Application of Fiber Undulation Model to Predict Oriented Strand Composite Elastic Properties

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ABSTRACT: The effects of strand undulation angles in wood-strand composites have often been ignored due to the virtual impossibility of experimental determination of their effects on composite material properties, and the difficulty in modeling localized deviations in angle along the path of a strand. The fiber undulation model (FUM), that has been previously verified, was applied in this study to predict the elastic constants of laboratory-manufactured oriented strand panels. A stochastic approach was incorporated where a series rule of mixtures with probability density functions of angle distributions was utilized in the model to transform the elastic constants in the constitutive matrix of the material for in- and out-of-plane strand deviations. Based on a theoretical approach, a reduction in E_x due to strand undulations averaged about 7 percent over all configurations of test panels, indicating that localized out-of-plane strand deviations in commercially manufactured wood-strand composites should not significantly affect longitudinal Young's modulus.

KEYWORDS: Wood strand, strand undulation, fiber undulation model, laminate effective properties, oriented strand composites

1 INTRODUCTION

Oriented strand composites (OSC), such as oriented strand board (OSB) and oriented strand lumber (OSL), are widely used wood-based composite structural materials for building construction. Properties of OSC depend on the properties of the constituent materials and their arrangement. Empirical, analytical, and theoretical models have been suggested by past researchers to predict their mechanical properties [1–11]; these models account for the effects of in-plane strand deviations with respect to the longitudinal axis of a panel, but generally ignore the out-of-plane strand deviations resulting from strands undulating through panel thickness. Variation in compaction of strands in the plane of the panel during the hot-pressing process leads to in-plane density variations; strands tend to conform to density variations throughout the panel resulting in waviness or undulation through the thickness of the composite panel.

Out-of-plane strand deviations, similar to in-plane deviations, reduce stiffness and strength of a composite and resistance to dimensional changes due to exposure to varying environmental conditions. Fiber undulation models (FUM) have been successfully used for a number of years that consider the effect of undulation on the elastic response of synthetic composites made using textile processes that produce fiber undulation that is akin to that observed in wood composites [12–15].

Yadama *et al.* [16] examined the effect that undulating strands have on the elastic behavior of wood-strand laminates with controlled levels of undulating strands and the ability of a fiber undulation model to estimate their elastic properties. The fiber undulation model (FUM) predictions of carefully manufactured wood-strand laminates indicated a nonlinear degradation in the Young's modulus of the composite as undulation angle increased. In compression, a maximum undulation angle of four degrees could reduce the Young's modulus by approximately 12%. However, the effect of strand undulation on the elastic properties of oriented strand composite, manufactured without any control on undulation angles, and validation of the FUM to predict its elastic properties, has not been performed. In another study, Yadama *et al.* [17]

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Figure 1 Strand undulations in the longitudinal direction.

characterized strand deviations in the longitudinal and transverse directions through the thickness of an oriented strand panel (Figure 1); strand undulation in the longitudinal direction was shown to range between 0 and 30 degrees, whereas in the transverse direction the undulation angle ranged between 0 and 60 degrees. Due to these relatively large undulation angles, behavior of a wood-strand composite, such as OSB, could be impacted because of the tendency of the strands to buckle more readily.

In this study, the investigators apply the fiber undulation model to predict the mechanical properties of laboratory manufactured oriented strand panels. This approach incorporates hot-pressing effects on strand properties [18], strand deviations in the plane of a panel, and strand undulations through the panel thickness in the longitudinal direction. The effects of strand undulation in the transverse direction are not included because of the assumption that the composite panels manufactured for this study are transversely isotropic [19].

The objective of this study is to confirm the validity of the fiber undulation model to predict the elastic properties of oriented strand composite panels. Specific tasks to achieve this objective are to:

1. Present a method to include the effects of out-of-plane strand undulations along with the in-plane strand deviations on the elastic behavior of a panel,
2. Predict the elastic properties, E_x, E_y, G_{xy}, and ν_{xy}, of panels fabricated with a range of strand geometries using the FUM and compare them with the experimental results compiled by Meyers [11].

2 EXPERIMENTAL EVALUATION OF TEST PANELS

Details regarding manufacturing of test panels can be found in Meyers [11]. Panels were manufactured for multiple geometries and levels of orientation. Three nominal strand lengths (100, 200 and 300 mm) and widths (12.5, 19 and 25 mm) were used with an average thickness of 0.76 mm. Two different vane spacings (38 and 76 mm) were used during hand forming of the mats to achieve different degrees of in-plane strand orientation. Panels made with 300 mm long strands were formed using only 76 mm vane spacing as forming with smaller vane spacing did not contribute to improving the alignment.

Tensile and compression specimens were cut from the test panels and their elastic constants, E_x and E_y, were evaluated and reported by Meyers [11]. Off-axis tensile tests were conducted [19] to compute shear moduli, G_{xy}, of the panels. Since ν_{xy} was not experimentally determined in this study, a value of 0.56 reported for aspen laminated strand lumber in a recent publication [20] was assumed.

2.1 Method to Include Strand Orientation Angles in FUM

The elastic properties of oriented strand panels predicted with the fiber undulation model include E_x, E_y, G_{xy}, and ν_{xy}. The panel and material coordinate axes of a strand with three-dimensional orientation in a composite panel are shown in Figure 2. The off-axis orientation of a strand in the plane of a panel is designated ϕ, and the strand undulation angle through panel depth in the longitudinal direction is designated θ. Strand undulation angle in the transverse direction is represented by ψ.

In a previous study [16], strand undulation paths were described with a discrete Fourier series expansion. These functions were subsequently utilized in the fiber undulation model to determine the effects of undulation angles on laminate elastic properties. This method, however, is not practical when a laminate consists of 35 to 45 layers and each layer could potentially be described by functions that involve over 10 terms of a Fourier series. A solution to this problem is to consider statistical distributions of strand angles in the fiber undulation model.

Meyers [11] previously characterized the distributions of in-plane orientation angle, ϕ, of strands for the wood-strand composite panels fabricated for this study; whereas, Yadama *et al.* [17] characterized the distributions of out-of-plane strand undulation angle, θ. It was shown that 2-parameter Weibull distribution describes both in-plane and out-of-plane orientation angle distributions accurately. Absolute values of all measured angles were taken to give a range of angles from 0 to 90 degrees.

A series rule of mixtures approach was considered to incorporate these statistical distributions into the FUM in determining the transformed stiffness matrix. The compliance matrix, **[S]**, was adjusted for strand

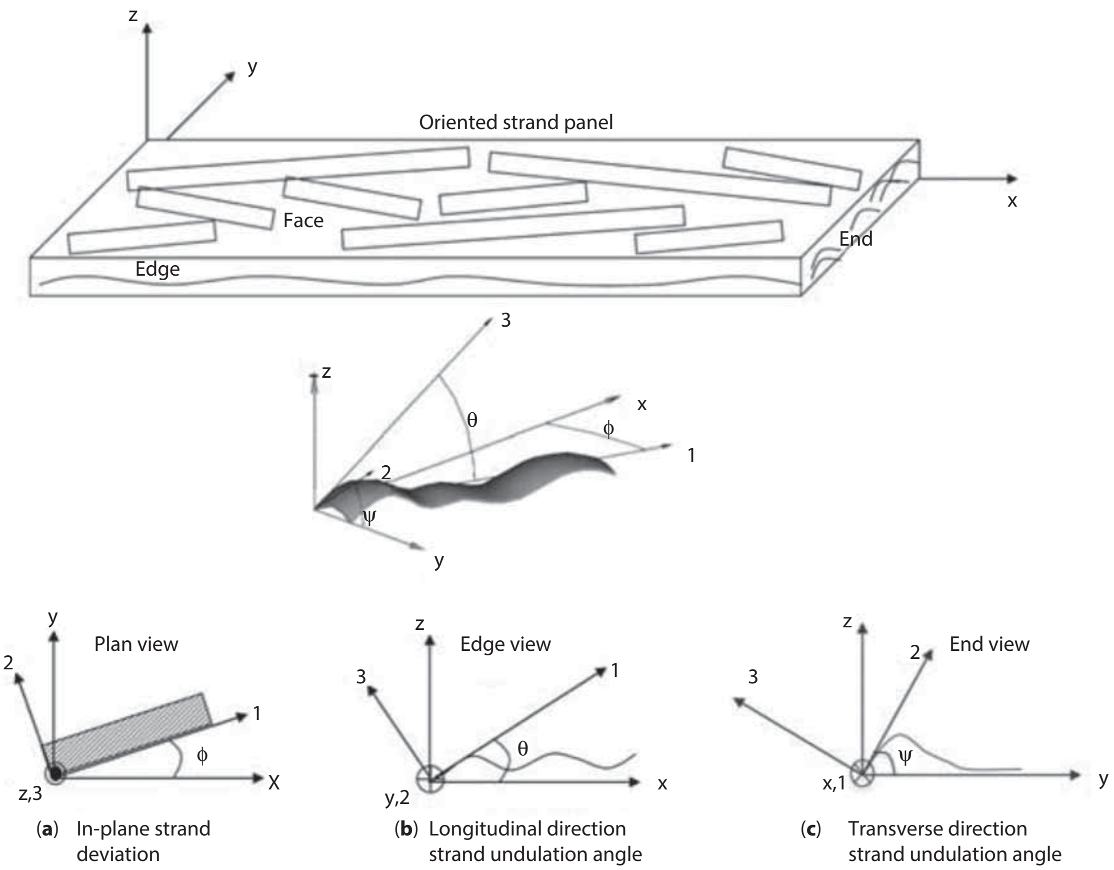

Figure 2 Panel coordinate system and strand orientation angles with respect to the panel coordinate axes.

undulation effects by calculating $\left[\bar{S}\right]_\theta^n$ at the mean angle for each n^th^ bin of the Weibull probability density distribution of an appropriate panel type (based on the strand geometry and vane spacing). The transformation matrix used to adjust for the strand undulation angle in the xz-plane is given as:

$$[T]_\theta = \begin{bmatrix} \cos(\theta)^2 & 0 & \sin(\theta)^2 & 0 & 2\cos(\theta)\sin(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin(\theta)^2 & 0 & \cos(\theta)^2 & 0 & -2\cos(\theta)\sin(\theta) & 0 \\ 0 & 0 & 0 & \cos(\theta) & 0 & -\sin(\theta) \\ -\cos(\theta)\sin(\theta) & 0 & \cos(\theta)\sin(\theta) & 0 & \cos(\theta)^2-\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \quad (1)$$

After determining the transformed compliance matrices for each two-degree wide bin, the compliance matrix of each bin was multiplied by the corresponding bin probability values to account for the bin's contribution to the undulation effects. The average transformed compliance matrix could then be determined by integrating over the range of the transformation angles (0 to 90 degrees in this study) as follows:

$$(\bar{S}_{ij})_\theta = \frac{2}{\pi}\int_0^{\pi/2} (\bar{S}_{ij})_\theta \Pr(\theta)d\theta \quad (2)$$

Since the transformed compliance matrix and Pr(θ) are constants within each bin, their product for each bin of a probability density distribution can be summed over the total number of bins, n, to determine an average transformed compliance matrix, $\left[\bar{S}\right]_\theta$, accounting for strand undulation effects as follows:

$$\left[\bar{S}\right]_\theta = \sum_{i=1}^{n} \left[\bar{S}\right]_\theta^i \Pr(\theta_i) \quad (3)$$

Then, $\left[\bar{S}\right]_\theta$ was transformed again to account for in-plane strand deviation angle, ϕ, in the plane of a panel to obtain $\left[\bar{S}\right]_{\theta\phi}$. Subsequently, $\left[\bar{S}\right]_{\theta\phi}$ may be inverted to obtain averaged stiffness matrix of the laminate to determine its effective elastic constants in the fiber undulation model. Thus, the effects of both undulation angle and in-plane deviation angle of a strand on

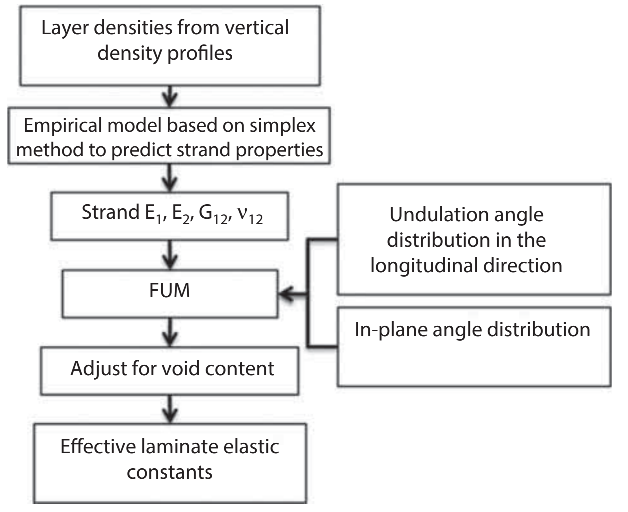

Figure 3 Model flow chart.

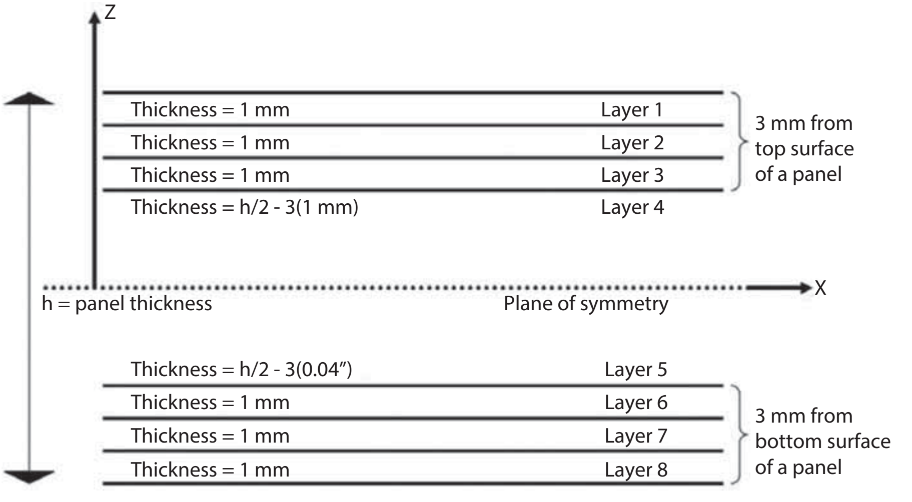

Figure 4 Composite panel considered as a laminate with eight layers.

the constitutive matrix of each lamina were included in the model.

2.2 Model Flow

A flow chart of the fiber undulation model to predict the elastic properties of the oriented strand panels is presented in Figure 3. Every panel was modeled as a symmetric laminate consisting of eight laminae (Figure 4). The laminate lay-up was determined based on vertical density profiles of the panels.

Close examination of the density profiles showed that each of the high-density regions on panel surfaces could be divided into three sections with the thickness of each section equal to 1 mm nominal. Densities within each of these sections were averaged to determine the mean density for each of the regions. Density values in the core section of the panels (between the outer 3.175 mm thick surface regions) were averaged to yield a mean value for the middle two laminae in each of the laminates. Based on the density profiles of a specimen from each panel group, average strand densities were assigned for the top four laminae. These values were also assigned to the bottom four laminae to produce a symmetric laminate.

Knowing the densities of each lamina, the volume fractions of the strand constituents, namely the cell wall and void content, were determined [19]. The volume fraction of resin was taken to be that of the target resin content, 0.06, and was subtracted from the void volume fraction to satisfy the condition that the sum of the three components (cell wall, voids, and resin) is one. The E_1 and ν_{12} of each lamina were computed using the empirical model developed using the simplex method

[18,19]. Based on the proportionality constants relating E_1 to E_2 and E_1 to G_{12} [19, 21], the corresponding material properties were computed. Values for ν_{23} and ν_{21} were taken to be 0.5 and 0.035 based on published values [21]. Based on the ratio of these properties found in the literature, G_{23} was taken to be one half of G_{12}. The remaining material properties were determined based on the assumption of transverse isotropy.

The model computes the transformed compliance and stiffness matrices and determines the effective elastic constants based on the laminate compliance matrix, **[a]**, as discussed by Yadama *et al.* [16]. These laminate elastic constants were further adjusted to account for between strand void volumes determined for these laminates [19]. Elastic properties, E_x, E_y, G_{xy}, and ν_{xy}, were predicted for panel groups with experimentally determined undulation angle distributions given in Yadama *et al.* [17]. Predicted properties were compared with experimentally determined values on specimens from panels tested in tension, compression, and shear by Meyers [11] and Yadama [19].

2.3 FUM Prediction Results and Discussion

Elastic constants were estimated by the fiber undulation model for all panel groups and compared with experimentally measured values for E_x and E_y by Meyers [11] (Table 1). To summarize experimentally determined undulation angle distributions given in Yadama *et al.* [17] used to predict the elastic properties, strand undulation angles in the longitudinal direction were found to range between 0 and 30 degrees in the core region and between 0 and 20 degrees in the surface regions; whereas, in the transverse direction, strand deviation angles ranged between 0 and 60 degrees for the core and between 0 and 50 degrees for surface strands.

Ratios of the predicted to measured values for each of the elastic properties are presented in Table 1. The model results are compared graphically with the experimental results for E_x and E_y in Figures 5 and 6.

For all panel groups but one, the FUM consistently under-predicted tensile E_x and over-predicted compressive E_x. The percent error ranged from 9 to 42 percent for tensile E_x and 1 to 53 percent for compression E_x. Close examination of Figure 5 shows that the majority of the FUM results fell within one standard deviation of compression E_x and within two standard deviations of tensile E_x. For E_y, the percent error between the FUM predictions and experimental values ranged from 2 to 132 percent for tensile E_y and from 9 to 66 percent for compression E_y. The FUM predictions were lower than tensile E_y and higher than compression E_y for the majority of the panel groups. The FUM consistently under-predicted both tension and compression E_y compared to the experimental values for panels manufactured with shorter strands (100 mm strands). Based on the model predictions, reduction in E_x due to strand undulations ranged from 4.5 to 8.8 percent depending on the panel group, with an average reduction of 6.9 percent. Therefore, results indicate that localized strand undulations in wood composites such as OSB and OSL do not significantly influence the elastic properties, especially the longitudinal modulus of elasticity. Variations in elastic properties due to undulations are masked by an inherent variation in strand properties due to material tropism and heterogeneity,

Table 1 Comparison of FUM predictions of panel elastic properties with experimental results.

Vane Spacing (mm)	Length (mm)	Width (mm)	Ratios of Predicted/Measured Values					
			Tensile Properties		Compressive Properties			
			E_x	E_y	E_x	E_y	G_{xy}	V_{xy}
38	100	19	0.58	1.23	1.12	0.79	0.44	0.70
	200	19	0.86	2.32	1.39	1.66	0.50	0.59
76	100	12.5	0.91	0.87	1.28	0.75	0.72	0.77
		19	0.73	0.78	1.34	0.76	0.68	0.79
		25	0.79	1.03	1.53	0.91	0.73	0.77
	200	19	0.67	0.98	1.01	0.78	0.49	0.68
	300	12.5	0.82	1.11	1.30	1.21	0.49	0.61
		19	0.63	1.36	0.87	0.82	0.42	0.63
		25	0.74	1.52	1.07	1.31	0.44	0.59

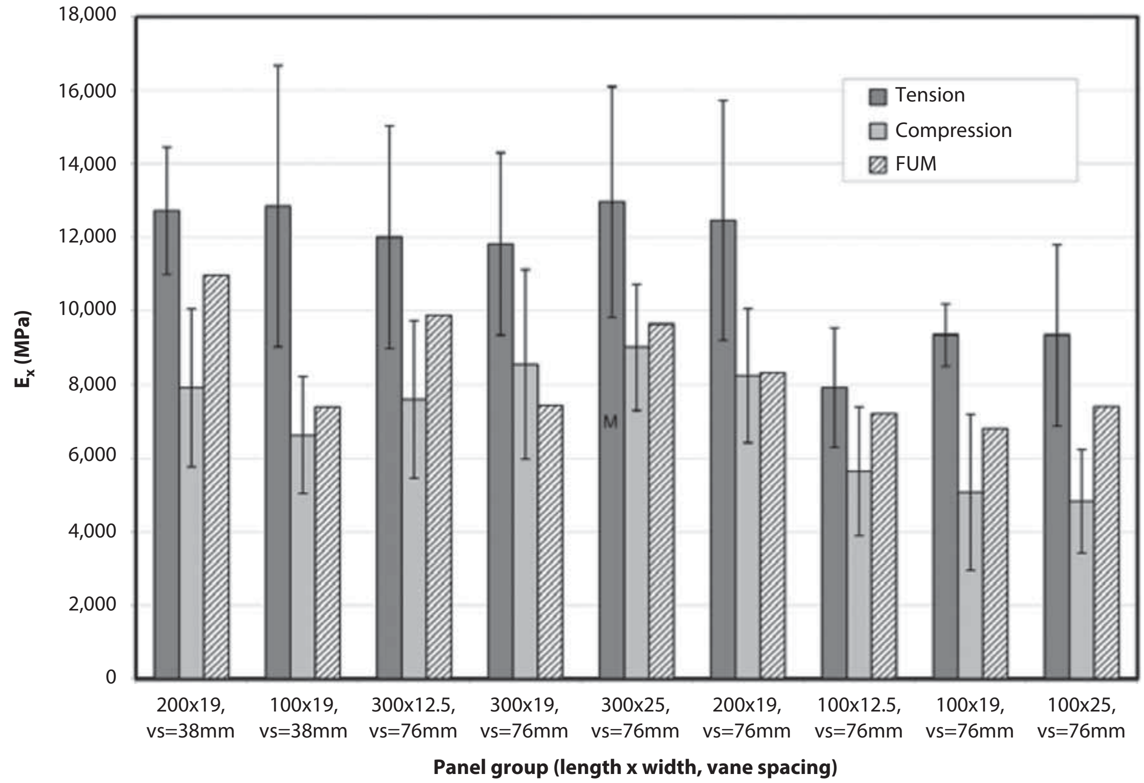

Figure 5 Comparison of tensile and compression experimental E_x against predicted E_x (vs = vane spacing).

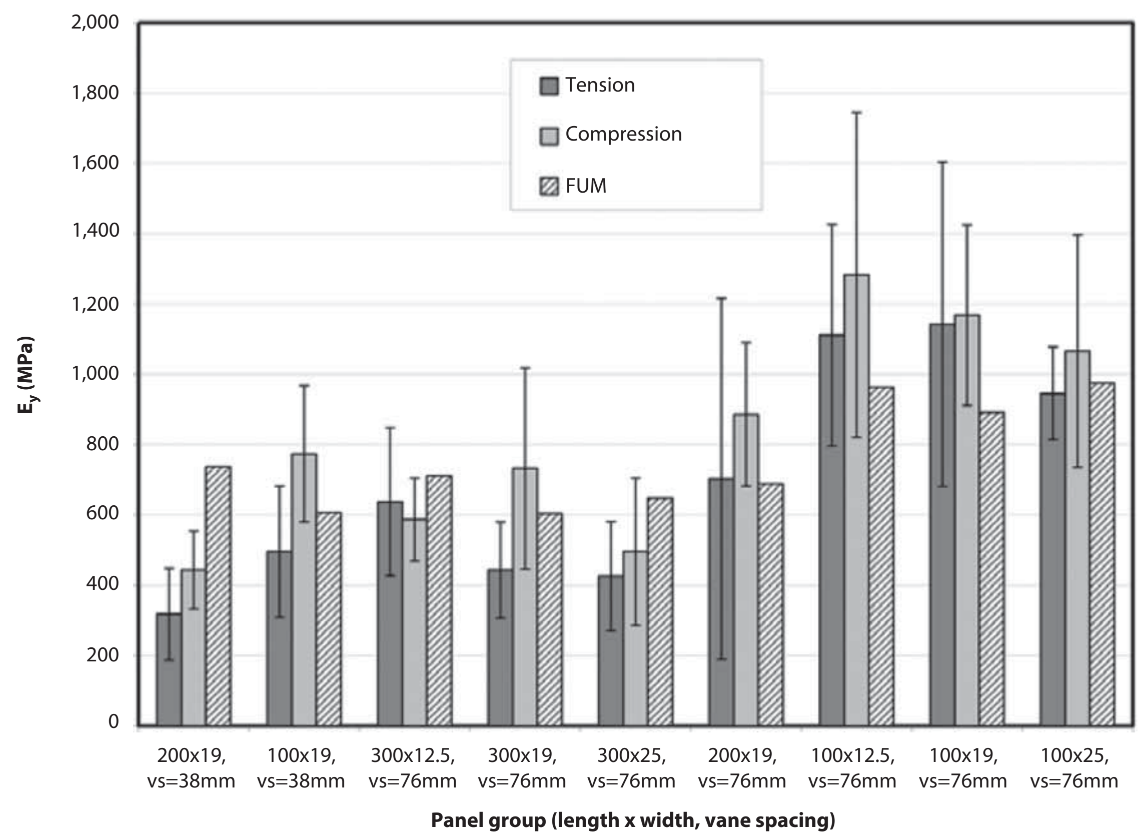

Figure 6 Comparison of experimental tension and compression E_y against predicted E_y (vs = vane spacing).

localized variations in density and void volume as a result of the hot-pressing process, and in-plane strand orientation; this is especially true as strand undulation is not constant over the entire length of a strand and, therefore, its effects are averaged and are not as significant as in-plane strand deviations.

Shear moduli values from off-axis tensile tests ranged from 758 MPa to 1972 MPa with a coefficient of variation of 30 percent. In general, panels produced from short strands yielded higher values for shear modulus, which could be attributed to a lower degree of strand alignment with shorter strands. Janowiak *et al.* [20] reported an average G_{xy} of 917 MPa for laminated strand lumber manufactured with aspen strands. Compared to the overall mean measured G_{xy} for all panel groups and published ν_{xy} of 0.56 [20], the FUM predictions were under-predicted by 23 to 56 percent. The full potential of the fiber undulation model could probably be harnessed when predicting the behavior of wood composite layers with pronounced undulations such as corrugated cores in sandwich panels [22,23].

3 SUMMARY AND CONCLUSIONS

Similar to in-plane strand deviations, strand undulation through the panel thickness could adversely affect the mechanical behavior, especially in the inelastic region, and failure mechanism of a wood-strand panel subjected to compression loads or repeated moisture cycling. Effects of strand undulation angles in modeling wood-strand composites have often been ignored due to the difficulty in including continuously varying undulation angle along the path of a strand. In this article, the fiber undulation model (FUM) that has been previously verified with carefully fabricated laminates [16], was applied to predict the elastic constants of laboratory manufactured oriented strand panels. A series rule of mixtures with probability density functions of angle distributions was utilized in the model to transform the elastic constants in the constitutive matrix of the material for in- and out-of-plane strand deviations. Even though only strand undulations in the longitudinal direction were accounted for in this study, a similar approach could be used to adjust the lamina properties for transverse strand undulations.

Due to very small undulations of strands in OSC, compared to the experimental values the model predictions of G_{xy} and ν_{xy} for all panel groups were conservative. Model predictions consistently under-predict tensile E_x and over-predict compression E_x. Based on theoretical prediction (FUM), reduction in E_x due to strand undulations averaged about 7 percent over all configurations of test panels. Therefore, it can be concluded that commonly occurring undulation in wood composites such as OSB or OSL do not significantly affect E_x. The presented theory to model the effect of undulation based on composite mechanics, is novel and can be used to effectively determine effective elastic constants of corrugated cores with pronounced undulations in wood composite sandwich panels as discussed by Garg, Yadama, and Cofer [24].

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