 SL, vol.10, no.2, pp.85-94, 2013

A New Solution Algorithm for Multi-Dimensional Reservoir Simulation

Boyun Guo[1] **and Yin Feng**[2]

Abstract: A new numerical procedure, called Semi-Alternating Direction Implicit (SADI), was developed for solving the Diffusivity Equation (DE) using the Finite Difference Method (FDM) in this study. The SADI procedure was compared with the Simultaneous Sweep (SS) procedure and the Alternating Direction Implicit (ADI) procedure in terms of numerical stability and computing speed in a 3-dimensional oil reservoir simulator. Results from the model-testing indicate that, due to different degrees of numerical dispersion, the function values generated by the SADI procedure are slightly greater than that by the SS procedure and the ADI procedure. The SADI procedure is as stable as the SS procedure, and is much more stable than the ADI procedure. The maximum stable time-step size with SADI is at least twice that with ADI. The SADI procedure is as fast as the ADI procedure and over a thousand times faster than the SS procedure. SADI's nature of increasing speed of computation does not diminish quickly as the time step size increases. The advantage of the SADI procedure in speeding up computation is more pronounced in large scale (large number of grid blocks) systems than in small scale systems. Although the SADI procedure was tested in an oil reservoir simulator that is used in the petroleum industry, the procedure is equally valid for any numerical simulations that involve solving the DE using the FDM.

1 Introduction

Numerical simulation has been widely adopted as a powerful tool in scientific and engineering system analyses. Finite difference method (FDM), an efficient procedure for solving partial differential equations (PDE), is very often used in numerical simulation. Application of the FEM to solving diffusivity equation (DE) was first found in heat conduction calculations [Carslaw and Jaeger (1959)]. It is also popular to use this technique in hydrology to simulate water flow in ground aquifers (McWhorter and Sunada, 1877). Numerical solution of DE with FDM is employed

[1] University of Louisiana at Lafayette.
[2] Louisiana State University.

in the petroleum engineering for planning the development of oil and natural gas reservoirs [Dake (1978)]. In the petroleum industry, numerical simulators fall into two categories: a) conventional "Black Oil" simulators, and b) special-purpose simulators. The former can model the flow of water, oil, and gas, accounting for pressure-dependent solubility of gas in oil, but cannot model the changes in oil and gas compositions. The latter can model compositional, thermal, and chemical processes in enhanced oil recovery (EOR) projects.

The technique of FDM applied to petroleum reservoir simulation is described by Thomas (1982). To accurately simulate fluid flow in a porous medium, the flow domain has to be divided into many grid blocks in which the values of the parameter of interest, e.g., pressure, are computed numerically. The size of the grid system depends on the scale of the flow domain and the required accuracy of simulation. Even using the modern computing technologies, numerical simulation with FDM is still a time-consuming task. Depending on the dimension of the simulation grids, the computing time required to complete a simulation job varies from a few days to a few weeks. The low-speed simulation runs hinder the application of the technology to large-dimension flow media.

Large-scale simulation problems have extremely large CPU and memory requirements. Traditional methods for solving such problems involve parallel processing, which adds cost and complexity in programming. Development of fast, robust and cost-effective computation technology is a necessity.

2 Current Practices

Commercial software packages used in the petroleum industry for reservoir simulation employ different algorithms to solve the DE for pressure and fluid distributions in the oil and gas reservoirs. The DE for a 3-dimensional problem is expressed as follows:

$$T_x\frac{\partial^2 p}{\partial x^2}+T_y\frac{\partial^2 p}{\partial y^2}+T_z\frac{\partial^2 p}{\partial z^2}+Q=\alpha\frac{\partial p}{\partial t} \tag{1}$$

where p is pressure and t is time. The T_x, T_y and T_z are fluid transmissibilities x, y and z directions. The Q is a source term and α is storage constant.

The most widely used solution algorithm is the implicit-pressure explicit-saturation (IMPES) algorithm. The speed of solution is controlled by the implicit procedure for computing the pressure distribution. This procedure involves solving a 3-band matrix for a 1-dimensional reservoir, a 5-band matrix for a 2-dimensional reservoir, and a 7-band matrix for a 3-dimensional reservoir. In order to increase the speed of solution for 2- and 3-dimensional reservoirs, an algorithm called Alternating Direction Implicit (ADI) procedure is utilized [McWhorter and Sinada (1977); Thomas

(1982)]. The ADI technique computes pressure distribution assuming alternating-flow in multiple directions. During each time-step, it computes pressures in sub-steps. In a 2-dimensional simulation, ADI computes pressures in two half-steps. In the first half-step, it computes pressures assuming flow occurs in the first direction only and no flow would occur in the second direction. In the second half-step, it computes pressures assuming flow occurs in the second direction only and no flow would occur in the first direction. In a 3-dimensional simulation, ADI computes pressures in three sub-steps. In the first 1/3 step, it computes pressures assuming flow occurs in the first direction only and no flow would occur in the other two directions. In the second 1/3 step, it computes pressures assuming flow occurs in the second direction only and no flow would occur in the other two directions. In the third 1/3 step, it computes pressures assuming flow occurs in the third direction only and no flow would occur in the other two directions. The procedure is illustrated in Figure 1(a).

The following notations are defined in numerical formulations:

$i =$ space index of grid block in x – direction

$j =$ space index of grid block in y – direction

$k =$ space index of grid block in z – direction

$n =$ time index for time t

$n+1/3 =$ time index for time $t+\Delta t/3$

$n+2/3 =$ time index for time $t+(2/3)\Delta t$

$n+1 =$ time index for time $t+\Delta t$

For a 3-dimensional problem, the numerical formulation in the first 1/3 step is

$$\begin{aligned}&\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+1/3}-2p_{w_{i,j,k}}^{n+1/3}+p_{w_{i+1,j,k}}^{n+1/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n}-2p_{i,j,k}^{n}+p_{i,j+1,k}^{n}\right)}{(\Delta y)^2}\\&+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n}-2p_{i,j,k}^{n}+p_{i,j,k+1}^{n}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+1/3}-p_{i,j,k}^{n}}{\Delta t/3}\end{aligned}\quad. \tag{2}$$

The numerical formulation in the second 1/3 step is

$$\begin{aligned}&\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+1/3}-2p_{w_{i,j,k}}^{n+1/3}+p_{w_{i+1,j,k}}^{n+1/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n+2/3}-2p_{i,j,k}^{n+2/3}+p_{i,j+1,k}^{n+2/3}\right)}{(\Delta y)^2}\\&+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n+1/3}-2p_{i,j,k}^{n+1/3}+p_{i,j,k+1}^{n+1/3}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+2/3}-p_{i,j,k}^{n+1/3}}{\Delta t/3}\end{aligned}\quad. \tag{3}$$

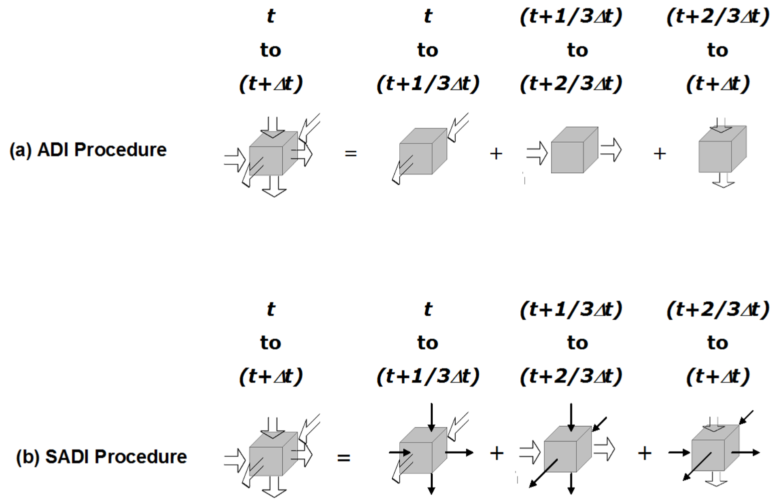

Figure 1: A graphical illustration of ADI and SADI procedures.

The numerical formulation in the third 1/3 step is

$$\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+2/3}-2p_{w_{i,j,k}}^{n+2/3}+p_{w_{i+1,j,k}}^{n+2/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n+2/3}-2p_{i,j,k}^{n+2/3}+p_{i,j+1,k}^{n+2/3}\right)}{(\Delta y)^2}$$
$$+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n+1}-2p_{i,j,k}^{n+1}+p_{i,j,k+1}^{n+1}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+1}-p_{i,j,k}^{n+2/3}}{\Delta t/3} \quad . \tag{4}$$

In a 2-dimensional simulation, the ADI procedure solves a 3-band matrix twice, instead of a 5-band matrix once, in each time step. In a 3-dimensional simulation, the ADI procedure solves a 3-band matrix three times, instead of a 7-band matrix once, in each time step. Because the ADI avoids solving large-size matrix, the time of computation is greatly reduced. However, the ADI technique become less table than the simultaneous sweep (SS) procedure as the time-step size gets larger, especially in simulation systems with non-uniform grid blocks.

3 New Algorithm SADI

A new algorithm called Semi Alternating Direction Implicit (SADI) procedure has been developed and tested in this study. It is a modification of the ADI procedure to make the ADI more stable while keeping its nature of fast solution. The SADI procedure is illustrated in Figure 1(b). The SADI technique also computes pressure distribution assuming alternating-flow in multiple directions. During each time-step, it computes pressures in sub-steps. In a 2-dimensional simulation, SADI computes pressures in two half-steps. In the first half-step, it computes pressures assuming a major flow occur in the first direction at new pressure levels and a minor flow occur in the second direction at a new pressure level in the central block and the old pressure levels in the up- and down-stream blocks. In the second half-step, it computes pressures assuming a major flow occurs in the second direction at new pressure levels and a minor flow occur in the first direction at a new pressure level in the central block and the old pressure levels in the up- and down-stream blocks. In a 3-dimensional simulation, ADI computes pressures in three sub-steps. In the first 1/3 step, it computes pressures assuming flow occurs in the first direction at new pressure levels and minor flow occur in the other two directions at a new pressure level in the central block and the old pressure levels in the up- and down-stream blocks. In the second 1/3 step, it computes pressures assuming flow occurs in the second direction at new pressure levels and minor flow occur in the other two directions at a new pressure level in the central block and the old pressure levels in the up- and down-stream blocks. In the third 1/3 step, it computes pressures assuming flow occurs in the third direction at new pressure levels and minor flow occur in the other two directions at a new pressure level in the central block and the old pressure levels in the up- and down-stream blocks.

For a 3-dimensional problem, the numerical formulation in the first 1/3 step is

$$\begin{aligned}&\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+1/3}-2p_{w_{i,j,k}}^{n+1/3}+p_{w_{i+1,j,k}}^{n+1/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n}-2p_{i,j,k}^{n+1/3}+p_{i,j+1,k}^{n}\right)}{(\Delta y)^2}\\&+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n}-2p_{i,j,k}^{n+1/3}+p_{i,j,k+1}^{n}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+1/3}-p_{i,j,k}^{n}}{\Delta t/3}\end{aligned}\quad. \tag{5}$$

The numerical formulation in the second 1/3 step is

$$\begin{aligned}&\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+1/3}-2p_{w_{i,j,k}}^{n+2/3}+p_{w_{i+1,j,k}}^{n+1/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n+2/3}-2p_{i,j,k}^{n+2/3}+p_{i,j+1,k}^{n+2/3}\right)}{(\Delta y)^2}\\&+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n+1/3}-2p_{i,j,k}^{n+2/3}+p_{i,j,k+1}^{n+1/3}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+2/3}-p_{i,j,k}^{n+1/3}}{\Delta t/3}\end{aligned}\quad. \tag{6}$$

The numerical formulation in the third 1/3 step is

$$\begin{aligned}&\frac{T_{x_{i,j,k}}\left(p_{i-1,j,k}^{n+2/3}-2p_{w_{i,j,k}}^{n+1}+p_{w_{i+1,j,k}}^{n+2/3}\right)}{(\Delta x)^2}+\frac{T_{y_{i,j,k}}\left(p_{i,j-1,k}^{n+2/3}-2p_{i,j,k}^{n+1}+p_{i,j+1,k}^{n+2/3}\right)}{(\Delta y)^2}\\&+\frac{T_{z_{i,j,k}}\left(p_{i,j,k-1}^{n+1}-2p_{i,j,k}^{n+1}+p_{i,j,k+1}^{n+1}\right)}{(\Delta z)^2}+Q_{i,j,k}=\alpha_{i,j,k}\frac{p_{i,j,k}^{n+1}-p_{i,j,k}^{n+2/3}}{\Delta t/3}\end{aligned}. \tag{7}$$

4 Testing of SADI

The new procedure SADI was tested for numerical stability and speed of computation by comparing with the SS and ADI procedures in solving a 2-dimensional problem and a 3-dimensional problem. Due to limited space, only the result from the 3-dimensional simulation is presented in this paper.

The 3-dimensionl problem is to compute pressure distribution in the oil reservoir described in Table 1 and Figure 2.

Table 1: Base parameter values used in the 3-dimensional reservoir simulation.

Reservoir area	367.31	acre
Pay zone thickness	150	ft
Simulation time	120	day
Reservoir Pressure	4,930	psia
Reservoir temperature	140	°F
Porosity	0.3	
Permeability	0.05	Darcy
Initial water saturation	0.5	
Water formation volume factor	1.05	rb/stb
Total compressibility	0.00003	psi^{-1}
API gravity	35	
Bubble point pressure	3000	psia
Gas specific gravity	0.7	air=1

The 3-dimensional oil reservoir was modeled using 4,800 grid blocks with Δx = 100 feet, Δy = 100 feet, and Δz = 50 feet. The numerical stabilities of the ADI, SADI, and SS procedures were tested in the model using various time step sizes from 1 day to 10 days. The stability test result is shown in Table 2. The data indicates that the ADI procedure is stable up to time step size of 4 days, while the

Table 2: Stability comparison of 3 procedures in a 3-dimensional simulation with 4800 grid blocks (Δx=100ft, Δy=100ft, Δz=50ft)

Time-step size	Numerical Stability of Procedure		
(day)	ADI	SADI	SS
1	Yes	Yes	Yes
2	Yes	Yes	Yes
3	Yes	Yes	Yes
4	Yes	Yes	Yes
5	No	Yes	Yes
10	No	Yes	Yes

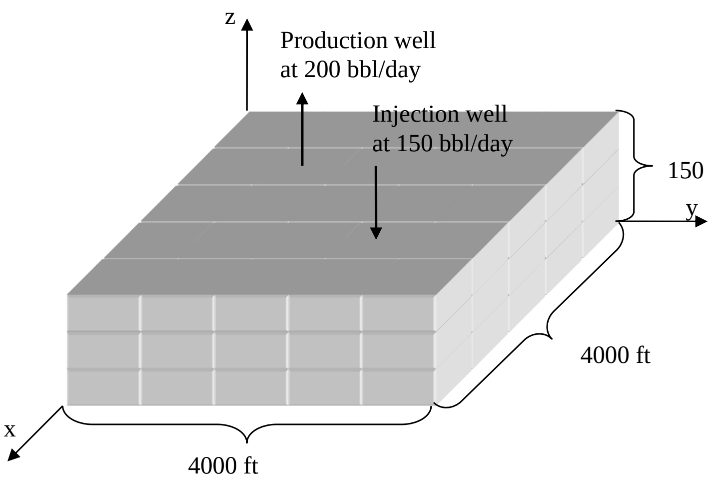

Figure 2: A 4000ft×4000ft×150ft reservoir model with one production well and one water injection well .

SADI procedure is as stable as the SS procedure for time step size of at least 10 days. This means that the SADI procedure is at least 2.5 time stable than the ADI procedure.

The speeds of computation with the ADI, SADI, and SS procedures were tested using 300, 1,200, and 4,800 grid-block models with time step sizes from 1 to 10 days (1 to 4 days for the ADI procedure). It was found that the SADI and ADI procedures are similar in terms of CPU time required to solve the problem. To minimize the effect of hardware on the comparison of computing speeds, fold of increase (FOI) in computing speed was defined as the CPU time required by the

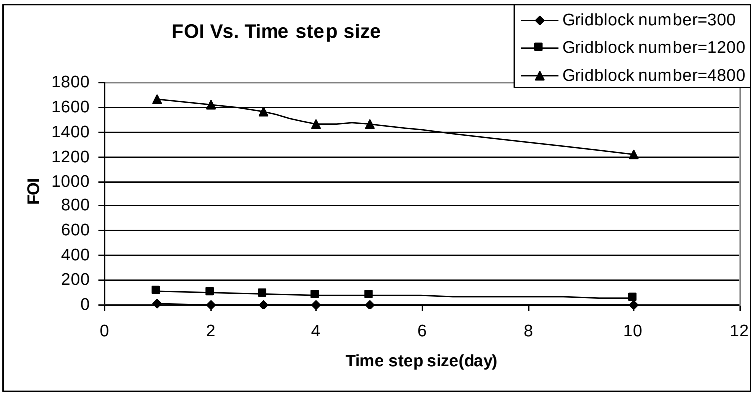

Figure 3: Fold of increase (FOI) in simulation speed for a 3-dimensional reservoir.

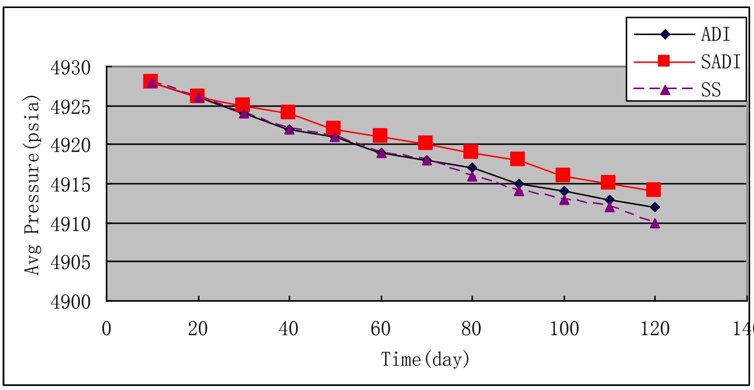

Figure 4: Comparison of results of numerical simulation with 3 procedures.

SS procedure over the CUP time required by the SADI procedure. The FOI data are plotted in Figure 3 for different numbers of grid blocks at various time step sizes. This figure indicates that the FOI declines slowly with the time step size and increases sharply with the number of grid blocks. The FOI decreases from 1,672 to 1,234 when the time step size increases from 1 day to 10 days. The FOI increases from 113 to 1,672 (nearly 15 times) when the number of grid blocks increases 4 times from 1,200 to 4,800 with time step size of 1 day.

It is understood that different numerical procedures will generate slightly different results due to different degrees of numerical dispersion. However, this does not affect application of numerical simulation because the numerical model has to be tuned with real data through history-match of well production data. Figure 4 presents the simulated average reservoir pressures given by the 3 procedures. It indicates that the pressure values generated by the SADI procedure are slightly higher than that by the SS procedure and the ADI procedure, although their trends are very similar.

5 Conclusions

A new-numerical procedure, called Semi-Alternating Direction Implicit (SADI), was developed for solving the Diffusivity Equation (DE) using the Finite Difference Method (FDM) in this study. The SADI procedure was compared with the Simultaneous Sweep (SS) procedure and the Alternating Direction Implicit (ADI) procedure in terms of numerical stability and computing speed in a 3-dimensional oil reservoir simulator. Results from the model-testing allows for drawing the following conclusions:

1. The SADI procedure is as stable as the SS procedure, and is much more stable than the ADI procedure. The maximum stable time-step size with SADI is at least twice that with ADI.

2. The SADI procedure is as fast as the ADI procedure and over a thousand times faster than the SS procedure. SADI's nature of increasing speed of computation does not diminish quickly as the time step size increases. The advantage of the SADI procedure in speeding up computation is more pronounced in large scale (large number of grid blocks) systems than in small scale systems.

3. The function values generated by the SADI procedure are slightly greater than that by the SS procedure and the ADI procedure. This is believed to be due to the different degrees of numerical dispersion associated with the different procedures.

4. Although the SADI procedure was tested in an oil reservoir simulator that is used in the petroleum industry, the procedure is equally valid for any numerical simulations that involve solving the Diffusivity Equation (DE) using the Finite Difference Method (FDM). Such numerical simulations are also found in computations of heat conduction in engineering systems and computations of fluid transport in underground aquifer.

References

Carslaw, H. S.; Jaeger, J. C. (1959): *Conduction of Heat in Solids.* Oxford Science Publications, Oxford.

Dake, L. P. (1978):*Fundamentals of Reservoir Engineering.* Elsevier Science Publishing Company, Amsterdam.

McWhorter, D. B.; Sunada, D. K. (1977): *Ground-Water Hydrology and Hydraulics.* Water Resources Publications, Fort Collins.

Thomas, G. W. (1982): *Principles of Hydrocarbon Reservoir Simulations.* International Human Resources Development Corporation, 2^{nd} Edition, Boston.