathrm{z} - \rho g$. From the above format, the coefficient of the pressure value of moment n + 1 can be gotten by the pressure value of the moment n, and it is linear difference format, then solve equations by appropriate numerical method, as for the stability of the format, we can use one dimensional two phase as an example to illustrate.

$$
\begin{cases}
\frac{\partial}{\partial x}[a_o(\frac{\partial p}{\partial x})^2 + b_o\frac{\partial p}{\partial x} + c_o] = \frac{\partial(\phi S_o)}{\partial x} \\
\frac{\partial}{\partial x}[a_w(\frac{\partial p}{\partial x})^2 + b_w\frac{\partial p}{\partial x} + c_w] = \frac{\partial(\phi S_w)}{\partial x}
\end{cases}
$$

two formulas additive:

$$\frac{\partial}{\partial x}[a_o(\frac{\partial p}{\partial x})^2+b_o\frac{\partial p}{\partial x}+c_o+a_w(\frac{\partial p}{\partial x})^2+b_w\frac{\partial p}{\partial x}+c_w]=0$$

This means that total speed is a constant, so we can get the stability conditions of this format by the stability conditions of the way of IMPES, just as formula (31), according to the formula, we can expand to three-dimensional.

$$\max_i\{\frac{\Delta t}{\Delta x}\cdot[(a_o+a_w)(\frac{p_i-p_{i+1}}{\Delta x})^2+(b_o+b_w)\frac{p_i-p_{i+1}}{\Delta x}+c_o+c_w]\}\leq 1 \tag{31}$$

3.2 Difference format of the fracture system

Using non-equidistant grid in the x direction of the fracture system, we can obtain the grid of the reservoir system and length crossed by the fracture system according to the crack length and orientation as well as coordinate of fracturing well, each grid of the reservoir system

corresponds to a grid of the fracture system, thus the interaction term between reservoir and fracture only exists in the corresponding grid, we can establish coupling equations by the interaction.

(1) When $Re_l\leq 0.3$, difference format for fracture system:

$$\frac{\gamma_{lxi+\frac{1}{2}}\frac{2(p_{li+1,j}^{n+1}-p_{li,j}^{n+1})}{\Delta x_i+\Delta x_{i+1}}-\gamma_{lxi-\frac{1}{2}}\frac{2(p_{li,j}^{n+1}-p_{li-1,j}^{n+1})}{\Delta x_i+\Delta x_{i-1}}}{\Delta x_i}+\frac{\gamma_{lzj+\frac{1}{2}}\frac{p_{lij+1}^{n+1}-p_{lij}^{n+1}}{\Delta z}-\gamma_{lzj-\frac{1}{2}}\frac{p_{lij}^{n+1}-p_{lij-1}^{n+1}}{\Delta z}}{\Delta z}$$
$$+(w_f\tau_{lmf})_{ij}+(w_f q_{lm})_{ij}=(w_f\beta_l)_{ij}\frac{p_{lij}^{n+1}-p_{lij}^{n}}{\Delta t}+(w_f\phi\rho_l)_{ij}\frac{S_{lij}^{n+1}-S_{lij}^{n}}{\Delta t} \tag{32}$$

Where $\gamma_l=w_f\frac{\rho_l k k_{rl}}{\mu_l}$

(2) When $Re_l>0.3$, partial derivative to both sides of formula (6) about x, getting:

$$\frac{\partial(v_{lx})}{\partial x}=\frac{1}{-\frac{\mu_l}{k_l}+2\beta\rho_l v_{lx}}\frac{\partial^2 p_l}{\partial x^2} \tag{33}$$

So difference format in x direction of the fracture system is formula (34):

$$\begin{aligned}
&\frac{\partial(w_f\rho_l v_{lx})}{\partial x}=w_f\rho_l\frac{\partial v_{lx}}{\partial x}+w_f v_{lx}\frac{\partial \rho_l}{\partial x}+\rho_l v_{lx}\frac{\partial w_f}{\partial x}\\
&\frac{\partial(w_f\rho_l v_{lx})}{\partial x}=\frac{w_f\rho_l}{-\frac{\mu_l}{k_l}+2\beta\rho_l v_{lx}}\frac{\partial^2 p_l}{\partial x^2}+cw_f v_{lx}\rho_l\frac{\partial p_l}{\partial x}+\rho_l v_{lx}\frac{\partial w_f}{\partial x}\\
&=\frac{w_f\rho_l}{-\frac{\mu_l}{k_l}+2\beta\rho_l v_{lxi,j}^n}\frac{\frac{2(p_{li+1,j}^{n+1}-p_{li,j}^{n+1})}{\Delta x_i+\Delta x_{i+1}}+\frac{2(p_{li,j}^{n+1}-p_{li-1,j}^{n+1})}{\Delta x_i+\Delta x_{i-1}}}{\Delta x_i}+\\
&cw_f v_{lx}\rho_l\frac{2(p_{li+1,j}^{n+1}-p_{li,j}^{n+1})}{\Delta x_i+\Delta x_{i+1}}+\rho_l v_{lx}\frac{2(w_{fi+1,j}^{n}-w_{fi,j}^{n})}{\Delta x_i+\Delta x_{i+1}}
\end{aligned}\tag{34}$$

So the difference format in z direction can be gotten in the same way, and we can get the difference format of high speed non-Darcy

seepage flow in the fracture by combining the difference of channeling, production and the right end item, the channeling interaction term only exists in the grid intersected between the reservoir system and fracture system, channeling item difference can be directly obtained by discreting as channeling formula.

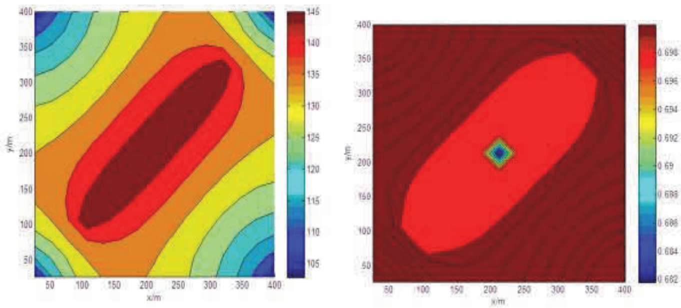
$$(\tau_{lmf})_{ijk}=\alpha_{lijk}(p_{lm,ijk}-p_{lf,ij})\tag{35}$$

Where $\alpha_l=\sigma\rho_l k_m k_{rl}/\mu_l$

4 Model results and correlation analysis

4.1 Model results

This paper adopts five point pattern, assume that only fracturing water injection well, not fracturing oil well, using hydraulic fracturing to the central water injection well, crack being wings symmetric and fracture propagation direction consistent with the maximum horizontal principal stress direction. Use programming to solve the mathematics model and to calculate the dynamic of the injection-production well group. After 90 days waterflood development, calculation results of pressure and saturation are shown in figure 1. As we can see from the results, pressure and saturation form contour around fracturing fracture and present symmetrical distribution characteristics along the crack flanks. Isobar is intensive near the crack, the further the isobar from the crack is, the more sparse it is; closer to the well bottom, more intensive isobar is. In the same way, Injection water also gradually spread along the fracture. Pressure and saturation distribution characteristics in the results conform to the actual situation, this means the model is correct.



(a)Pressure distribution (b) Oil saturation distribution

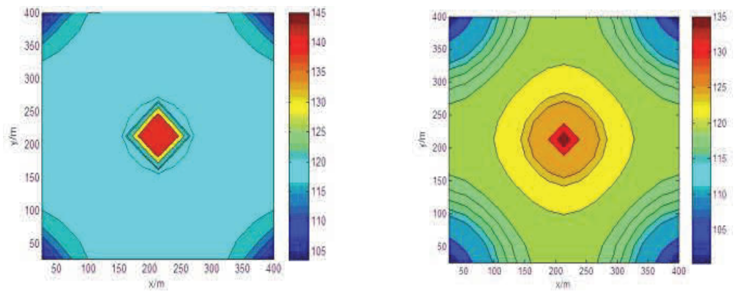
Figure 1: Computational result of fractured well group model

4.2 Considering the influence nonlinear section to the pressure distribution while not fracturing

There is nonlinear segment and quasi linear segment in low permeability reservoirs, in order to study the influence section describing seepage characteristics to injection-production dynamic, compare and analyze the results of using section description and using quasi linear description only, without considering injection-production well group fracturing here. On condition that bottom hole flowing pressure is constant, calculating results of two methods are shown respectively in figure 2(a) and 2(b). As we can see from the two figures, while only considering quasi linear seepage, there are many areas where fluid can not flow between water wells and oil wells, the main reason is that quasi linear seepage exaggerated start-up pressure gradient but ignore the nonlinear seepage in the case of low pressure gradient, this leads to a smaller flow area and causes well spacing design error, in this case, we can not achieve the best economic benefits. In addition, we can see from the two figures, when considering the nonlinear seepage, the pressure changes is more gentle than the one gotten in the case of considering quasi linear seepage only, this is consistent with the present conclusions.

4.3 Effect of fracture properties on injection-production well group dynamic

In order to study the influence of the fracture on injection-production production dynamic, this paper firstly calculated the injection-production dynamic of considering and not considering fracture. Figure 3 shows the oil production dynamic in injection-production well group in case of fracturing and not fracturing water injection well. When the water injection well is not hydraulic fractured, the daily oil production of wells declines rapidly and remains at a very low level; But when frac-



(a) Pressure distribution in the case of using single quasi linear segment

(b) Pressure distribution in the case of section calculation nonlinear segment

Figure 2: Pressure distribution in different seepage models

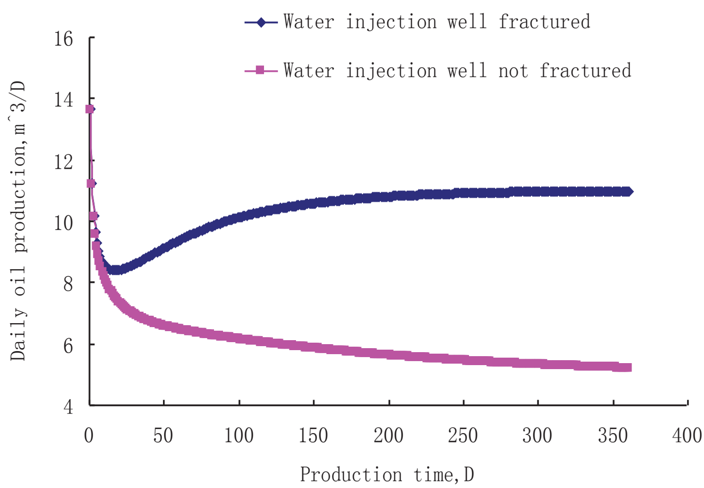


Figure 3: The effect of fracturing on production

tured, daily oil production decreases at first but rise later and it is obviously higher than the former. This shows that fracturing fracture has obvious effects on increasing daily oil production. The main reason is that fracturing fracture can obviously improve the injection index of injection well and the waterflood response.

4.4 Effect of fracture variable conductivity on injection-production well group dynamic

According to reference [Wen, Zhang, Wang, et al. (2005)], fracturing fracture conductivity is a function of formation closure pressure, in present the most think that fracture conductivity changes with time in the numerical simulation to fractured well, but in fact the

change of fracturing fracture conductivity mainly caused by the closure pressure. So this paper designs fracture conductivity into the function of pressure. Based on this, we compared the calculation results obtained by the methods of the references and this paper, as figure 4. From the calculation results, we see two curve with the same trend. But the production obtained by the methods referred in this paper is a little lower than those obtained by the methods referred in references. Because by the methods referred in references, well fracturing fracture conductivity gradually reduce with the extension of time; but by the methods referred in this paper, it depends on the change of fracturing fracture pressure: when the fracture pressure is greater than closure pressure, fracture conductivity decreases with the decreasing of fracture pressure; when the fracture pressure is less than closure pressure, the fracture conductivity is nearly zero. Well fracturing fracture conductivity determines the formation energy adding ability of the injection-production well group as well as influences the production change of the whole injection-production well group. Due to the above causes, calculation results of this paper and literature results have certain differences.

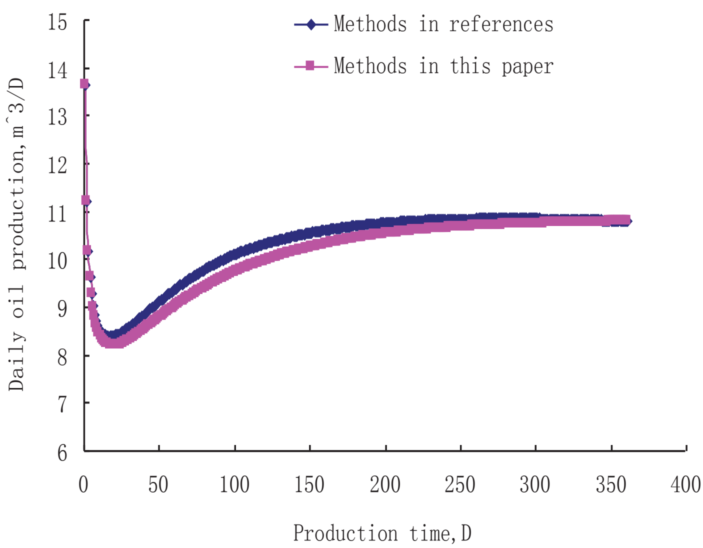


Figure 4: The effect of varying conductivity on production

5 Conclusions

(1) According to the characteristics of nonlinear seepage in low permeability reservoirs and the change rule of the fracturing fracture conductivity, this paper established reservoirs and fracture system coupled nonlinear mathematical model in low permeability reservoirs fracturing well group, comprehensively consider the rule of nonlinear seepage and quasi linear seepage in the low permeability reservoirs, as well as the rule of Darcy and non-Darcy seepage in fracturing fracture system, the simulation results accord with the results of actual formation, the model is correct and effective.

(2) This paper put forward a method to transform the nonlinear seepage equation of reservoirs system into linear difference equations by Taylor expansion and form a computer model through programming, the calculation results show that this method is correct and effective and it can be used to simulate the production dynamic in the low permeability reservoirs fracturing well group, in addition, it can solve questions more simply and quickly than other nonlinear model.

(3) In view of low permeability reservoirs with the characteristics of nonlinear seepage, comprehensive consideration of the nonlinear segment and quasi linear seepage is more reasonable than simple consideration of quasi linear seepage and it meets formation conditions better, so we can't ignore the influence of nonlinear section.

(4) Calculation results obtained by using five point pattern in low permeability reservoir fracturing well group show that hydraulic fracturing can cause significant effects on increasing production, fracturing fracture conductivity changes with formation closure pressure, the reducing of the conductivity can lead to the reducing of production of oil well.

Innovation: The coupled mathematical model established in this paper comprehensively consider the characteristics of nonlinear seepage in low permeability reservoir and Darcy or non-Darcy seepage in fracturing fracture, as well as the changes of the fracturing fracture conductivity. In addition, this paper used the method of Taylor expansion linearization, established the model of finite difference equation and well solved the coupled nonlinear seepage model of reservoir and fracture system, the calculation results can describe the distribution characteristics of formation pressure and saturation and the change rule of injection-production well group production dynamic more accurately.

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