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A Personalized Comprehensive Cloud-Based Method for Heterogeneous MAGDM and Application in COVID-19

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ABSTRACT

This paper proposes a personalized comprehensive cloud-based method for heterogeneous multi-attribute group decision-making (MAGDM), in which the evaluations of alternatives on attributes are represented by LTs (linguistic terms), PLTSs (probabilistic linguistic term sets) and LHFSs (linguistic hesitant fuzzy sets). As an effective tool to describe LTs, cloud model is used to quantify the qualitative evaluations. Firstly, the regulation parameters of entropy and hyper entropy are defined, and they are further incorporated into the transformation process from LTs to clouds for reflecting the different personalities of decision-makers (DMs). To tackle the evaluation information in the form of PLTSs and LHFSs, PLTS and LHFS are transformed into comprehensive cloud of PLTS (C-PLTS) and comprehensive cloud of LHFS (C-LHFS), respectively. Moreover, DMs' weights are calculated based on the regulation parameters of entropy and hyper entropy. Next, we put forward cloud almost stochastic dominance (CASD) relationship and CASD degree to compare clouds. In addition, by considering three perspectives, a comprehensive tri-objective programming model is constructed to determine the attribute weights. Thereby, a personalized comprehensive cloud-based method is put forward for heterogeneous MAGDM. The validity of the proposed method is demonstrated with a site selection example of emergency medical waste disposal in COVID-19. Finally, sensitivity and comparison analyses are provided to show the effectiveness, stability, flexibility and superiorities of the proposed method.

KEYWORDS

Heterogeneous MAGDM; regulation parameter; C-PLTS; C-LHFS; CASD

1 Introduction

Multi-attribute group decision-making (MAGDM) refers to a decision situation where a group of decision-makers (DMs) provide their own opinions on a given set of alternatives under a set of attributes, and then select the optimal alternative(s) by aggregating their opinions [1–6]. Since the real-life MAGDM problems often involve multiple different types of attributes, it is not easy for DMs to evaluate all attributes in only one form of evaluation information, which results in the appearance of heterogeneous MAGDM. In heterogeneous MAGDM process, the evaluations of different attributes can be expressed by qualitative and quantitative forms. For example, when a customer selects a car, a real number or an interval number can be used to evaluate its price,



but a LT (linguistic term) or its extended forms will be preferred than quantitative value to evaluate its safety. Due to the growing uncertainty of actual decision-making environments, it is more convenient and flexible for DMs to employ qualitative forms, e.g., LT, PLTS (probabilistic linguistic term set), LHFS (linguistic hesitant fuzzy set), to characterize the evaluation information of alternatives on attributes. Both PLTS and LHFS are two important extensions of LT. PLTS, proposed by Pang et al. [7], consists of LTs and their corresponding probabilities. LHFS, initiated by Meng et al. [8], contains LTs and their corresponding memberships. For example, a group of DMs are invited to select a site for emergency medical waste disposal during the outbreak of COVID-19. Five attributes, i.e., geographical location, equipment, process technologies, disposal capacity and transport capacity, are chosen to evaluate the alternatives. LTs are suitable to evaluate the geographical location. Since the evaluations for equipment and process technologies are divided into two parts: LTs and corresponding probabilities, PLTSs are suitable to evaluate the equipment and process technologies. Besides, it is easy for DMs to evaluate the disposal capacity and the transportation capacity by using LHFSs. Therefore, the site selection of emergency medical waste disposal is a typical problem of heterogeneous MAGDM with different types of qualitative evaluations. Currently, many scholars have studied heterogeneous MAGDM problems. Yu et al. [1] developed a fusion method based on trust and behavior analysis for heterogeneous MAGDM scenarios. Liu et al. [9] proposed a new axiomatic design-based mathematical programming method for heterogeneous MAGDM with linguistic fuzzy truth degrees. Gao et al. [10] provided a consensus model for heterogeneous MAGDM with several attribute sets. Wan et al. [11] initiated a new prospect theory based method for heterogeneous MAGDM with hybrid fuzzy truth degrees of alternative comparisons. With the in-depth study of previous literature, many heterogeneous MAGDM problems have been effectively solved. However, there is little research on heterogeneous MAGDM with multiple qualitative forms (especially LT, PLTS and LHFS). To fill the gap, this paper intends to use LTs, PLTSs and LHFSs to portray heterogeneous evaluations.

Qualitative evaluations are not easy to be computed directly, especially when DMs use diverse forms of qualitative evaluations. At present, some models have been developed to deal with the calculations of qualitative evaluations, such as linguistic symbolic model [12], two-tuple linguistic model [13], cloud model [14,15]. Linguistic symbolic model and two-tuple linguistic model deal with LTs by converting them into real numbers. Cloud model proposed by Li et al. [14,15] is a more effective tool to describe qualitative concepts since it has strong power in capturing the fuzziness and randomness of LTs, simultaneously. Based on the probability theory and fuzzy set theory, the cloud model utilizes three numerical characteristics, i.e., mathematical expectation Ex , entropy En and hyper entropy He , to realize the nimble and effective inter-transformation between qualitative evaluations and quantitative values. Cloud model has attracted extensive attention from scholars and has been successfully applied to various fields, such as behavioral analysis [16], artificial intelligence [17,18], system assessment [19], data mining [20], knowledge discovery [21] and decision-making [22–34], etc.

Although the above mentioned cloud-based methods [22–32] are efficient in handling various practical decision-making problems, there still exist some defects as follows:

- (1) Some previous studies [22–27,29,31,32] depicted the evaluations only with a single qualitative form, which might limit their applications in practical decision-making problems.
- (2) Few studies took DMs' personalities into account during the transformation process. Wang et al. [24] introduced overlap parameter into the transformation process to reflect the DMs'

personality and preference. But the determination of overlap parameter is a little subjective, which may lead to unreasonable decision results.

- (3) The comprehensive clouds in existing approaches [23,32] may cause the loss and distortion of evaluation information.
- (4) Methods in [22,24,26,29,32] used the expected score values of clouds to rank the alternatives, while methods in [23,27,30,33,34] utilized the closeness coefficient and priority vector to rank the alternatives. However, the expected score values of clouds sometimes are unstable since the expected score values are generated randomly. The closeness coefficient and priority vector depend on the distances between clouds, but different definitions of distance between clouds usually generate different ranking results.

To overcome the above limitations, this paper develops a personalized comprehensive cloud-based method for heterogeneous MAGDM, in which the evaluations of alternatives on attributes are represented as LTs, PLTSs and LHFSs. Regulation parameters of entropy and hyper entropy are proposed to reflect the DMs' personalities. Two approaches are put forward to transform PLTS and LHFS into comprehensive cloud of PLTS (C-PLTS) and comprehensive cloud of LHFS (C-LHFS), respectively. The cloud almost stochastic dominance (CASD) relationship and CASD degree are initiated to compare clouds and further rank the alternatives. In addition, a novel approach is presented to obtain DMs' weights and a comprehensive tri-objective programming model is constructed to determine the attribute weights. The proposed method is employed to the site selection of emergency medical waste disposal in COVID-19. Compared with existing studies, the major contributions of this paper are highlighted in the following four aspects:

- (1) Regulation parameters of entropy and hyper entropy are defined objectively. By incorporating regulation parameters into the transformation process, DMs' personalities are reflected well. Moreover, DMs' weights are objectively determined based on the proposed regulation parameters.
- (2) From the perspectives of probability and membership degree, two approaches are put forward to transform PLTS and LHFS into C-PLTS and C-LHFS, respectively. The modified ratios of LTs decrease the loss and distortion of evaluation information.
- (3) CASD relationship and CASD degree are defined and used to compare clouds. Based on the proposed comparison approach for clouds, the alternatives are ranked and the ranking results are stable and effective.
- (4) A comprehensive tri-objective programming model is constructed to determine the attribute weights. In this model, three perspectives are considered, including differentiation between evaluation values, relationship between attributes and the amount of information contained in evaluation values. The setting of balance coefficients enables DMs to make a tradeoff in the three perspectives, which can improve the flexibility of the proposed method.

The remainder of this paper is organized as follows: [Section 2](#) briefly introduces some concepts related to LTs and reviews cloud model as well as almost first-degree stochastic dominance (AFSD). [Section 3](#) describes the heterogeneous MAGDM problem and develops two novel transformation approaches from PLTS and LHFS to comprehensive clouds. In [Section 4](#), a personalized comprehensive cloud-based method is proposed for heterogeneous MAGDM problem. A numerical example and sensitivity analyses are conducted to illustrate the proposed method in [Section 5](#). [Section 6](#) performs some comparison analyses to explain the superiorities of the proposed method. Some conclusions are summarized in [Section 7](#).

2 Preliminaries

This section briefly introduces some concepts related to LTs and reviews cloud model as well as AFSD.

2.1 LT and Some Related Concepts

Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a finite and completely ordered discrete term set with odd cardinality [35], where τ is a nonnegative integer, and s_i represents a possible value for a LT. The set S is a linguistic term set (LTS) if $s_i, s_j \in S$ satisfy the following properties:

- (i) Ordered set: $s_i \leq s_j$ if and only if $i \leq j$;
- (ii) Negation operation: $neg(s_i) = s_j$, if $i + j = 2\tau + 1$.

In linguistic evaluation scales, the absolute deviation of semantics between any two adjacent LTs may increase, decrease or remain unchanged with increasing linguistic subscripts. To reflect various semantics deviation, linguistic scale functions (LSFs) [22] are used to flexibly portray evaluation scales according to specific semantic situations.

Definition 1. [22,36] Let $s_i \in S$ be a LT. When $\theta_i \in [0, 1]$ is a numerical value, the LSF is mapped from s_i to θ_i ($i = 1, 2, \dots, 2\tau + 1$) as follows:

$$F: s_i \rightarrow \theta_i, (i = 1, 2, \dots, 2\tau + 1)$$

where $0 \leq \theta_1 < \theta_2 < \dots < \theta_{2\tau+1} \leq 1$. θ_i represents the evaluation of DM when he/she chooses the LT s_i . As a result, the function F describes the semantics of s_i ($i = 1, 2, \dots, 2\tau + 1$). LSFs are strictly monotonously increasing with respect to the subscript i .

Three kinds of LSFs are shown below:

$$\text{LSF1: } F(s_i) = \theta_i = \frac{i-1}{2\tau}, (i = 1, 2, \dots, 2\tau + 1) \quad (1)$$

In LFS1, the absolute deviation between adjacent LTs remains unchanged with increasing linguistic subscripts. Take $\tau = 3$ as an example, and the LTs are graphically shown in Fig. 1.

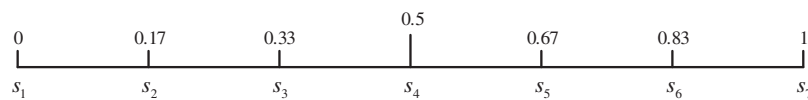


Figure 1: LSF1 ($\tau = 3$)

$$\text{LSF2: } F(s_i) = \theta_i = \begin{cases} \frac{a^\tau - a^{\tau-i+1}}{2(a^\tau - 1)}, & (i = 1, 2, \dots, \tau + 1) \\ \frac{a^\tau + a^{i-\tau-1} - 2}{2(a^\tau - 1)}, & (i = \tau + 2, \tau + 3, \dots, 2\tau + 1) \end{cases} \quad (2)$$

Lots of experimental studies [37] have illustrated that a generally lies in the interval [1.36, 1.4]. Moreover, a also can be determined by a subjective approach [22]. In LFS2, the absolute deviation between adjacent LTs gradually increases from the middle of the given LTs to both ends. If we take $\tau = 3$ and set $a = 1.36$, the LTs are graphically shown in Fig. 2.

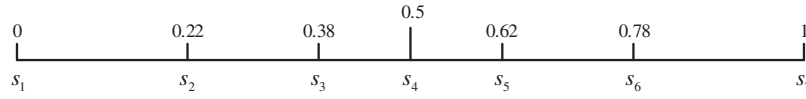


Figure 2: LSF2 ($\tau = 3$, $a = 1.36$)

$$\text{LSF3: } F(s_i) = \theta_i = \begin{cases} \frac{\tau^\alpha - (\tau - i + 1)^\alpha}{2\tau^\alpha}, & (i = 1, 2, \dots, \tau + 1) \\ \frac{\tau^\beta + (i - \tau - 1)^\beta}{2\tau^\beta}, & (i = \tau + 2, \tau + 3, \dots, 2\tau + 1) \end{cases}. \quad (3)$$

LSF3 is defined based on prospect theory's value function and the DMs' different sensation for the absolute deviation between adjacent linguistic subscripts. α and β ($\alpha, \beta \in [0, 1]$) represent the curvature of the subjective value function for gain and loss, respectively [38]. LSF3 reduces to LSF1 when $\alpha = \beta = 1$. The absolute deviation between adjacent LTs gradually decreases from the middle of the given LTs to both ends. If we take $\tau = 3$ and set $\alpha = \beta = 0.8$, the LTs are graphically shown in Fig. 3.

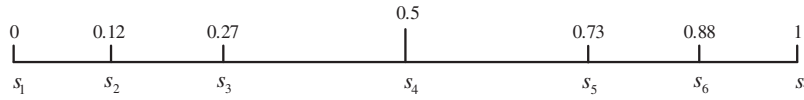


Figure 3: LSF3 ($\tau = 3$, $\alpha = \beta = 0.8$)

In order to save all of the given information and facilitate calculation, the aforesaid functions can be extended into $F^* : \tilde{S} \rightarrow R^+$, where $F^*(s_i) = \theta_i$ is a continuous and strictly monotonously increasing function.

Definition 2. [7] Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS. A PLTS $L(p)$ can be defined as $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S, p^{(l)} \geq 0, i = 1, 2, \dots, \#L(p), \sum_{l=1}^{\#L(p)} p^{(l)} \leq 1\}$, where $s^{(l)}(p^{(l)})$ is the LT $s^{(l)}$ associated with the probability $p^{(l)}$, and $\#L(p)$ denotes the number of all different LTs in $L(p)$. If $\sum_{l=1}^{\#L(p)} p^{(l)} < 1$, then $\tilde{p}^{(l)} = p^{(l)} / \sum_{l=1}^{\#L(p)} p^{(l)}$ is used to normalize the PLTS.

In this paper, it is assumed that all PLTSs have already been normalized.

Definition 3. [8] Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS. A LHFS LH in S is defined as $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$, where $lh(s^{(l)}) = \{r_1, r_2, \dots, r_{\#lh(s^{(l)})}\}$ is a set with $\#lh(s^{(l)})$ values in $(0, 1]$ and denotes the possible membership degrees of the element $s^{(l)} \in S$ to the set LH . $\#LH$ denotes the number of all different LTs in LH , and $\#lh(s^{(l)})$ represents the count of real numbers in $lh(s^{(l)})$.

2.2 Cloud Model

Definition 4. [14] Let U be the universe of discourse and T be a qualitative concept in U . If $x \in U$ is a random instantiation of concept T that satisfies $x \sim N(Ex, En'^2)$ and $En' \sim N(En, He^2)$, and $y \in [0, 1]$ is the certainty degree of x belonging to T that satisfies $y = \exp(-\frac{(x-Ex)^2}{2(En')^2})$, then the distribution of x in the universe U is defined as a normal cloud, and (x, y) represents a cloud drop.

For simplicity, normal cloud is called as cloud hereafter. The degree of certainty of x belonging to concept T is a probability distribution rather than a fixed number. Hence, $\forall x \in U$, $y = \exp(-\frac{(x-Ex)^2}{2(En')^2})$ is a one-to-many mapping.

There are two kinds of uncertainty: randomness and fuzziness. Randomness refers to the uncertainty contained in an event that has a clear definition but do not necessarily occur. Fuzziness refers to the uncertainty contained in an event that has appeared but it is difficult to define it accurately [39]. There is a practical demand to describe fuzziness and randomness inherent in LTs simultaneously. Cloud can perfectly depict the overall quantitative properties of a concept through three numerical characteristics: mathematical expectation Ex , entropy En and hyper entropy He , where Ex is the mathematical expectation of cloud drops belonging to a concept in the universe, and En reflects the uncertainty measurement of a qualitative concept, including randomness and fuzziness. From the perspective of probability theory, En is similar to standard variance of random variables. From the point of fuzzy set theory, En represents the scope in which cloud drops are accepted by the concept, and it indicates the support set of the concept with membership degrees larger than 0. As a result, En reflects randomness and fuzziness of a qualitative concept and their correlation, simultaneously. He represents the degree of uncertainty of En , i.e., the second-order entropy of the entropy [15,40]. A cloud can be described by Ex , En , He , and denoted by $C = (Ex, En, He)$.

Definition 5. [15] Given two clouds $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$, some operations of clouds are defined as follows:

- (1) $C_1 + C_2 = (Ex_1 + Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2})$;
- (2) $C_1 - C_2 = (Ex_1 - Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2})$;
- (3) $\gamma C_1 = (\gamma Ex_1, \sqrt{\gamma} En_1, \sqrt{\gamma} He_1)$, ($\gamma \geq 0$).

2.3 AFSD

The AFSD is used to compare two stochastic variables. It was proposed by Leshno and Levy [41]. Let X_1 and X_2 be two stochastic variables, where $G_1(x)$ and $G_2(x)$ denote two cumulative distribution functions, respectively. Let $\Omega = \{x | G_1(x) > G_2(x)\}$, $\Theta = \{x | G_2(x) > G_1(x)\}$ and $\|G_1(x) - G_2(x)\| = \int_{\Omega} G_1(x) - G_2(x) dx + \int_{\Theta} G_2(x) - G_1(x) dx$. Then, AFSD is defined below:

Definition 6. [41,42] For $0 < \delta < 0.5$, X_1 dominates X_2 by δ -AFSD if and only if $\int_{\Omega} G_1(x) - G_2(x) dx \leq \delta \|G_1(x) - G_2(x)\|$, where $\|G_1(x) - G_2(x)\|$ corresponds to the area between G_1 and G_2 , $\int_{\Omega} G_1(x) - G_2(x) dx$ corresponds to the area that G_1 is greater than G_2 , and δ denotes the degree of first-degree stochastic dominance violation allowed.

3 Heterogeneous MAGDM Problem and Comprehensive Cloud

This section describes the heterogeneous MAGDM problem and introduces the improved transformation approach between LT and cloud in detail. Particularly, we developed two novel transformation approaches from PLTS and LHFS to comprehensive clouds.

3.1 Description for Heterogeneous MAGDM Problem

A heterogeneous MAGDM problem is to find the best solution from all feasible alternatives assessed on multiple attributes by a group of DMs. The evaluation attributes in heterogeneous MAGDM can be classed into several subsets which are expressed by different kinds of forms.

For a heterogeneous MAGDM problem, suppose that DMs d_e ($e = 1, 2, \dots, k$) have to select the optimal alternative(s) from a group of alternatives x_u ($u = 1, 2, \dots, m$) or rank these alternatives based on attributes y_v ($v = 1, 2, \dots, n$). Denote an alternative set by $X = \{x_1, x_2, \dots, x_m\}$, an attribute set by $Y = \{y_1, y_2, \dots, y_n\}$, and a DM set by $D = \{d_1, d_2, \dots, d_k\}$. Denote $Y_1 = \{y_1, y_2, \dots, y_{v_1}\}$, $Y_2 = \{y_{v_1+1}, y_{v_1+2}, \dots, y_{v_2}\}$, $Y_3 = \{y_{v_2+1}, y_{v_2+2}, \dots, y_{v_3}\}$, respectively, where $1 \leq v_1 \leq v_2 \leq v_3 \leq n$. Namely, Y is divided into three subsets Y_t ($t = 1, 2, 3$), where Y_t ($t = 1, 2, 3$) are attribute subsets in which attribute values are expressed with LTs, PLTSs and LHFSs respectively. $Y_t \cap Y_l = \emptyset$ ($t, l = 1, 2, 3; t \neq l$), $\cup_{t=1}^3 Y_t = Y$, where \emptyset is an empty set. Denote $M = \{1, 2, \dots, m\}$, $N_1 = \{1, 2, \dots, v_1\}$, $N_2 = \{v_1 + 1, v_1 + 2, \dots, v_2\}$, $N_3 = \{v_2 + 1, v_2 + 2, \dots, n\}$, $N = \{1, 2, \dots, n\}$ and $K = \{1, 2, \dots, k\}$. Denote the DM weight vector by $\mathbf{w} = (\varpi_1, \varpi_2, \dots, \varpi_k)^T$, where ϖ_e is the weight of DM d_e , satisfying that $0 \leq \varpi_e \leq 1$ ($e = 1, 2, \dots, k$) and $\sum_{e=1}^k \varpi_e = 1$. Denote the attribute weight vector by $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$, where w_v is the weight of attribute y_v , satisfying that $0 \leq w_v \leq 1$ ($v = 1, 2, \dots, n$) and $\sum_{v=1}^n w_v = 1$ [43].

Let r_{uv}^e be the evaluation of an alternative x_u on attribute y_v given by DM d_e . If $v \in N_1$, r_{uv}^e is a LT, denoted by $s_i (s_i \in S)$; If $v \in N_2$, r_{uv}^e is a PLTS, denoted by $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}$; If $v \in N_3$, r_{uv}^e is a LHFS, denoted by $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$. After normalizing, the individual original normalized evaluation matrix $R^e = (r_{uv}^e)_{m \times n}$ can be obtained as

$$R^e = (r_{uv}^e)_{m \times n} = (r_{uv}^e)_{m \times n} = \begin{matrix} & y_1 & y_2 & \cdots & y_n \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} r_{11}^e & r_{12}^e & \cdots & r_{1n}^e \\ r_{21}^e & r_{22}^e & \cdots & r_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^e & r_{m2}^e & \cdots & r_{mn}^e \end{bmatrix} \end{matrix} \quad (e = 1, 2, \dots, k) \quad (4)$$

3.2 Transformation between LT and Cloud

Generally, two kinds of approaches have been proposed for transformation from LTs to clouds so far. One is based on the golden ratio [44], and the other is based on the LSF [22]. Wang et al. [24] introduced a parameter named overlapping degree into the transformation approach [22] to determining the degree of overlap between two adjacent clouds. With the parameter $\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]$, DMs could express their preference for the degree of overlap between two adjacent clouds. However, after processing the calculation formulae for three numerical characteristics, a problem emerges. That is, once the LSF and some related parameters are fixed in [24], $D(Ex_{i,i+1}) = \frac{Ex_{i+1} - Ex_i}{3}$, $D(Ex_{i-1,i+1}) = \frac{Ex_{i+1} - Ex_{i-1}}{6}$ and $D(Ex_{i-1,i}) = \frac{Ex_i - Ex_{i-1}}{3}$ are three fixed values. It is easy to see that the values of En_i and He_i are linearly dependent on $\frac{\varepsilon_{\max} + \varepsilon_{\min}}{2}$ and $\frac{\varepsilon_{\max} - \varepsilon_{\min}}{6}$, respectively. However, the determination of overlapping degree is entirely based on the subjective preference of DMs, which means the determination of En_i and He_i is also subjective. To overcome the above defects, two regulation parameters ς and ζ for entropy and hyper entropy are proposed in this paper. We improve the transformation approach in [24] by replacing $\frac{\varepsilon_{\max} + \varepsilon_{\min}}{2}$ ($\frac{\varepsilon_{\max} - \varepsilon_{\min}}{6}$) with ς (ζ) during the transformation process. Significantly, the determination of ς and ζ is totally objective and logical. The specific approaches to determining ς and ζ are stated in Sections 3.2.1 and 3.2.2.

Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS, $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}$ be a normalized PLTS, and $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$ be a LHFS. $L(p)_{uv}^e = \{s_{uv}^{(l)e}(p_{uv}^{(l)e})\}$ denotes the evaluation of an alternative x_u on attribute y_v ($v \in N_2$) given by the DM d_e , and $\#L(p)_{uv}^e \in [1, 2\tau + 1]$ denotes

the number of all different LTs in $L(p)_{uv}^e$. $LH_{uv}^e = \{(s^{(l)}, lh(s^{(l)})_{uv}^e) | s^{(l)} \in S\}$ indicates the evaluation of an alternative x_u on attribute y_v ($v \in N_3$) given by the DM d_e . $\#LH_{uv}^e \in [1, 2\tau + 1]$ signifies the number of all different LTs in LH_{uv}^e . $\#lh(s^{(l)})_{uv}^e$ represents the count of real numbers in $lh(s^{(l)})_{uv}^e = \{r_{uv,1}^e, r_{uv,2}^e, \dots, r_{uv,\#lh(s^{(l)})_{uv}^e}^e\}$. $lh(s^{(l)})_{uv}^e$ is a set with $\#lh(s^{(l)})_{uv}^e$ values in $(0, 1]$ and denotes the possible membership degrees of the element $s^{(l)} \in S$ to the set LH_{uv}^e .

3.2.1 Determination of the Regulation Parameter of Entropy

Entropy En reflects the uncertainty measurement of a qualitative concept, specifically randomness and fuzziness. From the perspective of fuzzy set theory, it represents the scope in which the cloud drops are accepted by the concept. It is common that the more elements are used in evaluations, the more hesitant the DM is. Hence, for a hesitant DM, larger En should be assigned to its LTs. Therefore, the regulation parameter of En should be determined according to DMs' hesitant degree.

Definition 7. For a heterogeneous MAGDM, the average number of LTs used by DM d_e at an evaluation in the form of PLTSs or LHFSSs, is defined as

$$\eta^e = \frac{1}{m(n-v_1)} \sum_{u=1}^m \left[\sum_{v=v_1+1}^{v_2} \#L(p)_{uv}^e + \sum_{v=v_2+1}^n \#LH_{uv}^e \right]. \quad (5)$$

It is obvious that $\eta^e \in [1, 2\tau + 1]$.

Definition 8. The hesitant degree of DM d_e is defined as

$$HD^e = \eta^e / (2\tau + 1). \quad (6)$$

Obviously, $HD^e \in [\frac{1}{2\tau+1}, 1]$.

The determination of DM's hesitant degree is based on the average number of LTs that DM uses at a single evaluation in the form of PLTSs or LHFSSs.

A great deal of experimental research has demonstrated that regulation parameter ς generally lies in $[1, 2]$. In fact, if the hesitant degree HD^e is $\frac{1}{2\tau+1}$, it means that only one LT is used by DM d_e at each evaluation in the form of PLTSs or LHFSSs. In this situation, DM d_e is regarded as a decisive and confident person. Accordingly, it is appropriate for DM d_e to take 1 as the value of ς . The more the LTs used by DM d_e , the bigger the value of ς is. Based on this premise, the regulation parameter ς of entropy is defined as follows:

Definition 9. Let $\rho_1 = 1 + \frac{2\tau}{2\tau+1}$. The regulation parameter of entropy for DM d_e is defined as

$$\varsigma^e = \log_{\rho_1} \left(\frac{\eta^e}{2\tau+1} + \frac{2\tau}{2\tau+1} \right) + 1 = \log_{\rho_1} \left(HD^e + \frac{2\tau}{2\tau+1} \right) + 1, \quad (7)$$

where $\eta^e \in [1, 2\tau + 1]$, $HD^e \in [\frac{1}{2\tau+1}, 1]$. Clearly, it holds that $\varsigma^e \in [1, 2]$.

In the following, an example is given to illustrate how to determine the value of ς .

Example 1. Let $S = \{s_i | i = 1, 2, \dots, 7\}$ be a LTS. There are three alternatives x_1, x_2 and x_3 . In order to select the optimal alternative, DM d_e gives evaluations for x_1, x_2, x_3 on three attributes

y_1, y_2, y_3 . The evaluations for y_1, y_2 and y_3 are expressed in the forms of LT, PLTS and LHFS, respectively. DM d_e gives an evaluation matrix as follows:

$$R^e = (r_{uv}^e)_{3 \times 3} = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} s_4 & \{s_6(0.1), s_7(0.9)\} & \{(s_3, 0.8, 0.9), (s_4, 0.6)\} \\ s_2 & \{s_3(0.1), s_4(0.2), s_5(0.7)\} & \{(s_5, 0.7)\} \\ s_6 & \{s_4(1)\} & \{(s_1, 0.8)\} \end{bmatrix} \end{matrix}.$$

Calculate the average number of LTs used by DM d_e at an evaluation in the form of PLTSs or LHFSs by Eq. (5):

$$\eta^e = \frac{1}{3 \times 2} \times [(2+2) + (3+1) + (1+1)] = 1.6667$$

Calculate the hesitant degree of DM d_e by Eq. (6):

$$HD^e = \frac{1.6667}{7} = 0.2381$$

Then, the value of ς^e is calculated by Eq. (7):

$$\varsigma^e = \log_{1+0.8571}(0.2381 + 0.8571) + 1 = 1.1470$$

3.2.2 Determination of the Regulation Parameter of Hyper Entropy

Hyper entropy He represents the degree of uncertainty of En , i.e., the second-order entropy of the entropy. The larger He is, the thicker the cloud is, and the wider the distribution of membership is. Thus, on the one hand, the information entropy as a very important concept to measure the uncertainty in evaluation information provided by DMs, the larger it is, the larger He should be. On the other hand, membership degree as an index to measure the degree that an element belongs to a certain concept, the lower it is, the larger the indeterminacy degree is. Furthermore, the larger indeterminacy degree in the decision matrix provided by DM, the larger He should be. With the above analysis, the regulation parameter of He should be determined according to indeterminacy degree and information entropy.

Definition 10. [45] The information entropy of $L(p)$ is defined as follows:

$$H(L(p)) = -\log_2 z \sum_{l=1}^{\#L(p)} (p^{(l)}) \log_2(p^{(l)}), \quad (8)$$

where z is a constant that is set to 1.28 in this paper as [45] sets. It is easily seen that $H(L(p)) \in [0, \log_2 z \cdot \log_2(2\tau + 1)]$.

Definition 11. The information entropy of DM d_e is defined as follows:

$$H^e = -\frac{\log_2 z}{m(v_2 - v_1)} \sum_{u=1}^m \sum_{v=v_1+1}^{v_2} \sum_{l=1}^{\#L(p)_{uv}^e} (p^{(l)}) \log_2(p^{(l)}), \quad (9)$$

where z is a constant that is set to 1.28 in this paper as [45] sets. Obviously, $H^e \in [0, \log_2 z \cdot \log_2(2\tau + 1)]$.

The closer the memberships for corresponding LTs are to 0, the larger indeterminacy degree DMs have for corresponding LT.

Definition 12. The indeterminacy degree of $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$ is defined as follows:

$$ID(LH) = \frac{1}{\#LH} \sum_{l=1}^{\#LH} \left[\frac{1}{\#lh(s^{(l)})} \sum_{r \in lh(s^{(l)})} (1-r) \right]. \quad (10)$$

Clearly, it holds that $ID(LH) \in [0, 1)$.

Definition 13. The indeterminacy degree of DM d_e is defined as follows:

$$ID^e = \frac{1}{m(n-v_2)} \sum_{u=1}^m \sum_{v=v_2+1}^n \left\{ \frac{1}{\#LH_{uv}^e} \sum_{l=1}^{\#LH_{uv}^e} \left[\frac{1}{\#lh(s^{(l)})_{uv}^e} \sum_{r_{uv}^e \in lh(s^{(l)})_{uv}^e} (1-r_{uv}^e) \right] \right\}. \quad (11)$$

It is easily seen that $ID^e \in [0, 1)$.

Definition 14. Let $\rho_2 = 1 + \log_2 z \cdot \log_2(2\tau + 1)$. The regulation parameter of hyper entropy for DM d_e is defined as

$$\zeta^e = \frac{1}{2} [\log_{\rho_2}(H^e + 1) + \log_2(ID^e + 1)], \quad (12)$$

where $H^e \in [0, \log_2 z \cdot \log_2(2\tau + 1)]$, $ID^e \in [0, 1)$. Obviously, $\zeta^e \in [0, 1)$.

Take $\tau = 3$ as an example. The graphical representation for the regulation parameter of hyper entropy is shown in Fig. 4.

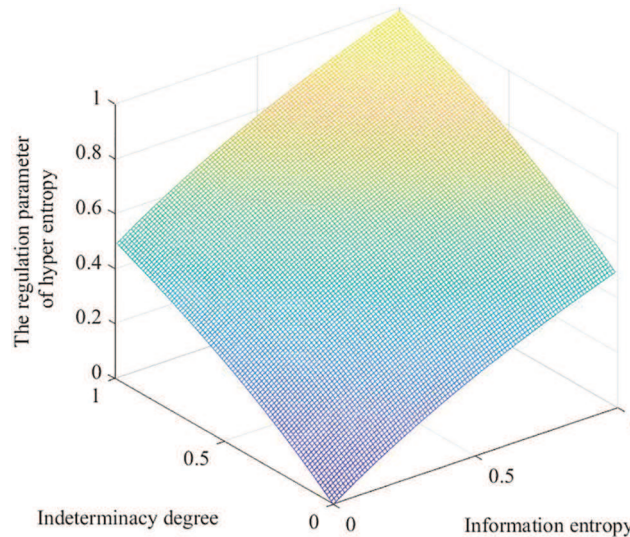


Figure 4: Graphical representation for the regulation parameter of hyper entropy ($\tau = 3$)

Example 2. Following Example 1, the value of ζ^e can be calculated as follows:

Calculate the information entropy of each evaluation in the form of PLTSs by Eq. (8):

$$H(\{s_6(0.1), s_7(0.9)\}) = -\log_2 1.28 \times (0.1 \times \log_2 0.1 + 0.9 \times \log_2 0.9) = 0.1670,$$

$$H(\{s_3(0.1), s_4(0.2), s_5(0.7)\}) = -\log_2 1.28 \times (0.1 \times \log_2 0.1 + 0.2 \times \log_2 0.2 + 0.7 \times \log_2 0.7) = 0.4120,$$

$$H(\{s_4(1)\}) = -\log_2 1.28 \times (1 \times \log_2 1) = 0$$

Calculate the information entropy of DM d_e by Eq. (9):

$$H^e = \frac{1}{3} \times (0.1670 + 0.4120 + 0) = 0.1930$$

Calculate the indeterminacy degree of each evaluation in the form of LHSs by Eq. (10):

$$ID(\{s_3(0.8, 0.9), s_4(0.6)\}) = \frac{1}{2} \times \left\{ \frac{1}{2} \times [(1 - 0.8) + (1 - 0.9)] + (1 - 0.6) \right\} = 0.275,$$

$$ID(\{s_5(0.7)\}) = 1 - 0.7 = 0.3,$$

$$ID(\{s_1(0.8)\}) = 1 - 0.8 = 0.2$$

Calculate the indeterminacy degree of DM d_e by Eq. (11):

$$ID^e = \frac{1}{3} \times (0.275 + 0.3 + 0.2) = 0.2583$$

Then, the value of ζ^e is calculated by Eq. (12):

$$\zeta^e = \frac{1}{2} \times [\log_{1.9998}(0.1930 + 1) + \log_2(0.2583 + 1)] = 0.2931$$

3.2.3 Specific Procedures for Transformation between LT and Cloud

Definition 15. [46] Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS, where τ is a positive integer. A valid universe $[X_{\min}, X_{\max}]$ is provided by DMs. Then, a LT s_i ($i = 1, 2, \dots, 2\tau + 1$) can be represented by the normal cloud $C_i = (Ex_i, En_i, He_i)$.

Then, the specific transformation procedures are shown as follows:

- (1) Calculate ς and ζ .

Determination approaches for ς and ζ are shown in Sections 3.2.1 and 3.2.2.

- (2) Calculate θ_i .

Map s_i to θ_i using LSFs.

LSF2 Eq. (2) is adopted in this paper, where $a = 1.36$.

- (3) Calculate Ex_i .

$$Ex_i = X_{\min} + \theta_i(X_{\max} - X_{\min}). \quad (13)$$

- (4) Calculate En_i .

Let (x, y) be a cloud drop, where $x \sim N(Ex, En^2)$. According to 3σ principle of the normal distribution, 99.7% cloud drops of C_i should be located in the interval $[Ex_{i-1}, Ex_{i+1}]$. However, since the distances between Ex_i and Ex_{i-1} are different from the distances between Ex_i and Ex_{i+1} ,

the entropy of C_i ($i = 2, 3, \dots, 2\tau$) are different for the right side and the left side. For simplicity, En_i ($i = 2, 3, \dots, 2\tau$) take the mean value of $\varsigma \underline{En}_i$ and $\varsigma \overline{En}_i$.

$$En_i = \begin{cases} \varsigma \overline{En}_i = \varsigma \frac{Ex_{i+1} - Ex_i}{3}, & i = 1 \\ \varsigma \frac{\overline{En}_i + \underline{En}_i}{2} = \varsigma \frac{Ex_{i+1} - Ex_{i-1}}{6}, & 2 \leq i \leq 2\tau, \\ \varsigma \underline{En}_i = \varsigma \frac{Ex_i - Ex_{i-1}}{3}, & i = 2\tau + 1 \end{cases} \quad (14)$$

where $\varsigma \underline{En}_i = \varsigma \frac{Ex_i - Ex_{i-1}}{3}$, $\varsigma \overline{En}_i = \varsigma \frac{Ex_{i+1} - Ex_i}{3}$, $\varsigma \underline{En}_i$ denotes En_i for the left side, and $\varsigma \overline{En}_i$ denotes En_i for the right side.

(5) Calculate He_i

$$He_i = \begin{cases} \varsigma \frac{Ex_{i+1} - Ex_i}{3}, & i = 1 \\ \varsigma \frac{Ex_{i+1} - Ex_{i-1}}{6}, & 2 \leq i \leq 2\tau. \\ \varsigma \frac{Ex_i - Ex_{i-1}}{3}, & i = 2\tau + 1 \end{cases} \quad (15)$$

Based on the above analyses, the corresponding cloud for LT s_i can be generated by Algorithm 1.

Algorithm 1: Transform LT into Cloud

Input: A valid universe $[X_{\min}, X_{\max}]$, a qualitative LT s_i in the LTS $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$, and DM's decision matrix.

Output: The corresponding cloud $C_i = (Ex_i, En_i, He_i)$

1. Calculate ς by Eqs. (5)–(7)
 2. Calculate ζ by Eqs. (8)–(12)
 3. Calculate θ_i by Eq. (1) or Eq. (2) or Eq. (3)
 4. Calculate Ex_i by Eq. (13)
 5. Calculate En_i by Eq. (14)
 6. Calculate He_i by Eq. (15)
 7. **Return** $C_i = (Ex_i, En_i, He_i)$
-

To illustrate the advantages of regulation parameters, an example is given below.

Example 3. Given a universe $[X_{\min}, X_{\max}]$ and a LTS $S = \{s_i | i = 1, 2, \dots, 7\}$. For alternatives x_1, x_2, x_3 regarding three attributes y_1, y_2, y_3 , DMs d_1, d_2 and d_3 give their evaluations. The evaluations for y_1, y_2 and y_3 are expressed in the forms of LT, PLTS and LHFS, respectively. DMs d_1, d_2 and d_3 give their evaluation matrices as follows:

$$R^1 = (r_{uv}^1)_{3 \times 3} = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} s_4 & \{s_6(1)\} & \{(s_3, 0.9)\} \\ s_2 & \{s_3(1)\} & \{(s_5, 1)\} \\ s_6 & \{s_4(1)\} & \{(s_1, 0.95)\} \end{bmatrix} \end{matrix}$$

$$R^2 = (r_{uv}^2)_{3 \times 3} = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} s_4 & \{s_6(0.1), s_7(0.9)\} & \{(s_3, 0.8, 0.9), (s_4, 0.6)\} \\ s_2 & \{s_3(0.1), s_4(0.2), s_5(0.7)\} & \{(s_5, 0.7)\} \\ s_6 & \{s_4(1)\} & \{(s_1, 0.8)\} \end{bmatrix} \end{matrix}$$

$$R^3 = (r_{uv}^3)_{3 \times 3} = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} s_4 & \{s_6(0.5), s_7(0.5)\} & \{(s_3, 0.2), (s_4, 0.1)\} \\ s_2 & \{s_3(0.3), s_4(0.3), s_5(0.4)\} & \{(s_5, 0.2)\} \\ s_6 & \{s_4(1)\} & \{(s_1, 0.1)\} \end{bmatrix} \end{matrix}$$

Based on Eqs. (5)–(12), the regulation parameters ς and ζ for each DM are calculated as follows:

$$\varsigma^1 = 1, \zeta^1 = 0.0352, \varsigma^2 = 1.1470, \zeta^2 = 0.2931, \varsigma^3 = 1.1470, \zeta^3 = 0.6359;$$

Based on Eqs. (2) and (13)–(15), θ_i , Ex_i , En_i , He_i ($i = 1, 2, \dots, 7$) for DMs d_1 , d_2 , d_3 are calculated and the results are shown in Tables 1–3, respectively.

Table 1: Transformation results for DM d_1

LT	s_1	s_2	s_3	s_4	s_5	s_6	s_7
θ_i	0	0.2197	0.3812	0.5	0.6188	0.7803	1
Ex_i	0.0000	2.1969	3.8122	5.0000	6.1878	7.8031	10.0000
En_i	0.7323	0.6354	0.4672	0.3959	0.4672	0.6354	0.7323
He_i	0.0258	0.0224	0.0164	0.0139	0.0164	0.0224	0.0258

Table 2: Transformation results for DM d_2

LT	s_1	s_2	s_3	s_4	s_5	s_6	s_7
θ_i	0	0.2197	0.3812	0.5	0.6188	0.7803	1
Ex_i	0.0000	2.1969	3.8122	5.0000	6.1878	7.8031	10.0000
En_i	0.8399	0.7288	0.5359	0.4541	0.5359	0.7288	0.8399
He_i	0.2146	0.1862	0.1369	0.1160	0.1369	0.1862	0.2146

Table 3: Transformation results for DM d_3

LT	s_1	s_2	s_3	s_4	s_5	s_6	s_7
θ_i	0	0.2197	0.3812	0.5	0.6188	0.7803	1
Ex_i	0.0000	2.1969	3.8122	5.0000	6.1878	7.8031	10.0000
En_i	0.8399	0.7288	0.5359	0.4541	0.5359	0.7288	0.8399
He_i	0.4657	0.4040	0.2971	0.2518	0.2971	0.4040	0.4657

Three sets of cloud generated by DMs d_1 , d_2 , d_3 are graphically shown in Figs. 5–7, respectively.

It can be seen from Figs. 5–7 that the cloud drops distribution varies in width and thickness for different DMs. In previous studies [22,23,25–31,46], clouds for the corresponding LTS are usually the same for all DMs. However, since different DMs have different personalities, knowledge

and experience, the width and thickness of clouds for the corresponding LTS should be different for different DMs. The more sure, confident and decisive the DM is for its evaluation matrix, the more concentrated and thinner the cloud distribution should be, vice versa. Unfortunately, most previous studies failed to notice this characteristic, whereas this paper sufficiently takes this characteristic into account. Although the overlap degree is considered in [24] to obtain personalized cloud sets for DMs, the determination of overlap degree depends on DMs' subjective intuition. This defect is overcome in this paper. The regulation parameters are determined according to the evaluation matrix given by DM, which means the determination of regulation parameters is objective and logical.

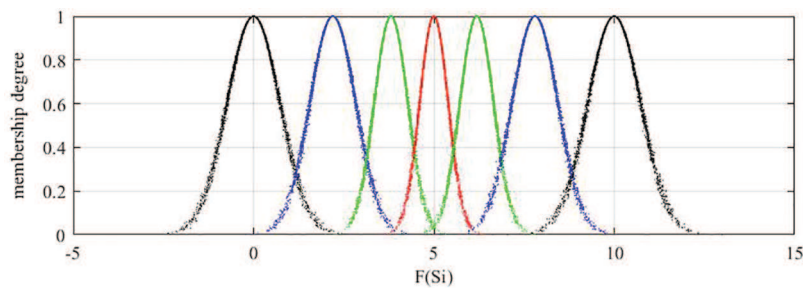


Figure 5: Clouds for the LTSs used by d_1

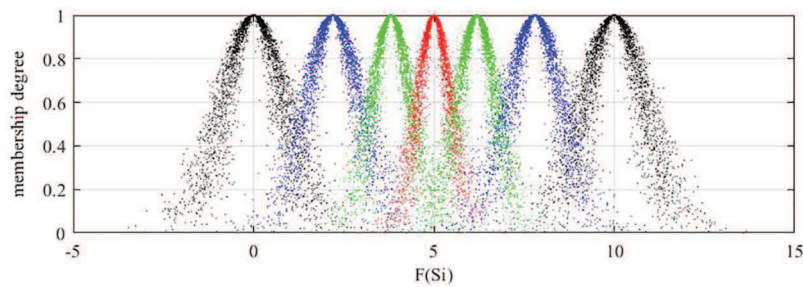


Figure 6: Clouds for the LTSs used by d_2

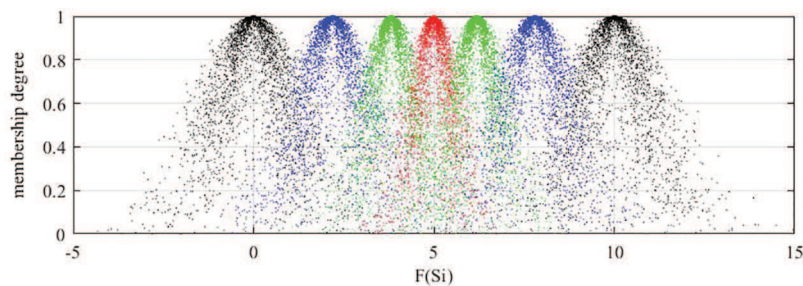


Figure 7: Clouds for the LTSs used by d_3

3.3 Transformation from PLTS and LHFS to Comprehensive Clouds

In this sub-section, two approaches are brought forward to transform PLTS and LHFS into comprehensive clouds, respectively.

3.3.1 Transformation from PLTS to Comprehensive Cloud

Definition 16. Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS. The valid universe is $[X_{\min}, X_{\max}]$. Let $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}$ be a normalized PLTS. The cloud $C_{s^{(l)}}(Ex_{s^{(l)}}, En_{s^{(l)}}, He_{s^{(l)}})$ represents LT $s^{(l)} \in S$. Then, $C_{L(p)}(Ex_{L(p)}, En_{L(p)}, He_{L(p)})$ can be defined as C-PLTS, which is characterized by three numerical characteristics $Ex_{L(p)}$, $En_{L(p)}$ and $He_{L(p)}$.

Definition 17. [24] Given a cloud $C = (Ex, En, He)$, if (x, y) is a cloud drop of C , x satisfies $x \sim N(Ex, En^2)$ and $En' \sim N(En, He^2)$. Then, the normal curve (NC) of all cloud drops can be defined as $f = \exp(-\frac{(x-Ex)^2}{2En^2})$.

In this paper, $g = \frac{1}{\sqrt{2\pi}En} \exp(-\frac{(x-Ex)^2}{2En^2})$ is utilized to represent the probability density function curve (PDFC) of C .

The specific procedures to determine $Ex_{L(p)}$, $En_{L(p)}$, $He_{L(p)}$ of C-PLTS are as follows:

- (1) Let $C_{s^{(l)}}$ be the corresponding clouds for $s^{(l)}$ ($l = 1, 2, \dots, \#L(p)$), $C_{s^{(l)}(p^{(l)})}$ be $C_{s^{(l)}}$ with corresponding probability $p^{(l)}$ and $\chi_{l,l+1} \in [Ex_{s^{(l)}}, Ex_{s^{(l+1)}}]$ be the abscissa value of intersection point between the PDFCs of $C_{s^{(l)}(p^{(l)})}$ and $C_{s^{(l+1)}(p^{(l+1)})}$. If $Ex_{s^{(l)}} + 3En_{s^{(l)}} > Ex_{s^{(l+1)}} - 3En_{s^{(l+1)}}$ ($l \in \{1, 2, \dots, \#L(p) - 1\}$), then Eq. (16) is used to calculate the value of $\chi_{l,l+1}$.

$$p^{(l)} \frac{1}{\sqrt{2\pi}En_{s^{(l)}}} \exp(-\frac{(\chi_{l,l+1} - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2}) = p^{(l+1)} \frac{1}{\sqrt{2\pi}En_{s^{(l+1)}}} \exp(-\frac{(\chi_{l,l+1} - Ex_{s^{(l+1)}})^2}{2(En_{s^{(l+1)}})^2}),$$

$$(\chi_{l,l+1} \in [Ex_{s^{(l)}}, Ex_{s^{(l+1)}}]). \quad (16)$$

- (2) Use Eq. (17) to calculate the area for the PDFC of $C_{s^{(l)}(p^{(l)})}$ from lower limit to upper limit, which can be denoted by $A_{s^{(l)}}$.

$$A_{s^{(l)}} = \begin{cases} p^{(l)} \frac{1}{\sqrt{2\pi}Ex_{s^{(l)}}} \int_{Ex_{s^{(l)}} - 3En_{s^{(l)}}}^{\chi_{l,l+1}} \exp\left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2}\right) dx, & l = 1 \\ p^{(l)} \frac{1}{\sqrt{2\pi}Ex_{s^{(l)}}} \int_{\chi_{l-1,l}}^{\chi_{l,l+1}} \exp\left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2}\right) dx, & l \in \{2, 3, \dots, \#L(p) - 1\} \\ p^{(l)} \frac{1}{\sqrt{2\pi}Ex_{s^{(l)}}} \int_{\chi_{l-1,l}}^{Ex_{s^{(l)}} + 3En_{s^{(l)}}} \exp\left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2}\right) dx, & l = \#L(p). \end{cases} \quad (17)$$

If $Ex_{s^{(l-1)}} + 3En_{s^{(l-1)}} \leq Ex_{s^{(l)}} - 3En_{s^{(l)}}$ ($l \in \{2, 3, \dots, \#L(p)\}$) ($Ex_{s^{(l)}} + 3En_{s^{(l)}} \leq Ex_{s^{(l+1)}} - 3En_{s^{(l+1)}}$ ($l \in \{1, 2, \dots, \#L(p) - 1\}$)), then $Ex_{s^{(l)}} - 3En_{s^{(l)}}$ ($Ex_{s^{(l)}} + 3En_{s^{(l)}}$) is substituted for $\chi_{l-1,l}$ ($\chi_{l,l+1}$).

- (3) The modified ratio of $s^{(l)}$, denoted by $t_{s^{(l)}}$, will be calculated by Eq. (18):

$$t_{s^{(l)}} = \frac{A_{s^{(l)}}}{\sum_{l=1}^{\#L(p)} A_{s^{(l)}}}, \quad (l = 1, 2, \dots, \#L(p)). \quad (18)$$

- (4) According to Definition 5, three numerical characteristics $Ex_{L(p)}$, $En_{L(p)}$, $He_{L(p)}$ of C-PLTS will be obtained as follows:

$$Ex_{L(p)} = \sum_{l=1}^{\#L(p)} (t_{s^{(l)}} \cdot Ex_{s^{(l)}}), \quad (19)$$

$$En_{L(p)} = \sqrt{\sum_{l=1}^{\#L(p)} [t_{s^{(l)}} \cdot (En_{s^{(l)}})^2]}, \quad (20)$$

$$He_{L(p)} = \sqrt{\sum_{l=1}^{\#L(p)} [t_{s^{(l)}} \cdot (He_{s^{(l)}})^2]}. \quad (21)$$

Based on the above analyses, the comprehensive cloud of PLTS $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}$ can be generated by Algorithm 2.

Algorithm 2: Transform PLTS into C-PLTS

Input: $L(p) = \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}$ and $C_{s^{(l)}}(Ex_{s^{(l)}}, En_{s^{(l)}}, He_{s^{(l)}})$

Output: C-PLTS $C_{L(p)}(Ex_{L(p)}, En_{L(p)}, He_{L(p)})$

1. Calculate χ by Eq. (16)
 2. Calculate $A_{s^{(l)}}$ by Eq. (17)
 3. Calculate $t_{s^{(l)}}$ by Eq. (18)
 4. Calculate $Ex_{L(p)}$ by Eq. (19)
 5. Calculate $En_{L(p)}$ by Eq. (20)
 6. Calculate $He_{L(p)}$ by Eq. (21)
 7. **Return** $C_{L(p)}(Ex_{L(p)}, En_{L(p)}, He_{L(p)})$
-

Example 4. Given a LTS $S = \{s_i | i = 1, 2, \dots, 7\}$ and a PLTS $L(p) = \{s_4(0.8), s_5(0.2)\}$, $C_4 = (5, 0.3959, 0.0396)$ is the corresponding cloud for LT s_4 and $C_5 = (6.1878, 0.4672, 0.0467)$ is the corresponding cloud for LT s_5 . The C-PLTS $C_{L(p)}(Ex_{L(p)}, En_{L(p)}, He_{L(p)})$ for $L(p) = \{s_4(0.8), s_5(0.2)\}$ can be obtained as follows:

- (1) Based on Eq. (16), the abscissa value of the intersection point χ is obtained:

Let $0.8 \times \frac{1}{\sqrt{2\pi} \times 0.3959} \times \exp(-\frac{(\chi_{4,5}-5)^2}{2 \times 0.3959^2}) = 0.2 \times \frac{1}{\sqrt{2\pi} \times 0.4672} \times \exp(-\frac{(\chi_{4,5}-6.1878)^2}{2 \times 0.4672^2})$ to solve the abscissa value of the intersection point: $\chi_{4,5} = 5.7788$.

- (2) Based on Eq. (17), the area for the PDFC of $C_{s_4(0.8)}$ from $Ex_4 - 3En_4$ to the intersection point and the area for the PDFC of $C_{s_5(0.2)}$ from the intersection point to $Ex_5 + 3En_5$ are obtained:

$$A_{s_4} = 0.8 \times \frac{1}{\sqrt{2\pi} \times 0.3959} \times \int_{3.8122}^{5.7788} \exp(-\frac{(x-5)^2}{2 \times 0.3959^2}) dx = 0.7792,$$

$$A_{s_5} = 0.2 \times \frac{1}{\sqrt{2\pi} \times 0.4672} \times \int_{5.7788}^{7.5893} \exp(-\frac{(x-6.1878)^2}{2 \times 0.4672^2}) dx = 0.1616$$

- (3) Based on Eq. (18), the modified ratios of s_4 and s_5 are obtained:

$$t_{s_4} = \frac{0.7792}{0.7792 + 0.1616} = 0.8282, \quad t_{s_5} = \frac{0.1616}{0.7792 + 0.1616} = 0.1718$$

(4) Based on Eqs. (19)–(21), three numerical characteristics are obtained:

$$Ex_{L(p)} = 0.8282 \times 5 + 0.1718 \times 6.1878 = 5.2040,$$

$$En_{L(p)} = \sqrt{0.8282 \times 0.3959^2 + 0.1718 \times 0.4672^2} = 0.4090$$

$$He_{L(p)} = \sqrt{0.8282 \times 0.0396^2 + 0.1718 \times 0.0467^2} = 0.0409$$

Finally, the C-PLTS of $L(p) = \{s_4(0.8), s_5(0.2)\}$ is $C_{\{s_4(0.8), s_5(0.2)\}}(5.2040, 0.4090, 0.0409)$.

The PDFCs and 5000 cloud drops of $C_{s_4(0.8)}$ and $C_{s_5(0.2)}$ are shown in Fig. 8. The areas for the PDFCs of $C_{s_4(0.8)}$ and $C_{s_5(0.2)}$ are shown in Fig. 9.

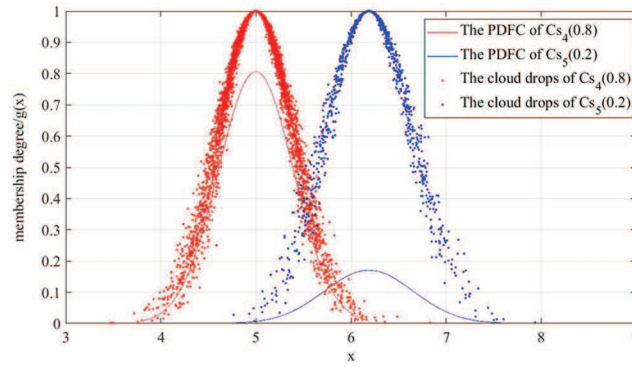


Figure 8: PDFCs and 5000 cloud drops of $C_{s_4(0.8)}$ and $C_{s_5(0.2)}$

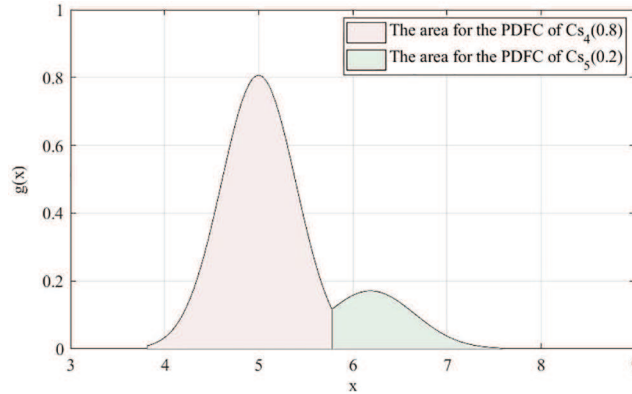


Figure 9: Areas for the PDFCs of $C_{s_4(0.8)}$ and $C_{s_5(0.2)}$

From $L(p) = \{s_4(0.8), s_5(0.2)\}$, we can know that the proportions of LTs s_4 and s_5 are 0.8 and 0.2, respectively. If a DM uses $L(p) = \{s_4(0.8), s_5(0.2)\}$ to evaluate an alternative, it can be assumed that the DM uses 5000 cloud drops to express his/her opinion, then he/she will place 4000 cloud drops in the s_4 region and 1000 cloud drops in s_5 region. However, it can be seen from Fig. 8 that parts of the 4000 cloud drops belonging to s_4 region will overlap with cloud drops belonging to s_5 region, and parts of the 1000 cloud drops belonging to s_5 region will overlap with

cloud drops belonging to s_4 region. In order to eliminate the information distortion caused by the overlapping part, this paper eliminates the overlapped cloud drops from the cloud drops originally allocated and recalculates the proportions that belong to each region. PLTS contains LTs and their corresponding probabilities. Thus the intersection point between the PDFC of $C_{s_4(0.8)}$ and the PDFC of $C_{s_5(0.2)}$ is taken as the boundary to recalculate the proportions of cloud drops distributed in the two regions, which are shown in Fig. 9. From the perspective of probability, the C-PLTS is obtained. In the meanwhile, the modified ratios of LTs decrease the loss and distortion of evaluation information.

3.3.2 Transformation from LHFS to Comprehensive Cloud

Definition 18. Let $S = \{s_i | i = 1, 2, \dots, 2\tau + 1\}$ be a LTS. The valid universe is $[X_{\min}, X_{\max}]$. Let $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$ be a LHFS. The cloud $C_{s^{(l)}}(Ex_{s^{(l)}}, En_{s^{(l)}}, He_{s^{(l)}})$ represents LT $s^{(l)} \in S$. Then, $C_{LH}(Ex_{LH}, En_{LH}, He_{LH})$ can be defined as C-LHFS, which is characterized by three numerical characteristics Ex_{LH} , En_{LH} and He_{LH} .

The specific procedures to determine Ex_{LH} , En_{LH} , He_{LH} of C-LHFS are as follows:

- (1) Let $C_{s^{(l)}}$ be the corresponding clouds for $s^{(l)}$ ($l = 1, 2, \dots, \#LH$), $C_{(s^{(l)}, lh(s^{(l)}))}$ be $C_{s^{(l)}}$ with corresponding average value of membership degrees $\frac{1}{\#lh(s^{(l)})} \sum_{r \in lh(s^{(l)})} r$ and $\chi_{l,l+1} \in$

$[Ex_{s^{(l)}}, Ex_{s^{(l+1)}}]$ be the abscissa value of intersection point between the NCs of $C_{(s^{(l)}, lh(s^{(l)}))}$ and $C_{(s^{(l+1)}, lh(s^{(l+1)}))}$. If $Ex_{s^{(l)}} + 3En_{s^{(l)}} > Ex_{s^{(l+1)}} - 3En_{s^{(l+1)}}$ ($l \in \{1, 2, \dots, \#LH - 1\}$), then Eq. (22) is used to calculate the value of $\chi_{l,l+1}$.

$$\begin{aligned} & \left(\frac{1}{\#lh(s^{(l)})} \sum_{r \in lh(s^{(l)})} r \right) \cdot \exp \left(-\frac{(\chi_{l,l+1} - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2} \right) \\ &= \left(\frac{1}{\#lh(s^{(l+1)})} \sum_{r \in lh(s^{(l+1)})} r \right) \cdot \exp \left(-\frac{(\chi_{l,l+1} - Ex_{s^{(l+1)}})^2}{2(En_{s^{(l+1)}})^2} \right), \quad (\chi_{l,l+1} \in [Ex_{s^{(l)}}, Ex_{s^{(l+1)}}]). \end{aligned} \quad (22)$$

- (2) Use Eq. (23) to calculate the area for the NC of $C_{(s^{(l)}, lh(s^{(l)}))}$ from lower limit to upper limit, which can be denoted by $A_{s^{(l)}}$.

$$A_{s^{(l)}} = \begin{cases} \frac{1}{\sqrt{2\pi}En_{s^{(l)}}} \int_{Ex_{s^{(l)}} - 3En_{s^{(l)}}}^{\chi_{l,l+1}} \exp \left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2} \right) dx, & l = 1 \\ \frac{1}{\sqrt{2\pi}En_{s^{(l)}}} \int_{\chi_{l-1,l}}^{\chi_{l,l+1}} \exp \left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2} \right) dx, & l \in \{2, 3, \dots, \#LH - 1\} \\ \frac{1}{\sqrt{2\pi}En_{s^{(l)}}} \int_{\chi_{l-1,l}}^{Ex_{s^{(l)}} + 3En_{s^{(l)}}} \exp \left(-\frac{(x - Ex_{s^{(l)}})^2}{2(En_{s^{(l)}})^2} \right) dx, & l = \#LH. \end{cases} \quad (23)$$

If $Ex_{s^{(l-1)}} + 3En_{s^{(l-1)}} \leq Ex_{s^{(l)}} - 3En_{s^{(l)}}$ ($l \in \{2, 3, \dots, \#LH\}$) ($Ex_{s^{(l)}} + 3En_{s^{(l)}} \leq Ex_{s^{(l+1)}} - 3En_{s^{(l+1)}}$ ($l \in \{1, 2, \dots, \#LH - 1\}$)), then $Ex_{s^{(l)}} - 3En_{s^{(l)}}$ ($Ex_{s^{(l)}} + 3En_{s^{(l)}}$) is substituted for $\chi_{l-1,l}$ ($\chi_{l,l+1}$).

(3) The modified ratio of $s^{(l)}$, denoted by $t_{s^{(l)}}$, will be calculated by Eq. (24):

$$t_{s^{(l)}} = \frac{A_{s^{(l)}}}{\sum_{l=1}^{\#LH} A_{s^{(l)}}}, (l = 1, 2, \dots, \#LH). \quad (24)$$

(4) According to Definition 5, three numerical characteristics Ex_{LH} , En_{LH} , He_{LH} of C-LHFS will be obtained as follows:

$$Ex_{LH} = \sum_{l=1}^{\#LH} t_{s^{(l)}} \cdot Ex_{s^{(l)}}, \quad (25)$$

$$En_{LH} = \sqrt{\sum_{l=1}^{\#LH} [t_{s^{(l)}} \cdot (En_{s^{(l)}})^2]}, \quad (26)$$

$$He_{LH} = \sqrt{\sum_{l=1}^{\#LH} [t_{s^{(l)}} \cdot (He_{s^{(l)}})^2]}. \quad (27)$$

Based on the above analyses, the comprehensive cloud of LHFS $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$ can be generated by Algorithm 3.

Algorithm 3: Transform LHFS into C-LHFS

Input: $LH = \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}$ and $C_{s^{(l)}}(Ex_{s^{(l)}}, En_{s^{(l)}}, He_{s^{(l)}})$

Output: C-LHFS $C_{LH}(Ex_{LH}, En_{LH}, He_{LH})$

1. Calculate χ by Eq. (22)
 2. Calculate $A_{s^{(l)}}$ by Eq. (23)
 3. Calculate $t_{s^{(l)}}$ by Eq. (24)
 4. Calculate Ex_{LH} by Eq. (25)
 5. Calculate En_{LH} by Eq. (26)
 6. Calculate He_{LH} by Eq. (27)
 7. **Return** $C_{LH}(Ex_{LH}, En_{LH}, He_{LH})$
-

Example 5. Given a LTS $S = \{s_i | i = 1, 2, \dots, 7\}$ and a LHFS $LH = \{(s_5, 0.6), (s_6, 0.9)\}$, $C_5 = (6.1878, 0.4672, 0.0467)$ is the corresponding cloud for the LT s_5 and $C_6 = (7.8031, 0.6354, 0.0635)$ is the corresponding cloud for the LT s_6 . The C-LHFS $C_{LH}(Ex_{LH}, En_{LH}, He_{LH})$ for $LH = \{(s_5, 0.6), (s_6, 0.9)\}$ can be obtained as follows:

(1) Based on Eq. (22), the abscissa value of the intersection point χ is obtained:

Let $0.6 \times \exp\left(-\frac{(\chi_{5,6} - 6.1878)^2}{2 \times 0.4672^2}\right) = 0.9 \times \exp\left(-\frac{(\chi_{5,6} - 7.8031)^2}{2 \times 0.6354^2}\right)$ to solve the abscissa value of the intersection point: $\chi_{5,6} = 6.7966$.

(2) Based on Eq. (23), the area for the NC of $C_{(s_5, 0.6)}$ from $Ex_5 - 3En_5$ to the intersection point and the area for the NC of $C_{(s_6, 0.9)}$ from the intersection point to $Ex_6 + 3En_6$ are obtained:

$$A_{s_5} = \frac{1}{\sqrt{2\pi} \times 0.4672} \times \int_{4.7862}^{6.7966} \exp\left(-\frac{(x-6.1878)^2}{2 \times 0.4672^2}\right) dx = 0.9024,$$

$$A_{s_6} = \frac{1}{\sqrt{2\pi} \times 0.6354} \times \int_{6.7966}^{9.7092} \exp\left(-\frac{(x-7.8031)^2}{2 \times 0.6354^2}\right) dx = 0.9421$$

(3) Based on Eq. (24), the modified ratios of s_5 and s_6 are obtained:

$$t_{s_5} = \frac{0.9024}{0.9024 + 0.9421} = 0.4892, \quad t_{s_6} = \frac{0.9421}{0.9024 + 0.9421} = 0.5108$$

(4) Based on Eqs. (25)–(27), three numerical characteristics are obtained:

$$Ex_{LH} = 0.4892 \times 6.1878 + 0.5108 \times 7.8031 = 7.0128,$$

$$En_{LH} = \sqrt{0.4892 \times 0.4672^2 + 0.5108 \times 0.6354^2} = 0.5594,$$

$$He_{LH} = \sqrt{0.4892 \times 0.0467^2 + 0.5108 \times 0.0635^2} = 0.0559$$

Finally, the C-LHFS of $LH = \{(s_5, 0.6), (s_6, 0.9)\}$ is $C_{\{(s_5, 0.6), (s_6, 0.9)\}} = (7.0128, 0.5594, 0.0559)$.

The NCs and 5000 cloud drops of $C_{(s_5, 0.6)}$ and $C_{(s_6, 0.9)}$ are shown in Fig. 10. The areas for the PDFCs of $C_{(s_5, 0.6)}$ and $C_{(s_6, 0.9)}$ are shown in Fig. 11.

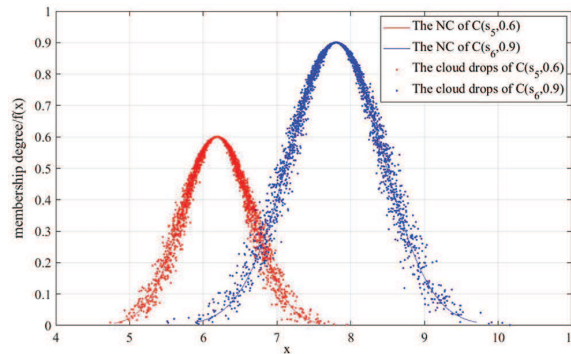


Figure 10: NCs and 5000 cloud drops of $C_{(s_5, 0.6)}$ and $C_{(s_6, 0.9)}$

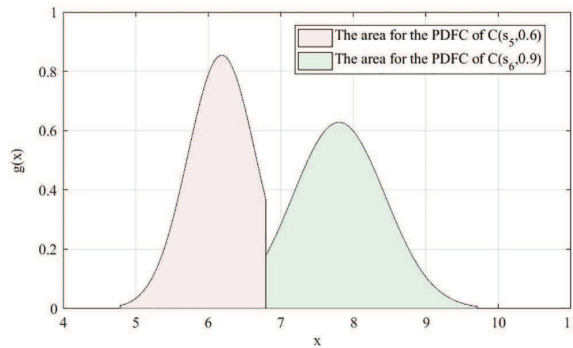


Figure 11: Areas for the PDFCs of $C_{(s_5, 0.6)}$ and $C_{(s_6, 0.9)}$

From $LH = \{(s_5, 0.6), (s_6, 0.9)\}$, we can know that the membership degrees of LTs s_5 and s_6 are 0.6 and 0.9 respectively. If a DM uses $LH = \{(s_5, 0.6), (s_6, 0.9)\}$ to evaluate an alternative, it can be assumed that the DM uses 5000 cloud drops to express his/her opinion, then he/she will place 2500 cloud drops in the s_5 region and 2500 cloud drops in s_6 region. Since the membership degree of LT s_5 is 0.6, the maximum membership degree of cloud drops belonging to s_5 region should be adjusted to 0.6. For the same reason, the maximum membership degree of cloud drops belonging to s_6 region should be adjusted to 0.9. Similar to the process of C-PLTS, parts of the 2500 cloud drops belonging to s_5 region will overlap with cloud drops belonging to s_6 region, and parts of the 2500 cloud drops belonging to s_6 region will overlap with cloud drops belonging to s_5 region, which can be seen from Fig. 10. In order to eliminate the information distortion caused by the overlapping part, this paper eliminates the overlapped cloud drops from the cloud drops originally allocated and recalculates the proportions that belong to each region. LHFS contains LTs and their corresponding membership degree. Thus the intersection point between the NC of $C_{(s_5, 0.6)}$ and the NC of $C_{(s_6, 0.9)}$ is taken as the boundary to recalculate the proportions of cloud drops distributed in the two regions, which are shown in Fig. 11. From the perspective of membership degree, the C-LHFS is obtained. In the meanwhile, the modified ratios of LTs decrease the loss and distortion of evaluation information.

Up till now, heterogeneous MAGDM matrices in which attribute values are expressed with LTs, PLTSs and LHFSs can be transformed into homogeneous MAGDM cloud matrices. For simplicity, homogeneous MAGDM cloud matrix is called as cloud matrix hereafter. Then, the individual cloud matrix $C^e = (C_{uv}^e)_{m \times n}$ can be elicited as

$$C^e = (C_{uv}^e)_{m \times n} = \begin{matrix} & \begin{matrix} y_1 & y_2 & \cdots & y_n \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} C_{11}^e & C_{12}^e & \cdots & C_{1n}^e \\ C_{21}^e & C_{22}^e & \cdots & C_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1}^e & C_{m2}^e & \cdots & C_{mn}^e \end{bmatrix} \end{matrix} \quad (e = 1, 2, \dots, k). \quad (28)$$

4 Cloud-Based Heterogeneous MAGDM

In this section, some related techniques are introduced, such as the comparison approach for clouds, the determination approaches of DM weight vector and attribute weight vector. Significantly, a personalized comprehensive cloud-based method for heterogeneous MAGDM problem is proposed.

4.1 Determination of DM Weight Vector

As mentioned above, the regulation parameter ς of entropy is determined according to DMs' hesitant degree and the regulation parameter ζ of hyper entropy is determined according to DMs' indeterminacy degree and information entropy. Therefore, the larger the two parameters are, the smaller weight should be given to the DM. Assume that there are a series of regulation parameters $\varsigma^e \in [1, 2]$ of entropy and a series of regulation parameters $\zeta^e \in [0, 1)$ of hyper entropy for DM d_e . Based on the regulation parameters ς^e and ζ^e , the weight of DM d_e can be calculated by

$$\varpi_e = \frac{3 - \varsigma^e - \zeta^e}{\sum_{e=1}^k (3 - \varsigma^e - \zeta^e)} \quad (e = 1, 2, \dots, k). \quad (29)$$

Solving Eq. (29), the DM weight vector $\mathbf{v} = (\varpi_1, \varpi_2, \dots, \varpi_k)^T$ is obtained.

Example 6. Following Example 3, $\varsigma^1 = 1$, $\zeta^1 = 0.0352$; $\varsigma^2 = 1.1470$, $\zeta^2 = 0.2931$; $\varsigma^3 = 1.1470$, $\zeta^3 = 0.6359$.

By Eq. (29), the DM weight vector $\mathbf{v} = (\varpi_1, \varpi_2, \varpi_3)^T = (0.4144, 0.3290, 0.2567)^T$ is obtained.

Definition 19. [22] Assume that \aleph is the set of all clouds and $C^e(Ex^e, En^e, He^e)$ ($e = 1, 2, \dots, k$) is a subset of \aleph . A mapping CWAA: $\aleph^m \rightarrow \aleph$ is defined as the cloud-weighted arithmetic averaging (CWAA) operator so that the following is true:

$$CWAA_{\mathbf{v}}(C^1, C^2, \dots, C^k) = \sum_{e=1}^k \varpi_e C^e, \quad (30)$$

where $\mathbf{v} = (\varpi_1, \varpi_2, \dots, \varpi_k)^T$ is the associated weight vector of $C^e(Ex^e, En^e, He^e)$ satisfying that $0 \leq \varpi_e \leq 1$ and $\sum_{e=1}^k \varpi_e = 1$.

Based on Eqs. (29) and (30) and basic operations of clouds in Definition 5, the individual cloud matrices $C^e = (C_{uv}^e)_{m \times n}$ ($e = 1, 2, \dots, k$) can be aggregated into a collective cloud matrix $C^g = (C_{uv}^g)_{m \times n}$ as

$$C^g = (C_{uv}^g)_{m \times n} = \begin{matrix} & \begin{matrix} y_1 & y_2 & \cdots & y_n \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} C_{11}^e & C_{12}^e & \cdots & C_{1n}^e \\ C_{21}^e & C_{22}^e & \cdots & C_{2n}^e \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1}^e & C_{m2}^e & \cdots & C_{mn}^e \end{bmatrix} \end{matrix}. \quad (31)$$

4.2 Pairwise Comparisons of Clouds

The evaluations from DMs have been transformed to clouds. As mentioned above, if (x, y) is a cloud drop of $C = (Ex, En, He)$, it is easily known that $g = \frac{1}{\sqrt{2\pi}En} \exp(-\frac{(x-Ex)^2}{2En^2})$ is the PDFC of C . Let $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$ be two clouds, then $g_{C_1} = \frac{1}{\sqrt{2\pi}En_1} \exp(-\frac{(x-Ex_1)^2}{2En_1^2})$ and $g_{C_2} = \frac{1}{\sqrt{2\pi}En_2} \exp(-\frac{(x-Ex_2)^2}{2En_2^2})$ are the PDFCs for C_1 and C_2 . $G_{C_1}(x)$ and $G_{C_2}(x)$ are the corresponding distribution functions respectively. Motivated by the comparison approach for linguistic distributions in [42], a new comparison approach for clouds is presented in the following.

4.2.1 CASD Relationship

According to the characteristics of cloud, AFSD theory is used to compare the dominance relationship between clouds with characteristics Ex, En in $C = (Ex, En, He)$.

Definition 20. Let $\Omega = \{x | G_{C_1}(x) > G_{C_2}(x)\}$, $\Theta = \{x | G_{C_2}(x) > G_{C_1}(x)\}$, and $\|G_{C_1}(x) - G_{C_2}(x)\| = \int_{\Omega} G_{C_1}(x) - G_{C_2}(x) dx + \int_{\Theta} G_{C_2}(x) - G_{C_1}(x) dx$. Let

$$D_{21} = \frac{\int_{\Omega} G_{C_1}(x) - G_{C_2}(x) dx}{\|G_{C_1}(x) - G_{C_2}(x)\|}. \quad (32)$$

If $D_{21} < 0.5$, then C_1 CASD C_2 . It is easily seen that $D_{12} = 1 - D_{21}$. Thus, if $D_{12} > 0.5$, C_1 CASD C_2 can be obtained as well.

4.2.2 CASD Degree

As mentioned above, the CASD relationship is adapted to compare two clouds. However, this relationship cannot quantify the degree for one cloud over another. To quantify the dominance degree, CASD degree is put forward.

Let μ_1 be the threshold for the deviation between Ex_1 and Ex_2 , μ_2 be the threshold for the deviation between En_1 and En_2 , and μ_3 be the threshold for the deviation between He_1 and He_2 . Let q_{12} denote the CASD degree for C_1 over C_2 . If C_1 CASD C_2 , q_{12} can be calculated by dividing into the cases in Table 4. If C_1 CASD C_2 , but Ex_1 , Ex_2 , En_1 , En_2 , He_1 and He_2 do not satisfy cases in Table 4, then $q_{12} = 0.5$. If C_1 CASD C_2 is not verified, then $q_{12} = 1 - q_{21}$.

Table 4: Calculation approach of CASD degree

Case	Comparison for characteristics	CASD degree
Case 1	$Ex_1 - Ex_2 \geq \mu_1$	$q_{12} = 1$
Case 2.1	$\mu_1 > Ex_1 - Ex_2 > 0$ $En_2 - En_1 \geq \mu_2$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times [0.5 \times (1 + 1)] \right\} = 0.5 + 0.5 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right $
Case 2.2.1	$\mu_1 > Ex_1 - Ex_2 > 0$ $\mu_2 > En_2 - En_1 > 0$ $He_2 - He_1 \geq \mu_3$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left[0.5 \times \left(1 + \left \frac{En_1 - En_2}{\mu_2} \right \times (0.5 \times (1 + 1)) \right) \right] \right\}$ $= 0.5 + 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right + 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right $
Case 2.2.2	$\mu_1 > Ex_1 - Ex_2 > 0$ $\mu_2 > En_2 - En_1 > 0$ $\mu_3 > He_2 - He_1 > 0$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left[0.5 \times \left(1 + \left \frac{En_1 - En_2}{\mu_2} \right \times \left(0.5 \times \left(1 + \left \frac{He_1 - He_2}{\mu_3} \right \right) \right) \right) \right] \right\}$ $= 0.5 + 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right + 0.125 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right + 0.125 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right \times \left \frac{He_1 - He_2}{\mu_3} \right $
Case 2.2.3	$\mu_1 > Ex_1 - Ex_2 > 0$ $\mu_2 > En_2 - En_1 > 0$ $\mu_3 > He_1 - He_2 \geq 0$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left[0.5 \times \left(1 + \left \frac{En_1 - En_2}{\mu_2} \right \times \left(0.5 \times \left(1 - \left \frac{He_1 - He_2}{\mu_3} \right \right) \right) \right) \right] \right\}$ $= 0.5 + 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right + 0.125 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right - 0.125 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right \times \left \frac{He_1 - He_2}{\mu_3} \right $
Case 2.2.4	$\mu_1 > Ex_1 - Ex_2 > 0$ $\mu_2 > En_2 - En_1 > 0$ $He_1 - He_2 \geq \mu_3$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left[0.5 \times \left(1 + \left \frac{En_1 - En_2}{\mu_2} \right \times (0.5 \times (1 - 1)) \right) \right] \right\} = 0.5 + 0.5 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right $
Case 2.3	$\mu_1 > Ex_1 - Ex_2 > 0$ $\mu_2 > En_1 - En_2 \geq 0$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left[0.5 \times \left(1 - \left \frac{En_1 - En_2}{\mu_2} \right \right) \right] \right\}$ $= 0.5 + 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right - 0.25 \times \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times \left \frac{En_1 - En_2}{\mu_2} \right $
Case 2.4	$\mu_1 > Ex_1 - Ex_2 > 0$ $En_1 - En_2 \geq \mu_2$	$q_{12} = 0.5 \times \left\{ 1 + \left \frac{Ex_1 - Ex_2}{\mu_1} \right \times [0.5 \times (1 - 1)] \right\} = 0.5$

To rank the alternatives and select the optimal alternative, the comparison approach for clouds is applied to the collective cloudmatrix. Alternatives x_u ($u = 1, 2, \dots, m$) are compared in pair on

each attribute y_v ($v = 1, 2, \dots, n$). In this paper, the values of μ_1 , μ_2 and μ_3 are determined as follows:

$$\mu_1 = \frac{2}{m(m+1)} \sum_{u=1}^m \sum_{o=u+1}^m d(Ex_u, Ex_o), \quad (33)$$

$$\mu_2 = \frac{2}{m(m+1)} \sum_{u=1}^m \sum_{o=u+1}^m d(En_u, En_o), \quad (34)$$

$$\mu_3 = \frac{2}{m(m+1)} \sum_{u=1}^m \sum_{o=u+1}^m d(He_u, He_o), \quad (35)$$

Based on the comparison approach mentioned above, the CASD degree for the alternatives x_u over x_o ($u, o = 1, 2, \dots, m$; $u \neq o$) with respect to attributes y_v can be calculated, denoted by $q_{uo,v}$. At the same time, the collective CASD degree matrix $Q_v = (q_{uo,v})_{m \times m}$ on each attribute y_v is obtained:

$$Q_v = (q_{uo,v})_{m \times m} = \begin{matrix} & \begin{matrix} x_1 & x_2 & \cdots & x_m \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} - & q_{12,v} & \cdots & q_{1m,v} \\ q_{21,v} & - & \cdots & q_{2m,v} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1,v} & q_{m2,v} & \cdots & - \end{bmatrix} \end{matrix} \quad (v = 1, 2, \dots, n). \quad (36)$$

Let q_{uv} denote the collective overall CASD degree for alternatives x_u over other alternatives with respect to attribute y_v , where

$$q_{uv} = \frac{1}{m-1} \sum_{o=1, o \neq u}^m q_{uo,v}. \quad (37)$$

Then, the collective overall CASD degree matrix $Q = (q_{uv})_{m \times n}$ can be obtained

$$Q = (q_{uv})_{m \times n} = \begin{matrix} & \begin{matrix} y_1 & y_2 & \cdots & y_n \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{bmatrix} \end{matrix}. \quad (38)$$

4.3 Determination of Attribute Weight Vector

As mentioned in Section 3.1, the attribute weight vector is denoted by $w = (w_1, w_2, \dots, w_n)^T$, satisfying $0 \leq w_v \leq 1$ ($v = 1, 2, \dots, n$) and $\sum_{v=1}^n w_v = 1$. In this paper, three perspectives are considered to obtain the attribute weights, which are differentiation between evaluation values, relationship between attributes and the amount of information contained in evaluation values.

4.3.1 From the Perspective of Differentiation between Evaluation Values

Let $DEV_{uv} = \frac{1}{m-1} \sum_{o=1, o \neq u}^m d(q_{uv}, q_{ov})$ be the deviation between alternative x_u and other alternatives on attribute y_v , where $d(q_{uv}, q_{ov}) = |q_{uv} - q_{ov}|$ indicates the distance between q_{uv} and q_{ov} .

Next, let $DEV_v = \frac{2}{m(m-1)} \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov})$ be the total deviation of alternative x_u on attribute y_v .

According to maximizing deviation approach [47], an attribute with a larger deviation value among alternatives should be assigned a larger weight, and vice versa. Thus, Model 1 is constructed as follows:

Model 1

$$\begin{aligned} \max Z_1(w) &= \sum_{v=1}^n DEV_v w_v \\ \text{s.t. } &\begin{cases} \sum_{v=1}^n (w_v)^2 = 1 \\ w_v \geq 0, \quad (v = 1, 2, \dots, n) \end{cases} \end{aligned} \quad (39)$$

4.3.2 From the Perspective of Relationship between Attributes

Let $REL_{vp} = [\sum_{u=1}^m (q_{uv} \cdot q_{up})] / [\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}]$ be the correlation coefficient between attribute y_v and attribute y_p ($p \neq v$).

Then, let $REL_v = \frac{1}{n-1} \sum_{p=1, p \neq v}^n [\sum_{u=1}^m (q_{uv} \cdot q_{up})] / [\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}]$ be the correlation coefficient between attribute y_v and all the other attributes.

From the perspective of correlation coefficient [48], larger REL_v means the elimination of attribute y_v has less influence on ordering and attribute y_v should be assigned a smaller weight, and vice versa. Based on correlation coefficient, Model 2 is built as follows:

Model 2

$$\begin{aligned} \max Z_2(w) &= \sum_{v=1}^n (1 - REL_v) w_v \\ \text{s.t. } &\begin{cases} \sum_{v=1}^n (w_v)^2 = 1 \\ w_v \geq 0, \quad (v = 1, 2, \dots, n) \end{cases} \end{aligned} \quad (40)$$

4.3.3 From the Perspective of the Amount of Information Contained in Evaluation Values

Let $ENT_v = -\frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv})$ be the information entropy of attribute y_v , where $\ell_{uv} = q_{uv} / \sum_{u=1}^m q_{uv}$.

It has been mentioned in [Section 3.2.2](#) that information entropy is an important tool to measure the uncertainty of the evaluation information. It is easily known that if the information entropy of evaluations on attribute y_v is small, the difference degree contained in evaluations on attribute y_v is great, which means the evaluations on attribute y_v are informative and attribute y_v should be assigned a large weight, and vice versa [49]. Therefore, Model 3 is constructed as follows:

Model 3

$$\begin{aligned} \max Z_3(\mathbf{w}) &= \sum_{v=1}^n (1 - ENT_v) w_v \\ s.t. \quad &\begin{cases} \sum_{v=1}^n (w_v)^2 = 1 \\ w_v \geq 0, \quad (v = 1, 2, \dots, n) \end{cases} \end{aligned} \quad (41)$$

4.3.4 A Comprehensive Tri-Objective Optimization Model

Combining [Eqs. \(39\)–\(41\)](#), a comprehensive tri-objective optimization model is built as

$$\begin{aligned} \max Z_1(\mathbf{w}) &= \sum_{v=1}^n DEV_v w_v \\ \max Z_2(\mathbf{w}) &= \sum_{v=1}^n (1 - REL_v) w_v \\ \max Z_3(\mathbf{w}) &= \sum_{v=1}^n (1 - ENT_v) w_v \\ s.t. \quad &\begin{cases} \sum_{v=1}^n (w_v)^2 = 1 \\ w_v \geq 0, \quad (v = 1, 2, \dots, n) \end{cases} \end{aligned} \quad (42)$$

To solve the comprehensive tri-objective optimization model, we add three balance coefficients ψ_1 , ψ_2 and ψ_3 into [Eq. \(42\)](#) and convert it into a single-objective optimization model as:

Model 4

$$\begin{aligned} \max Z(\mathbf{w}) &= \psi_1 \frac{2}{m(m-1)} \sum_{v=1}^n \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov}) w_v + \psi_2 \sum_{v=1}^n \left(1 - \frac{1}{n-1} \sum_{p=1, p \neq v}^n \frac{\sum_{u=1}^m (q_{uv} \cdot q_{up})}{\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}} \right) w_v \\ &\quad + \psi_3 \sum_{v=1}^n \left(1 + \frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv}) \right) w_v \end{aligned}$$

$$s.t. \begin{cases} \sum_{v=1}^n (w_v)^2 = 1 \\ w_v \geq 0, \quad (v = 1, 2, \dots, n) \end{cases}, \quad (43)$$

where ψ_1 , ψ_2 and ψ_3 are the balance coefficients, satisfying $0 \leq \psi_1, \psi_2, \psi_3 \leq 1$ and $\psi_1 + \psi_2 + \psi_3 = 1$. The values of ψ_1 , ψ_2 and ψ_3 could be given by DMs in advance, according to the actual situation and personal preference.

To solve Eq. (43), a Lagrange function is constructed as

$$L(\mathbf{w}, \lambda) = \psi_1 \frac{2}{m(m-1)} \sum_{v=1}^n \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov}) w_v + \psi_2 \sum_{v=1}^n \left(1 - \frac{1}{n-1} \sum_{p=1, p \neq v}^n \frac{\sum_{u=1}^m (q_{uv} \cdot q_{up})}{\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}}\right) w_v \\ + \psi_3 \sum_{v=1}^n \left(1 + \frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv})\right) w_v + \frac{\lambda}{2} \left(\sum_{v=1}^n (w_v)^2 - 1\right), \quad (44)$$

where λ is a real number, denoting the Lagrange multiplier.

The global optimal solution can be derived by taking partial derivatives of w_v and λ in Eq. (44), such that

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial w_v} = \psi_1 \frac{2}{m(m-1)} \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov}) + \psi_2 \left(1 - \frac{1}{n-1} \sum_{p=1, p \neq v}^n \frac{\sum_{u=1}^m (q_{uv} \cdot q_{up})}{\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}}\right) \\ + \psi_3 \left(1 + \frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv})\right) + \lambda w_v = 0, \quad (45)$$

$$\frac{\partial L(\mathbf{w}, \lambda)}{\partial \lambda} = \frac{1}{2} \left(\sum_{v=1}^n (w_v)^2 - 1\right) = 0. \quad (46)$$

By solving Eqs. (45) and (46), the solution can be obtained

$$w_v^* = \frac{\psi_1 \frac{2}{m(m-1)} \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov}) + \psi_2 \left(1 - \frac{1}{n-1} \sum_{p=1, p \neq v}^n \frac{\sum_{u=1}^m (q_{uv} \cdot q_{up})}{\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}}\right) + \psi_3 \left(1 + \frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv})\right)}{\sqrt{\sum_{v=1}^n \left(\psi_1 \frac{2}{m(m-1)} \sum_{u=1}^m \sum_{o=u+1}^m d(q_{uv}, q_{ov}) + \psi_2 \left(1 - \frac{1}{n-1} \sum_{p=1, p \neq v}^n \frac{\sum_{u=1}^m (q_{uv} \cdot q_{up})}{\sqrt{\sum_{u=1}^m (q_{uv})^2} \cdot \sqrt{\sum_{u=1}^m (q_{up})^2}}\right) + \psi_3 \left(1 + \frac{1}{\ln m} \sum_{u=1}^m (\ell_{uv} \ln \ell_{uv})\right)\right)^2}}. \quad (47)$$

After normalizing w_v^* ($v = 1, 2, \dots, n$) in Eq. (47), we can obtain the attribute weight vector

$$\mathbf{w} = (w_1, w_2, \dots, w_n)^T, \text{ where } w_v = w_v^* / \sum_{v=1}^n w_v^*.$$

Model 4 enables DMs to make a tradeoff in the above three aspects. Multifaceted considerations enhance the stability of the proposed method and the setting of balance coefficients improves the flexibility of the proposed method.

4.4 Obtaining the Ranking of Alternatives

Up till now, the collective overall CASD degree matrix $Q = (q_{uv})_{m \times n}$ and the attribute weight vector $w = (w_1, w_2, \dots, w_n)^T$ have been obtained. Thus, the total CASD degree of x_u can be calculated as

$$q_u = \sum_{v=1}^n q_{uv} w_v \quad (u = 1, 2, \dots, m). \quad (48)$$

Based on the values of q_u ($u = 1, 2, \dots, m$), the ranking of alternatives is obtained. The larger q_u , the better the alternative x_u .

4.5 Decision Steps for the Personalized Comprehensive Cloud-Based Method

A personalized comprehensive cloud-based method for heterogeneous MAGDM problem is proposed in this sub-section. Particularly, the resolution procedures of the proposed method are depicted in Fig. 12.

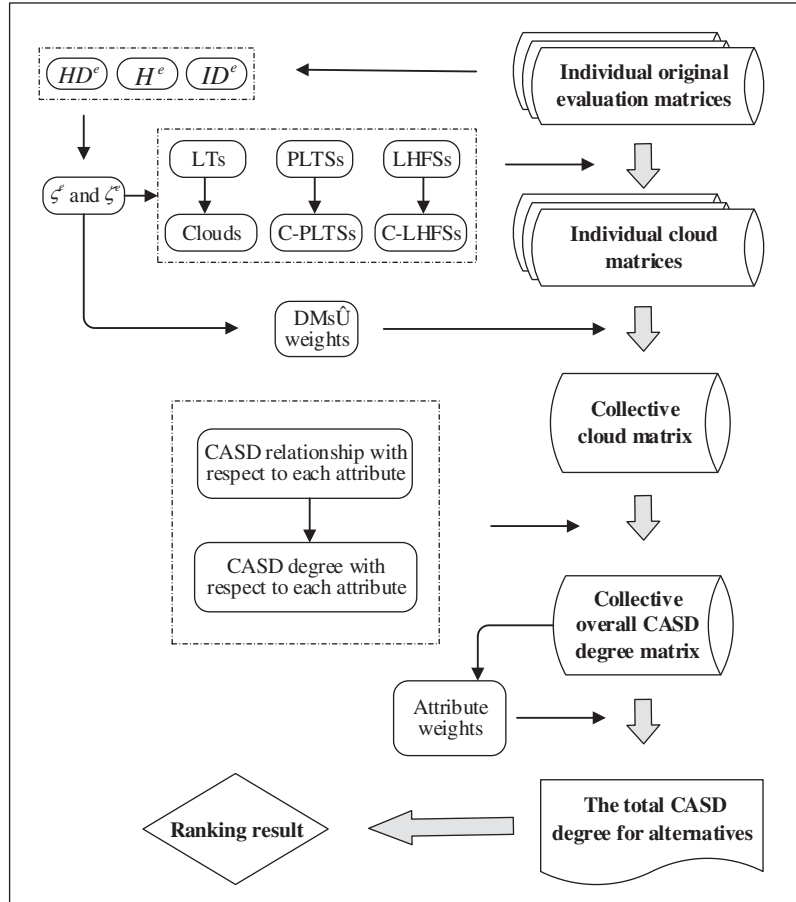


Figure 12: Resolution procedures of the proposed method

As depicted in Fig. 12, the proposed method mainly includes five steps below:

Step 1. Construct the individual original normalized evaluation matrix $R^e = (r_{uv}^e)_{m \times n}$ as Eq. (4).

DMs identify the feasible alternatives x_u ($u = 1, 2, \dots, m$) and determine related attributes y_v ($v = 1, 2, \dots, n$) and their evaluation forms, such as LT, PLTS, or LHFS. Each DM gives original evaluation matrix $\hat{R}^e = (\hat{r}_{uv}^e)_{m \times n}$. Obtain the individual original normalized evaluation matrix $R^e = (r_{uv}^e)_{m \times n}$ by using $\tilde{p}^{(l)} = p^{(l)} / \sum_{l=1}^{\#L(p)} p^{(l)}$, where

$$R^e = (r_{uv}^e)_{m \times n} = \begin{cases} s_i, & \text{if } y_v \text{ is benefit attribute} \\ \text{neg}(s_i), & \text{if } y_v \text{ is cost attribute} \\ \{s^{(l)}(p^{(l)}) | s^{(l)} \in S\}, & \text{if } y_v \text{ is benefit attribute} \\ \{\text{neg}(s^{(l)}(p^{(l)})) | s^{(l)} \in S\}, & \text{if } y_v \text{ is cost attribute} \\ \{(s^{(l)}, lh(s^{(l)})) | s^{(l)} \in S\}, & \text{if } y_v \text{ is benefit attribute} \\ \{(\text{neg}(s^{(l)}), lh(s^{(l)})) | s^{(l)} \in S\}, & \text{if } y_v \text{ is cost attribute} \end{cases}.$$

Step 2. Transform the individual original normalized evaluation matrix $R^e = (r_{uv}^e)_{m \times n}$ to the individual cloud matrix $C^e = (C_{uv}^e)_{m \times n}$ as Eq. (28).

Hesitant degree HD^e , information entropy H^e and indeterminacy degree ID^e for DM d_e are calculated according to the individual original evaluation matrix $R^e = (r_{uv}^e)_{m \times n}$ based on Eqs. (5), (6) and (8)–(11). Then, the regulation parameters of entropy and hyper entropy for each DM are calculated by Eqs. (7) and (12), respectively. LTs, PLTSs and LHFSs are transformed into clouds, C-PLTSs and C-LHFSs by the improved transformation approaches in Sections 3.2 and 3.3, respectively.

Step 3. Aggregate the individual cloud matrices $C^e = (C_{uv}^e)_{m \times n}$ ($e = 1, 2, \dots, k$) into the collective cloud matrix $C^g = (C_{uv}^g)_{m \times n}$ as Eq. (31) with basic operations of clouds and CWA operator.

Based on Eq. (29), DM weight vector $\mathbf{v} = (\varpi_1, \varpi_2, \dots, \varpi_k)^T$ can be acquired according to the regulation parameters ζ^e and ξ^e of each DM.

Step 4. Transform the collective cloud matrix $C^g = (C_{uv}^g)_{m \times n}$ to the collective overall CASD degree matrix $Q = (q_{uv})_{m \times n}$ as Eq. (38).

Firstly, pairwise comparisons are made to judge the CASD relationships for alternatives x_u ($u = 1, 2, \dots, m$) on each attribute y_v according to Eq. (32). Then, the thresholds for Ex , En and He are calculated based on Eqs. (33)–(35) and the CASD degrees are calculated according to Table 4. At the same time, the collective CASD degree matrices $Q_v = (q_{uo,v})_{m \times m}$ ($v = 1, 2, \dots, n$) on different attributes y_v ($v = 1, 2, \dots, n$) are obtained, as Eq. (36). Finally, calculate the collective overall CASD degrees q_{uv} for alternatives x_u ($u = 1, 2, \dots, m$) over other alternatives on each attribute y_v by Eq. (37) and generate the collective overall CASD degree matrix $Q = (q_{uv})_{m \times n}$.

Step 5. Generate the ranking order of all alternatives x_u according to the decreasing order of the total CASD degrees q_u ($u = 1, 2, \dots, m$).

Set the balance coefficients ψ_1, ψ_2, ψ_3 for Eq. (43). After obtaining the attribute weight vector $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$ by Model 4, the total CASD degrees of x_u ($u = 1, 2, \dots, m$) can be calculated by Eq. (48). Based on the values of q_u ($u = 1, 2, \dots, m$), the ranking of alternatives is obtained.

5 Illustrative Example

In this section, the proposed method is applied to an example of emergency medical waste disposal site selection in COVID-19. Furthermore, sensitivity analyses are conducted to demonstrate the stability and flexibility of the proposed method.

5.1 Illustration of the Proposed Method

5.1.1 Background Description

At the end of 2019, COVID-19 broke out in various provinces and cities in China. The amount of medical waste kept rising along with the number of confirmed cases. The explosive growth of medical waste occurred in many cities, and the lack of disposal capacity made the situation more serious. In such an emergency situation, medical waste disposal becomes a special battlefield in the fight against pneumonia. If these massive amounts of medical waste that may carry the virus were not disposed in a safe and timely way, it was likely to cause secondary infections and further spread of COVID-19, which may result in a series of unimaginable aftermaths. Generally, qualified medical waste disposal companies existed previously were completely at full capacity in many cities during the outbreak of COVID-19. In order to cope with the increasing amount of medical waste, many local governments adopted a series of emergency measures. One of these measures was converting other waste disposal companies, such as industrial hazardous waste disposal companies and solid waste disposal companies, to medical waste disposal sites temporarily for emergency disposal of medical waste. The selection for emergency medical waste disposal sites can be regarded as a heterogeneous MAGDM problem.

To select a suitable emergency medical waste disposal site from five alternatives $\{x_1, x_2, x_3, x_4, x_5\}$, a panel of four experts $\{d_1, d_2, d_3, d_4\}$ were appointed to evaluate the five alternatives on five attributes: geographical location (y_1), equipment (y_2), process technologies (y_3), disposal capacity (y_4) and transport capacity (y_5). The five attributes all are qualitative benefit attributes. The LTS is predefined as $S = \{s_1 : \text{very bad}; s_2 : \text{bad}; s_3 : \text{a little bad}; s_4 : \text{medium}; s_5 : \text{a little good}; s_6 : \text{good}; s_7 : \text{very good}\}$. The evaluations for geographical location (y_1) can be evaluated by LTs. PLTSs are used to evaluate equipment (y_2) and process technologies (y_3). LHFSSs are used to evaluate disposal capacity (y_4) and transport capacity (y_5). The evaluations of all alternatives on the five attributes given by the four DMs are normalized and listed in Table 5.

5.1.2 Resolution Process by Using the Proposed Method of This Paper

The procedures are summarized in the following steps:

Step 1. The individual original normalized evaluation matrices $R^e = (r_{uv}^e)_{5 \times 5}$ ($e = 1, 2, 3, 4$) have been obtained and presented in Table 5.

Step 2. Based on Eqs. (5)–(12), Hesitant degree HD , information entropy H , indeterminacy degree ID , regulation parameters ς and ζ for each DM are calculated and presented in Table 6.

According to the proposed transformation approaches from LTs, PLTSs and LHFSSs to clouds, C-PLTSs and C-LHFSSs, the individual original normalized evaluation matrices $R^e = (r_{uv}^e)_{5 \times 5}$ ($e = 1, 2, 3, 4$) have been transformed into individual cloud matrices $C^e = (C_{uv}^e)_{5 \times 5}$ ($e = 1, 2, 3, 4$). LSF2 Eq. (2) and $\alpha = 1.36$ are adopted in this paper. All the individual cloud matrices are listed in Table 7.

Table 5: Individual original normalized evaluation matrices for different DMs

DM	Alternative	Attribute				
		y_1	y_2	y_3	y_4	y_5
d_1	x_1	s_6	$\{s_4(0.6), s_5(0.4)\}$	$\{s_5(0.7), s_6(0.3)\}$	$\{(s_4, 0.4), (s_5, 0.5)\}$	$\{(s_6, 0.8)\}$
	x_2	s_4	$\{s_5(0.1), s_6(0.2), s_7(0.7)\}$	$\{s_6(1)\}$	$\{(s_5, 0.2), (s_6, 0.3)\}$	$\{(s_4, 0.3), (s_5, 0.2)\}$
	x_3	s_5	$\{s_5(0.5), s_6(0.5)\}$	$\{s_2(0.1), s_3(0.65), s_4(0.25)\}$	$\{(s_4, 0.5), (s_5, 0.4)\}$	$\{(s_1, 0.3), (s_2, 0.2)\}$
	x_4	s_3	$\{s_3(0.6), s_4(0.4)\}$	$\{s_6(0.25), s_7(0.75)\}$	$\{(s_6, 0.2), (s_7, 0.4)\}$	$\{(s_3, 0.5), (s_4, 0.3)\}$
	x_5	s_4	$\{s_6(0.7), s_7(0.3)\}$	$\{s_4(0.4), s_5(0.6)\}$	$\{(s_5, 0.3), (s_6, 0.3), (s_7, 0.2)\}$	$\{(s_4, 0.4), (s_5, 0.7)\}$
d_2	x_1	s_6	$\{s_7(1)\}$	$\{s_4(0.6), s_5(0.4)\}$	$\{(s_4, 0.7)\}$	$\{(s_4, 0.7)\}$
	x_2	s_7	$\{s_6(1)\}$	$\{s_6(0.55), s_7(0.45)\}$	$\{(s_6, 0.5), (s_7, 0.7)\}$	$\{(s_6, 0.8), (s_7, 0.4)\}$
	x_3	s_5	$\{s_6(1)\}$	$\{s_3(0.2), s_4(0.8)\}$	$\{(s_5, 0.6)\}$	$\{(s_3, 0.9), (s_4, 0.6)\}$
	x_4	s_3	$\{s_2(0.6), s_3(0.4)\}$	$\{s_6(1)\}$	$\{(s_5, 0.5), (s_6, 0.8)\}$	$\{(s_6, 0.8)\}$
	x_5	s_2	$\{s_6(0.3), s_7(0.7)\}$	$\{s_6(0.1), s_7(0.9)\}$	$\{(s_6, 0.7), (s_7, 0.5)\}$	$\{(s_5, 0.7), (s_6, 0.8)\}$
d_3	x_1	s_6	$\{s_4(1)\}$	$\{s_5(0.1), s_6(0.9)\}$	$\{(s_3, 1)\}$	$\{(s_7, 0.9)\}$
	x_2	s_7	$\{s_4(0.2), s_5(0.8)\}$	$\{s_6(1)\}$	$\{(s_5, 0.8), (s_6, 0.9)\}$	$\{(s_6, 1)\}$
	x_3	s_5	$\{s_4(1)\}$	$\{s_3(1)\}$	$\{(s_4, 1)\}$	$\{(s_4, 1)\}$
	x_4	s_3	$\{s_3(1)\}$	$\{s_6(0.9), s_7(0.1)\}$	$\{(s_6, 0.9)\}$	$\{(s_3, 1)\}$
	x_5	s_2	$\{s_6(1)\}$	$\{s_7(1)\}$	$\{(s_6, 0.9)\}$	$\{(s_5, 0.9), (s_6, 0.9)\}$
d_4	x_1	s_4	$\{s_3(0.6), s_4(0.4)\}$	$\{s_4(0.6), s_5(0.4)\}$	$\{(s_3, 0.8)\}$	$\{(s_3, 1)\}$
	x_2	s_5	$\{s_5(0.6), s_6(0.4)\}$	$\{s_6(1)\}$	$\{(s_5, 0.9)\}$	$\{(s_3, 0.3), (s_4, 0.6)\}$
	x_3	s_6	$\{s_6(1)\}$	$\{s_3(0.55), s_4(0.45)\}$	$\{(s_4, 0.5), (s_5, 0.6)\}$	$\{(s_1, 0.7), (s_2, 0.6)\}$
	x_4	s_4	$\{s_3(0.8), s_4(0.2)\}$	$\{s_5(0.3), s_6(0.7)\}$	$\{(s_6, 0.7)\}$	$\{(s_2, 0.5)\}$
	x_5	s_5	$\{s_7(1)\}$	$\{s_7(1)\}$	$\{(s_7, 0.6, 0.7)\}$	$\{(s_5, 0.8), (s_6, 0.7)\}$

Table 6: Calculation results of some related indexes for DMs

DM	η	HD	H	ID	ς	ζ
d_1	2.05	0.2929	0.3163	0.6033	1.2258	0.5388
d_2	1.6	0.2286	0.1783	0.325	1.1329	0.3214
d_3	1.25	0.1786	0.0591	0.055	1.0567	0.08
d_4	1.5	0.2143	0.1962	0.305	1.1115	0.3213

Table 7: Individual cloud matrices for different DMs

DM	Alternative	Attribute				
		y_1	y_2	y_3	y_4	y_5
d_1	x_1	(7.8031, 0.7788, 0.3423)	(5.4413, 0.5195, 0.2283)	(6.6099, 0.6331, 0.2783)	(5.6086, 0.5319, 0.2338)	(7.8031, 0.7788, 0.3423)
	x_2	(5, 0.4853, 0.2133)	(9.293, 0.854, 0.3754)	(7.8031, 0.7788, 0.3423)	(7.0278, 0.6877, 0.3023)	(5.5683, 0.5289, 0.2325)
	x_3	(6.1878, 0.5727, 0.2517)	(6.9734, 0.6808, 0.2992)	(3.9486, 0.5742, 0.2524)	(5.5797, 0.5298, 0.2329)	(1.0657, 0.8421, 0.3702)
	x_4	(3.8122, 0.5727, 0.2517)	(4.2722, 0.5405, 0.2376)	(9.5102, 0.8726, 0.3835)	(8.9592, 0.8435, 0.3707)	(4.3713, 0.5334, 0.2344)
	x_5	(5, 0.4853, 0.2133)	(8.4004, 0.8129, 0.3573)	(5.7278, 0.5405, 0.2376)	(7.9979, 0.7609, 0.3344)	(5.6323, 0.5336, 0.2345)

(Continued)

Table 7 (continued)

DM	Alternative	Attribute				
		y_1	y_2	y_3	y_4	y_5
d_2	x_1	(7.8031, 0.7198, 0.2042)	(10, 0.8296, 0.2354)	(5.4474, 0.4806, 0.1363)	(5, 0.4485, 0.1272)	(5, 0.4485, 0.1272)
	x_2	(10, 0.8296, 0.2354)	(7.8031, 0.7198, 0.2042)	(8.7712, 0.7701, 0.2185)	(8.9227, 0.7777, 0.2206)	(8.8599, 0.7746, 0.2197)
	x_3	(6.1878, 0.5293, 0.1502)	(7.8031, 0.7198, 0.2042)	(4.8092, 0.4625, 0.1312)	(6.1878, 0.5293, 0.1502)	(4.384, 0.4921, 0.1396)
	x_4	(3.8122, 0.5293, 0.1502)	(2.8373, 0.651, 0.1847)	(7.8031, 0.7198, 0.2042)	(7.0257, 0.6353, 0.1802)	(7.8031, 0.7198, 0.2042)
	x_5	(2.1969, 0.7198, 0.2042)	(9.3789, 0.8001, 0.227)	(9.8327, 0.8218, 0.2331)	(8.8812, 0.7756, 0.22)	(7.0036, 0.6327, 0.1795)
d_3	x_1	(7.8031, 0.6714, 0.0508)	(5, 0.4184, 0.0317)	(7.6838, 0.6599, 0.05)	(3.8122, 0.4937, 0.0374)	(10, 0.7738, 0.0586)
	x_2	(10, 0.7738, 0.0586)	(5.9868, 0.4818, 0.0365)	(7.8031, 0.6714, 0.0508)	(7.0012, 0.5899, 0.0447)	(7.8031, 0.6714, 0.0508)
	x_3	(6.1878, 0.4937, 0.0374)	(5, 0.4184, 0.0317)	(3.8122, 0.4937, 0.0374)	(5, 0.4184, 0.0317)	(5, 0.4184, 0.0317)
	x_4	(3.8122, 0.4937, 0.0374)	(3.8122, 0.4937, 0.0374)	(7.9834, 0.6804, 0.0515)	(7.8031, 0.6714, 0.0508)	(3.8122, 0.4937, 0.0374)
	x_5	(2.1969, 0.6714, 0.0508)	(7.8031, 0.6714, 0.0508)	(10, 0.7738, 0.0586)	(7.8031, 0.6714, 0.0508)	(6.9955, 0.5893, 0.0446)
d_4	x_1	(5, 0.4401, 0.1272)	(4.2757, 0.4899, 0.1416)	(5.4488, 0.4716, 0.1363)	(3.8122, 0.5193, 0.1501)	(3.8122, 0.5193, 0.1501)
	x_2	(6.1878, 0.5193, 0.1501)	(6.7982, 0.5968, 0.1725)	(7.8031, 0.7062, 0.2041)	(6.1878, 0.5193, 0.1501)	(4.4396, 0.4791, 0.1385)
	x_3	(7.8031, 0.7062, 0.2041)	(7.8031, 0.7062, 0.2041)	(4.345, 0.4854, 0.1403)	(5.6031, 0.4819, 0.1393)	(1.0895, 0.7624, 0.2204)
	x_4	(5, 0.4401, 0.1272)	(4.0077, 0.5071, 0.1466)	(7.346, 0.6587, 0.1904)	(7.8031, 0.7062, 0.2041)	(2.1969, 0.7062, 0.2041)
	x_5	(6.1878, 0.5193, 0.1501)	(10, 0.8139, 0.2353)	(10, 0.8139, 0.2353)	(10, 0.8139, 0.2353)	(6.988, 0.619, 0.1789)

Step 3. DMs' weights are calculated by Eq. (29). With basic operations of clouds, CWAA operator, and DM weight vector $\mathbf{w} = (w_1, w_2, w_3, w_4)^T = (0.1989, 0.2488, 0.3, 0.2523)^T$, the collective cloud matrix $C^g = (C_{uv}^g)_{5 \times 5}$ is obtained and shown in Table 8.

Step 4. Compare the alternatives x_u ($u = 1, 2, \dots, 5$) in pair and determinate the CASD relationships on each attribute y_v by Eq. (32). The CASD degrees are calculated according to Eqs. (33)–(35) and Table 4. The collective CASD degree matrices $Q_v = (q_{uo,v})_{5 \times 5}$ ($v = 1, 2, \dots, 5$) are shown in Table 9. Subsequently, the collective overall CASD degrees for alternative x_u ($u = 1, 2, \dots, 5$) over other alternatives on each attribute y_v are calculated by Eq. (37). The collective overall CASD degree matrix $Q = (q_{uv})_{5 \times 5}$ is presented in Table 10.

Step 5. Balance coefficients $\psi_1 = \frac{1}{3}$, $\psi_2 = \frac{1}{3}$, $\psi_3 = \frac{1}{3}$ are set for Eq. (43) in this example. By Model 4, the attribute weight vector is obtained as $\mathbf{w} = (w_1, w_2, w_3, w_4, w_5)^T = (0.2368, 0.1886, 0.1664, 0.2036, 0.2046)^T$.

Table 8: Collective cloud matrix

Alternative	Attribute				
	y_1	y_2	y_3	y_4	y_5
x_1	(7.0959, 0.6585, 0.1963)	(6.1492, 0.5814, 0.1718)	(6.3498, 0.5691, 0.1596)	(4.4651, 0.4975, 0.1449)	(6.7577, 0.6468, 0.1845)
x_2	(8.0437, 0.6825, 0.1719)	(7.301, 0.6584, 0.2152)	(8.044, 0.7275, 0.2156)	(7.2794, 0.6457, 0.1912)	(6.773, 0.6309, 0.1685)
x_3	(6.5953, 0.5778, 0.1707)	(6.7972, 0.632, 0.1975)	(4.2219, 0.5013, 0.1495)	(5.563, 0.4864, 0.1469)	(3.0776, 0.6328, 0.2113)
x_4	(4.1119, 0.5069, 0.1507)	(3.7104, 0.5491, 0.1599)	(8.0814, 0.7271, 0.2227)	(7.8396, 0.7092, 0.2161)	(4.509, 0.62, 0.1796)
x_5	(3.7613, 0.6157, 0.1609)	(8.8683, 0.7701, 0.2301)	(9.1087, 0.7569, 0.1994)	(8.6644, 0.7531, 0.2214)	(6.7245, 0.5976, 0.1662)

Table 9: Collective CASD degree matrices on different attributes

	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5
	y_1					y_2					y_3				
x_1	-	0.4259	0.5036	1.0000	1.0000	-	0.4676	0.4638	1.0000	0.0000	-	0.5000	0.6141	0.5000	0.0000
x_2	0.5741	-	0.5000	1.0000	1.0000	0.5324	-	0.5409	1.0000	0.5000	0.5000	-	1.0000	0.4959	0.4097
x_3	0.4964	0.5000	-	1.0000	1.0000	0.5362	0.4591	-	1.0000	0.5000	0.3859	0.0000	-	0.0000	0.0000
x_4	0.0000	0.0000	0.0000	-	0.5759	0.0000	0.0000	0.0000	-	0.0000	0.5000	0.5041	1.0000	-	0.4132
x_5	0.0000	0.0000	0.0000	0.4241	-	1.0000	0.5000	0.5000	1.0000	-	1.0000	0.5903	1.0000	0.5868	-
	y_4					y_5									
x_1	-	0.0000	0.3669	0.0000	0.0000	-	0.4968	1.0000	1.0000	0.5000					
x_2	1.0000	-	0.5000	0.4624	0.4547	0.5032	-	1.0000	1.0000	0.5000					
x_3	0.6331	0.5000	-	0.0000	0.0000	0.0000	0.0000	-	0.2076	0.0000					
x_4	1.0000	0.5376	1.0000	-	0.4319	0.0000	0.0000	0.7924	-	0.0000					
x_5	1.0000	0.5453	1.0000	0.5681	-	0.5000	0.5000	1.0000	1.0000	-					

Table 10: Collective overall CASD degree matrix

	y_1	y_2	y_3	y_4	y_5
x_1	0.7324	0.4828	0.4035	0.0917	0.7492
x_2	0.7685	0.6433	0.6014	0.6043	0.7508
x_3	0.7491	0.6238	0.0965	0.2833	0.0519
x_4	0.1440	0.0000	0.6043	0.7424	0.1981
x_5	0.1060	0.7500	0.7943	0.7784	0.7500

With the obtained attribute weight vector, the total CASD degrees of q_u ($u = 1, 2, \dots, 5$) are calculated by Eq. (48) and the results are listed as follows:

$$q_1 = 0.5036, q_2 = 0.68, q_3 = 0.3794, q_4 = 0.3263, q_5 = 0.6107$$

Thus, the ranking order is $x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$ and x_2 is the optimal alternative.

5.2 Sensitivity Analyses

LSFs are strictly monotonously increasing with respect to the subscript i . In linguistic evaluation scales, the absolute deviation of semantics between any two adjacent LTs may increase, decrease or remain unchanged with increasing linguistic subscripts. DMs can choose different LSFs according to the actual situation and personal preference. In Eq. (43), ψ_1, ψ_2, ψ_3 are considered as the balance coefficients for each perspective of attribute weights obtaining, satisfying $0 \leq \psi_1, \psi_2, \psi_3 \leq 1$ and $\psi_1 + \psi_2 + \psi_3 = 1$. The values of ψ_1, ψ_2, ψ_3 are given by DMs in advance. This sub-section takes different LSFs and different balance coefficients to solve the above example. The corresponding decision results are listed in Table 11 and shown in Fig. 13. The average differences of total CASD degrees between two adjacent alternatives in ranking results are shown in Fig. 14.

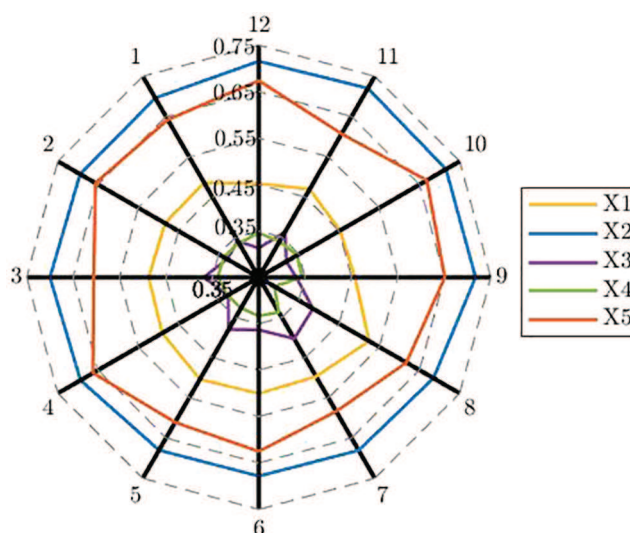
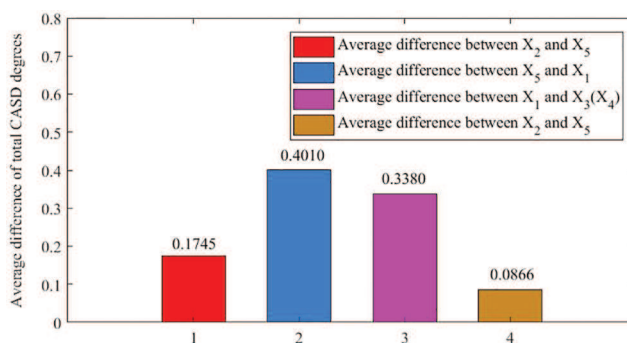
Table 11: Decision results for different LSFs and balance coefficients

No.	LFS	Balance coefficients	Total CASD degrees	Ranking of alternatives
1	LFS1	$\psi_1 = \frac{1}{3}, \psi_2 = \frac{1}{3}, \psi_3 = \frac{1}{3}$	$q_1 = 0.4857, q_2 = 0.6960,$ $q_3 = 0.3404, q_4 = 0.3356,$ $q_5 = 0.6421$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
2		$\psi_1 = 1, \psi_2 = 0, \psi_3 = 0$	$q_1 = 0.4832, q_2 = 0.6946,$ $q_3 = 0.3287, q_4 = 0.3370,$ $q_5 = 0.6564$	$x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$
3		$\psi_1 = 0, \psi_2 = 1, \psi_3 = 0$	$q_1 = 0.4871, q_2 = 0.7,$ $q_3 = 0.3688, q_4 = 0.3383,$ $q_5 = 0.6056$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
4		$\psi_1 = 0, \psi_2 = 0, \psi_3 = 1$	$q_1 = 0.4902, q_2 = 0.6936,$ $q_3 = 0.3263, q_4 = 0.3274,$ $q_5 = 0.6624$	$x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$
5	LFS2 ($a = 1.36$ is set in this example)	$\psi_1 = \frac{1}{3}, \psi_2 = \frac{1}{3}, \psi_3 = \frac{1}{3}$	$q_1 = 0.5036, q_2 = 0.68,$ $q_3 = 0.3794, q_4 = 0.3263,$ $q_5 = 0.6107$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
6		$\psi_1 = 1, \psi_2 = 0, \psi_3 = 0$	$q_1 = 0.5004, q_2 = 0.6782,$ $q_3 = 0.363, q_4 = 0.3333,$ $q_5 = 0.6251$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
7		$\psi_1 = 0, \psi_2 = 1, \psi_3 = 0$	$q_1 = 0.4967, q_2 = 0.6804,$ $q_3 = 0.4027, q_4 = 0.3365,$ $q_5 = 0.5836$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
8		$\psi_1 = 0, \psi_2 = 0, \psi_3 = 1$	$q_1 = 0.5247, q_2 = 0.6846,$ $q_3 = 0.3847, q_4 = 0.2888,$ $q_5 = 0.6173$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
9	LFS3 ($\alpha = \beta = 0.8$ is set in this example)	$\psi_1 = \frac{1}{3}, \psi_2 = \frac{1}{3}, \psi_3 = \frac{1}{3}$	$q_1 = 0.4574, q_2 = 0.7171,$ $q_3 = 0.3310, q_4 = 0.3436,$ $q_5 = 0.6509$	$x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$
10		$\psi_1 = 1, \psi_2 = 0, \psi_3 = 0$	$q_1 = 0.4522, q_2 = 0.7159,$ $q_3 = 0.3183, q_4 = 0.3453,$ $q_5 = 0.6682$	$x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$

(Continued)

Table 11 (continued)

No. LFS	Balance coefficients	Total CASD degrees	Ranking of alternatives
11	$\psi_1 = 0, \psi_2 = 1, \psi_3 = 0$	$q_1 = 0.4694, q_2 = 0.7202, q_3 = 0.3637, q_4 = 0.3395, q_5 = 0.6071$	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
12	$\psi_1 = 0, \psi_2 = 0, \psi_3 = 1$	$q_1 = 0.4522, q_2 = 0.7155, q_3 = 0.3131, q_4 = 0.3455, q_5 = 0.6737$	$x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$

**Figure 13:** Demonstration of ranking results**Figure 14:** Average differences of total CASD degrees

As can be seen from Table 11 and Fig. 13, the ranking order of alternatives is $x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$ or $x_2 \succ x_5 \succ x_1 \succ x_4 \succ x_3$. Besides, it is easy to discover from Table 11 that the top three alternatives are always x_2 , x_5 and x_1 , which indicates that the alteration of LSFs and balance

coefficients has only little impact on the ranking order of the alternatives. Therefore, the proposed method has high stability in determining the optimal alternative.

Furthermore, the proposed method can handle various decision situations and meet different DMs' preferences by taking different LSFs and balance coefficients. Thus, the flexibility of the proposed method can be reflected by the acquired ranking results derived by various selections of LSFs and balance coefficients.

6 Comparison Analyses and Discussion

To justify the advantages of our proposal, comparison analyses with methods based on cloud and other classical MAGDM methods are conducted. Besides, a summary of transformation approaches with different evaluation forms is presented.

6.1 Comparison with Methods Based on Cloud

Peng et al. [23] proposed a new method based on cloud to handle MAGDM problems with PLTSs. Hu [32] proposed two methods based on comprehensive cloud aggregation operator to solve MAGDM problems with LHFSSs. This paper proposes a novel method based on cloud for heterogeneous MAGDM, which could handle MAGDM problems with LT, PLTS, LHFSS or one of them. Obviously, the aforesaid methods all are based on cloud. The proposed method could handle MAGDM problems in [23,32], while Peng et al.'s method [23] and Hu's methods [32] could not solve the problem of this paper. Thus, the proposed method has wider applicability. Except for wider applicability, other important distinguishing factors and superiorities of the proposed method are stated as follows:

- (1) The cloud in [23] contains five characteristics, yet the values of left and right entropy are averaged into one in this paper, which greatly reduce the complexity of the following calculation. The transformation from LTs to clouds in [32] is based on the golden ratio, while it is based on LSFs in this paper. The selection for LSFs and its related parameters makes the transformation more flexible and practical. In addition, this paper proposes regulation parameters for entropy and hyper entropy. DMs' personalities can be reflected with the incorporation of regulation parameters in the transformation from LTs to clouds. Moreover, the modified ratios of LTs decrease the loss and distortion of evaluation information in the transformation from PLTSs (LHFSSs) to C-PLTSs (C-LHFSSs).
- (2) The method in [23] determines the attribute weights only via maximizing deviation, while three perspectives are considered to obtain the attribute weights in this paper. Besides, the setting of balance coefficients enhances the flexibility of the proposed method. Moreover, three steps are needed to obtain attribute weights in [23], including determining the individual weights of criteria, determining the weights associated with groups (equivalent to DMs in this paper) and calculating the overall weights of attributes. By contrast, only one step is needed to obtain attribute weights by the proposed method, which reduces the complexity of the calculation greatly.
- (3) The proposed approach to determining DMs' weights is superior to [32]. Hu [32] and this paper both consider the number of LTs and corresponding indeterminacy degree in LHFSS. However, if all the DMs only use one LT with different memberships in all LHFSSs, the determination approach of DMs' weights in [32] becomes invalid. For example, d_1 and d_2 give their evaluation matrices as follows:

$$R^1 = \begin{matrix} & y_1 & y_2 \\ x_1 & \{(s_6, 1)\} & \{(s_3, 0.9)\} \\ x_2 & \{(s_3, 1)\} & \{(s_1, 0.95)\} \end{matrix} \quad R^2 = \begin{matrix} & y_1 & y_2 \\ x_1 & \{(s_5, 0.1)\} & \{(s_3, 0.2)\} \\ x_2 & \{(s_4, 0.05)\} & \{(s_1, 0.1)\} \end{matrix}$$

It can be seen that d_1 and d_2 does not have the same confidence for their own evaluation matrices, but they will be assigned the same weight in [32]. However, d_1 and d_2 will be assigned different weights in this paper for their different membership for LTs. Obviously, the determination approach of DMs' weights in this paper is more reasonable.

- (4) The ranking of alternatives is based on the expected score values of clouds in [32]. The expected score values of clouds sometimes are unstable and may lead to inaccurate decision results. By contrast, the ranking of alternatives is based on the total CASD degree in this paper. Obviously, the ranking approach of this paper is more stable.

6.2 Comparison with Other Classical MAGDM Methods

Lin et al. [50] put forward two novel TOPSIS-ScoreC-PLTS and VIKOR-ScoreC-PLTS methods to handle MAGDM problems with PLTSs. This paper proposes a personalized comprehensive cloud-based method for heterogeneous MAGDM, which could handle MAGDM problems with LT, PLTS, LHFS or one of them. To justify the advantages of our proposal, comparison analyses with Lin et al.'s method [50] are conducted as follows:

- (1) Solve the adapted example of this paper by the methods in [50]

Since Lin et al.'s method just cloud handle the MAGDM problems with PLTSs, we only retain the evaluations on y_1 , y_2 and y_3 in the site selection example of emergency medical waste disposal and replace Eq. (12) by $\zeta^e = \log_{\rho_2}(H^e + 1)$. The adapted problem is dealt with the TOPSIS-ScoreC-PLTS method in [50], the VIKOR-ScoreC-PLTS method in [50] and the proposed method, respectively. The ranking results are displayed in Table 12.

Table 12: Ranking results with different methods

Method	Ranking result
TOPSIS-ScoreC-PLTS method in [50]	$x_2 \succ x_5 \succ x_1 \succ x_3 \succ x_4$
VIKOR-ScoreC-PLTS method in [50]	$x_2 \succ x_1 \succ x_3 \succ x_5 \succ x_4$
Proposed method of this paper	$x_2 \succ x_1 \succ x_5 \succ x_3 \succ x_4$

It is easy to find that x_2 is always the optimal alternative and x_4 is always the worst alternative, which shows the effectiveness of the proposed method.

- (2) Solve the example in [50] by the proposed method of this paper

The proposed method could handle MAGDM problems with LT, PLTS, LHFS or one of them. As a result, the proposed method could settle the example in [50] directly and the calculation results are as follows:

DM weight vector: $\mathbf{v} = (w_1, w_2, w_3, w_4)^T = (0.2415, 0.2399, 0.2545, 0.2641)^T$.

Attribute weight vector: $\mathbf{w} = (w_1, w_2, w_3, w_4, w_5)^T = (0.2071, 0.2331, 0.1871, 0.2205, 0.1523)^T$.

The total CASD degrees: $q_1 = 0.6808$, $q_2 = 0.4606$, $q_3 = 0.3932$, $q_4 = 0.4654$.

Therefore, the final ranking order is $x_1 \succ x_4 \succ x_2 \succ x_3$ and x_1 is the optimal alternative.

The ranking order by method in [50] is $x_1 \succ x_2 \succ x_3 \succ x_4$ and $x_1 \succ x_3 \succ x_2 \succ x_4$. No matter by method in [50] or the proposed method, x_1 is the best alternative. However, the ranking of the rest alternatives differs greatly. Obviously, the most remarkable difference is the ranking order of x_4 . After analysis, it is easily found that equal weights are given to each DM directly in [50], while higher weights are given to DMs with more informative evaluations in this paper. DMs d_3 and d_4 are given higher weights for their informative evaluations and they give overwhelming evaluations to x_4 on y_1 . Thus, the ranking of x_4 improves a lot by the proposed method. It is clear that the proposed method of this paper is more objective and practical.

6.3 Comparison with Other Transformation Approaches

Previous studies [22–27,30–34] have proposed a lot of transformation approaches from LT and its' extended forms to cloud and comprehensive clouds. A specific summary is shown in Table 13.

Table 13: A summary of transformation approaches with different evaluation forms

Approach	The form of evaluation	Probability	Membership	Interval concept	Personality of DMs
Wang et al. [22]	LT	-	-	-	-
Wang et al. [24]	LT	-	-	-	Yes
Zhu et al. [25] and Hu [32]	LT and LHFS	-	Yes	-	-
Peng et al. [23]	LT and PLTS	Yes	-	-	-
Zhou et al. [31]	LT and HFLTS	-	-	-	-
Mao et al. [26]	LT and Interval-Valued Hesitant Fuzzy Linguistic Set		Yes	Yes	
Peng et al. [27]	LT and Uncertain Z-number	-	-	Yes	-
Jia et al. [30]	LT, Atanassov's interval-valued Intuitionistic fuzzy sets and Z-numbers	-	Yes	Yes	-
Wang et al. [33]	LT and Unbalanced linguistic distribution assessments	Yes	-	-	-
Wang et al. [34]	LT and PLTS	Yes	-	-	-
Transformation approaches of this paper	LT, PLTS and LHFS	Yes	Yes	-	Yes

In summary, we find that most existing studies can only process LTs, or LTs with probability, or LTs with membership or LTs with interval concept. However, this paper provides the transformation approaches for LTs, LTs with probability and LTs with membership, simultaneously. Moreover, there are few studies that take DMs' personalities into account during the transformation process. Although Wang et al. [24] introduced overlap parameter into the transformation process to reflect the DMs' personality, the determination of overlap parameter is a little subjective. This paper proposes regulation parameters for entropy and hyper entropy and further incorporates them into the transformation process from LTs to clouds to reflect the different personalities of DMs. It is worth emphasizing that the determination of regulation parameters is totally objective. Apparently, the proposed transformation approaches of this paper are more applicable and effective.

7 Conclusion

This paper develops a personalized comprehensive cloud-based method for heterogeneous MAGDM, in which the evaluations of alternatives on attributes are represented as LTs, PLTSs and LHFSs. The validity of the proposed method is demonstrated with a site selection example of emergency medical waste disposal in COVID-19. The effectiveness, stability, flexibility and superiorities of the proposed method are proven by sensitivity and comparison analyses, respectively. Compared with the existing methods, the proposed method of this paper has the following prominent superiorities:

- (1) With the proposed regulation parameters, the width and thickness of clouds for the corresponding LTS are different for different DMs, which makes the DMs' personalities can be reflected in clouds. Besides, a novel approach to obtaining DM weight vector is constructed based on the proposed regulation parameters.
- (2) The new transformation approaches from PLTS and LHFS to C-PLTS and C-LHFS decrease the loss and distortion of evaluation information.
- (3) CASD relationship and CASD degree are initiated in this paper to compare clouds. With CASD relationship and CASD degree, alternatives in the form of clouds can be ranked and the ranking results are stable and effective. This innovation provides new perspective for pairwise comparisons of clouds.
- (4) The comprehensive tri-objective programming constructed in this paper enables DMs to make a tradeoff among three different aspects. Multifaceted considerations enhance the stability of the proposed method and the setting of balance coefficients improves the flexibility of the proposed method.

Although an example of emergency medical waste disposal site selection in COVID-19 is illustrated to the effectiveness of the proposed method, and it is expected to be applied to more real-life decision-making problems, such as investment selection, supply chain management, and so on. More effective transformation approaches for other evaluation forms, especially LTs with interval concept are waiting for us to come up with and apply them to heterogeneous MAGDM problems. Additionally, how to extend some classical decision-making methods to heterogeneous MAGDM based on cloud is also very interesting and deserves to be studied in the future.

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