

**PROCEEDINGS****Aeroelastic Analysis of a Supercritical Airfoil with Free-Play in Transonic Flow****Shun He<sup>1,\*</sup> and Gaowei Cui<sup>2</sup>**<sup>1</sup> School of Aerospace Science and Technology, Xidian University, Xi'an, 710126, China<sup>2</sup> Beijing Institute of Structure and Environment Engineering, Beijing, 100076, China

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**ABSTRACT**

An investigation has been made into the nonlinear aeroelastic behavior of a supercritical airfoil (NLR 7301) considering free-play in transonic flow. Computational Fluid Dynamics (CFD) based on Navier-Stokes equations is implemented to calculate unsteady aerodynamic forces. A loosely coupled scheme with steady CFD and a graphic method are developed to obtain the static aeroelastic position. Frequency domain flutter solution with transonic aerodynamic influence coefficient is used to capture the linear flutter characteristic at different angle of attack (AoA). Time marching approach based on CFD is used to calculate the nonlinear aeroelastic response. The bifurcation diagram of pitching motion shows that as the airspeed increases, the limit cycle oscillation (LCO) appears then quenches, forming the first LCO branch; LCO occurs again and sustains until the divergence of the response, forming the second LCO branch. This phenomenon is essentially induced by the combined action of the static aeroelastic position caused by the camber effect of the airfoil and the S shape flutter boundary with respect to AoA.

**KEYWORDS**

Transonic flutter; supercritical airfoil; free-play; limit cycle oscillation

**Introduction**

Supercritical airfoil is widely used on business jets, airliners and transport aircrafts to weaken the effects of shock wave in transonic regime. It is characterized by large leading edge radius, flattened upper surface and highly cambered rear section. Such geometry of a supercritical airfoil may induce strong aerodynamic nonlinearity in transonic viscous flow [1]. Generally, the nonlinearities involved with aeroelastic instability in transonic flow can arise from two aspects: the aerodynamic and the structural. On the one hand, shock wave, flow separation and the shock-boundary layer interaction may contribute to the nonlinear effect of aerodynamics [1–3]. Various nonlinear phenomena, e.g. LCOs and bifurcations, have been observed in transonic flow even though only aerodynamic nonlinearity is involved. On the other hand, linear structure is usually assumed in a conventional aeroelastic analysis. In reality, however, the structural system of an aeroelastic airfoil can be subject to nonlinear stiffness such as free-play, which significantly affects its aeroelastic behavior in subsonic [4,5] and supersonic [6] air flow. Therefore, expecting strong interaction between aerodynamic and structural nonlinearities, the investigation of the aeroelastic response of an airfoil with nonlinear stiffness in transonic flow is of particular interest.

As a high-fidelity technique to capture shock wave and flow separation, CFD method has been widely applied to carry out the aeroelastic response of the nonlinear structural model particularly



in transonic air flow. The boundary layer of a cambered airfoil is prone to separation with the interaction with shock in transonic flow, which may lead to a strong aerodynamic nonlinearity. Note that symmetrical airfoils are usually employed in aforementioned researches considering both aerodynamic and structural nonlinearities, even though the camber of an airfoil can cause strong aerodynamic nonlinearity. Meanwhile, the transonic aeroelastic analysis on a cambered airfoil such as NLR 7301 was always limited to a linear structure. There have been few reports on the aeroelastic behavior of the unsymmetrical airfoil system with nonlinear structural stiffness in the existing literature. Therefore, the aim of present paper is to examine the aeroelastic behavior of a supercritical airfoil with free-play in transonic flow.

## Formulas and Numerical Method

### Governing Equations for an Aeroelastic Airfoil

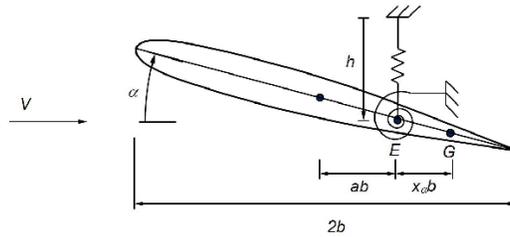
Fig. 1 shows a sketch of an aeroelastic airfoil with plunging ( $h$ ) and pitching ( $\alpha$ ) DOFs. The elastic axis of the airfoil (E point) is located at a distance of  $ab$  rear of the mid-chord point, the gravity center (G point) is located at  $x_\alpha b$  behind the elastic axis, where  $b$  is the half-chord length. The mass per unit span is  $m$ , the first moment of inertia about the elastic axis is  $S_\alpha = mx_\alpha b$ , and the moment of inertia about the elastic axis is  $I_\alpha = mr_\alpha^2 b^2$ . The bending stiffness and torsion stiffness are modelled by springs attached to the elastic axis. A linear spring is considered in plunging DOF, and the plunging stiffness coefficient is  $K_h = m\omega_h^2$ . While a free-play nonlinearity is assumed in the pitching DOF, and the nonlinear structural restoring moment can be described as

$$M(\alpha) = \begin{cases} K_\alpha(\alpha - \delta) & \alpha \geq \delta \\ 0 & -\delta < \alpha < \delta \\ K_\alpha(\alpha + \delta) & \alpha \leq -\delta \end{cases} = K_\alpha \alpha + K_\alpha \begin{cases} -\delta & \alpha \geq \delta \\ -\alpha & -\delta < \alpha < \delta \\ \delta & \alpha \leq -\delta \end{cases} = K_\alpha \alpha + K_\alpha f_{non} \quad (1)$$

where  $\delta$  denotes the measurement of free-play,  $K_\alpha = I_\alpha \omega_\alpha^2$  is the torsion stiffness coefficient.

The governing equations of motion for the linear structure were derived from the Lagrange equations according to Dowell et al. [7]. The nonlinear structural restoring moment from the spring with free-play in pitching DOF is considered in the present study. The nonlinear governing equations can be expressed as

$$\begin{cases} m\ddot{h} + S_\alpha \ddot{\alpha} + D_h \dot{h} + K_h h = -L \\ S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + D_\alpha \dot{\alpha} + M(\alpha) = M_{e\alpha} \end{cases} \quad (2)$$



**Figure 1:** An aeroelastic airfoil in transonic air flow

Introducing non-dimensional time  $\tau = \omega_\alpha t$  and mass ratio  $\mu = m/\pi\rho b^2$ , Eq. (2) can be written as

$$\mathbf{M}\xi'' + \mathbf{D}\xi' + \mathbf{K}\xi = \frac{U^2}{\pi\mu} \mathbf{f}_a + \mathbf{F}_{non} \quad (3)$$

By defining the structural state vector  $\mathbf{x}_s = \{\xi \ \xi'\}^T$ , the governing equations of the aeroelastic system can be written as

$$\mathbf{x}'_s = \mathbf{A}_s \mathbf{x}_s + \frac{U^2}{\pi\mu} \mathbf{B}_s \mathbf{f}_a + \mathbf{B}_s \mathbf{F}_{non} \quad (4)$$

### ***Time Marching Approach***

As mentioned in Introduction, the time marching approach based on CFD technique is a high fidelity tool to calculate the aeroelastic response in transonic air flow. Nowadays many commercial software packages are capable of conducting fluid-structure interaction simulations directly or via user-defined functions. In the current investigation, Fluent is used to carry out the aeroelastic response due to its high flexibility of using User-Defined Function (UDF) to incorporate with the structural model in CFD simulation.

Fluent is a general purpose CFD program, which can be used to model a wide range of incompressible and compressible air flow. In the present study, the pressure-based coupled algorithm is applied to solve the fluid governing equations. In Fluent, a control-volume-based technique is employed to convert the general scalar transport equation to an algebraic equation, which is solved by using a point implicit (Gauss-Seidel) linear equation solver in conjunction with an algebraic multigrid (AMG) method. A classical one-equation turbulence model, the S-A model, is used to deal with viscous flow problems. For spatial discretization, the second-order upwind scheme is utilized to interpolate the convection terms. In terms of temporal discretization, a technique called bounded second order implicit time integration is employed in Fluent for real-time advancement to carry out the unsteady flow field.

As we know, the aerodynamic forces are determined by time and airfoil motion in time marching CFD simulations. It is reasonable to assume that the aerodynamic forces are continuously changing over time. Therefore, theoretically speaking, it is feasible to predict the aerodynamic forces by implementing an interpolation method. An RK4 scheme with aerodynamic interpolation technique can be expressed as

$$\begin{cases} \mathbf{x}_{s,n+1} = \mathbf{x}_{s,n} + \frac{k_1+2k_2+2k_3+k_4}{6} \\ k_1 = \Delta\tau(\mathbf{A}_s\mathbf{x}_{s,n} + \frac{U^2}{\pi u}\mathbf{B}_s\mathbf{f}_a(\mathbf{x}_{s,n},\tau) + \mathbf{B}_s\mathbf{F}_{non}(\mathbf{x}_{s,n})) \\ k_2 = \Delta\tau(\mathbf{A}_s(\mathbf{x}_{s,n} + \frac{k_1}{2}) + \frac{U^2}{\pi u}\mathbf{B}_s\mathbf{f}_a(\tau + \frac{\Delta\tau}{2}) + \mathbf{B}_s\mathbf{F}_{non}(\mathbf{x}_{s,n} + \frac{k_1}{2})) \\ k_3 = \Delta\tau(\mathbf{A}_s(\mathbf{x}_{s,n} + \frac{k_2}{2}) + \frac{U^2}{\pi u}\mathbf{B}_s\mathbf{f}_a(\tau + \frac{\Delta\tau}{2}) + \mathbf{B}_s\mathbf{F}_{non}(\mathbf{x}_{s,n} + \frac{k_2}{2})) \\ k_4 = \Delta\tau(\mathbf{A}_s(\mathbf{x}_{s,n} + k_3) + \frac{U^2}{\pi u}\mathbf{B}_s\mathbf{f}_a(\tau + \Delta\tau) + \mathbf{B}_s\mathbf{F}_{non}(\mathbf{x}_{s,n} + k_3)) \end{cases} \quad (5)$$

where  $\mathbf{f}_a(\tau + \Delta\tau/2)$  and  $\mathbf{f}_a(\tau + \Delta\tau)$  can be obtained by using a second order interpolation on the aerodynamic forces at previous time steps  $\mathbf{f}_a(\tau)$ ,  $\mathbf{f}_a(\tau - \Delta\tau)$  and  $\mathbf{f}_a(\tau - 2\Delta\tau)$ .

### **Conclusions**

The bifurcation diagram of pitching shows that with the increasing of airspeed, the simple LCO appears firstly and then quenches; when the airspeed is increased further, the second LCO branch occurs until the divergence of the response. To explain this observation,

The static aeroelastic position is obtained. The magnitude of static aeroelastic pitching angle increases quadratically with the increasing of the airspeed.

The flutter speed at different angle of attack is examined. An S shape flutter boundary with respect to AoA is observed.

The appearance and quenching of LCOs are essentially induced by the static aeroelastic position and the S shape flutter boundary. As the airspeed increases, the static aeroelastic position changes, hence the stability of aeroelastic system alters. When the aeroelastic system leaves the first stable region and enters the unstable region, the first LCO branch appears. With further increasing of airspeed, the aeroelastic system departs from the unstable region and falls into the second stable region, so that LCOs quench in this region. When the airspeed is increased large enough, the system becomes unstable again and the second branch of LCO takes forth.

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