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K-Banhatti Sombor Invariants of Certain Computer Networks

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Abstract: Any number that can be uniquely determined by a graph is called a graph invariant. During the last twenty years' countless mathematical graph invariants have been characterized and utilized for correlation analysis. However, no reliable examination has been embraced to decide, how much these invariants are related with a network graph or molecular graph. In this paper, it will discuss three different variants of bridge networks with good potential of prediction in the field of computer science, mathematics, chemistry, pharmacy, informatics and biology in context with physical and chemical structures and networks, because k-banhatti sombor invariants are freshly presented and have numerous prediction qualities for different variants of bridge graphs or networks. The study solved the topology of a bridge graph/networks of three different types with two invariants K-Banhatti Sombor Indices and its reduced form. These deduced results can be used for the modeling of computer networks like Local area network (LAN), Metropolitan area network (MAN), and Wide area network (WAN), backbone of internet and other networks/structures of computers, power generation, bio-informatics and chemical compounds synthesis.

Keywords: Bridge networks; invariants; k-banhatti; sombor indices; maple; network graph; molecular graph

1 Introduction

Mansour and Schork [1] introduce the idea of bridge graphs which is a combination of networks bridge together in a single network. A bridge graph is a graph obtain from number of graphs

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G1,G2,G3, . . . Gm by associate the vertices vi and vi + 1 by an edge∀, i = 1, 2, . . . , m − 1 [2]. On the other hand, Gutman in 2021, define the idea of sombor indices. A new vertex degree-based invariant graph named Sombor Index is used to capture the sharp lower and upper bounds of the connected network and the characteristics of the network reaching the boundaries [3]. There are two variants of k-banhatti sombor indices, the first one is k-banhatti sombor index and second is its reduced version [4]. A K-Banhatti Sombor Index is a topological index which is a number associated with a network graph that captures the symmetry of the network structure and provides a scientific language for predicting the characteristics of the network.

Mostly networks of different topologies are performed well and efficient separately but with the combination of two or more efficiency compromised. For the formal reason, study discusses and solves topology of bridge networks with the help of graph theory numerically.

As another arising science is developed by the help of computer sciences, mathematics and chemistry called cheminformatics, whose significant segments incorporate Quantitative structure-activity relationship (QSAR) and Quantitative structure-property relationships (QSPR) and the segments can add to the examination on physicochemical characteristics of synthetic mixtures. QSAR is a modeling tool used to solve the topology of networks or structure of compounds and modeled the efficient and best performer networks or structures. QSPR is also a modeling tool which correlates the properties of a network structures with the help of mathematical equation or expression. It also provides the quantitative relationship between properties of networks or chemical structures [5].

Its first notable application in chemistry was the investigation of paraffin edges of boiling over by wiener. Different topological indices were presented following this examination that clarified physico-synthetic properties. The headway of large-scale integrated circuit innovation has empowered the development of interconnected networks which are complex in nature. Graph theory gives a key apparatus to designing and evaluating such networks. Connected networks and graph theory give a detailed comprehension of these connected themes. Chemical graph theory is a part of numerical science where in it applies devices of graph theory to demonstrate the chemical wonder numerically. In the architecture of network interconnection point addresses processor and an edge addresses a network path through which transmission completed. In network vertex is addresses network hub like Personal computer (PC), switch, some other gadget and an edge addresses a network path through which transmission completed. This theory contributes a noticeable job in the fields of chemical sciences [6].

A sub-atomic or chemical graph is a basic finite graph in which vertices indicate the molecules and edges signify the chemical bonds in the basic chemical construction. The topological indices are really a numeric amount related with chemical constitution indicating for connection of the chemical construction with numerous properties of physiochemical nature, reactivity of chemicals, and organic exercises. These topological indices or invariants are also numeric values related with computer networks, their interconnections and their properties etc. This examination gives a basis to understand the profound topologies of some important bridge networks and how these networks can be developed on the basis of best topological properties. This feature also gives potential assistance to scholars to contemplate networks characteristics better. For additional work, if the relating networks are replaced by different networks, this study can also calculate and get the comparing formulas [3].

An interconnection network's structure can be mathematically demonstrated by a graph. The geography of a graph decides the way where vertices are associated by edges. From the geography of a network certain properties can easily be resolved. Maximum distance is resolved between any two hubs in the network. The level of hub is identified by the number of connections attached with

it. Computer networks from intranet to world wide networks, electric power interconnection, social networks, sexual sickness of networks of transmission, and genome networks are comparable with graph theory with the help of complex networks analysis apparatus. All these networks are at peak level of their use and diversified. In this load of cases, this study can compute boundaries called Topological invariants (TIs) that mathematically depicted the connectedness designs (structure) between the hubs or entertainers in a network. So this study can construct a brain shocking network of general sets of laws partner laws (hubs) that direct typical organic subjects for instance. QSAR and QSPR are providing the foundation for these models. A final remark is that the utilization of the measurement in the network plane facilitates a quantitative evaluation of various geography safeguarding mapping algorithms [4].

To increase the effectiveness of observing and conservation endeavors, it is of key importance to foster sound quantitative techniques that are able to indicate which key areas and landscape components play noticeable and critical job in the working of territory mosaics. In this article, new emerging study will compute different indices of bridge networks. In the end formulate a mathematical formula to construct computer network or processor design or chemical compound, check the properties of concerning before and also check the feasibility of any one which discuss earlier [6].

The study has implications in the fields of computer science, physics, chemistry, mathematics and bio-informatics for modeling purpose of networks, network interconnections, power generation interconnection networks and chemical compounds. K-Banhatti Sombor topological invariants allow us to accumulate information about algebraic structures and mathematically predict hidden properties of various structures such as bridge networks.

2 Literature Review

The study is defined and calculated different kinds of indices in context with topology like degree based, distance based and counting related topological indices etc. The molecular and atomic structures of lattice are similar to hexagonal structures; honey comb structures mesh structures and grid structures networks. The Atom-bond connectivity (ABC) and Geometric arithmetic (GA) correlate different physical and chemical properties of chemicals having above structures, like solidness and strain energy and boiling point etc. Group theory and construction of graphs can use the Cayley graphs (CG) of groups for determining properties. In this study, they compute equations of general Randi' c index for various upsides of α, first Zagreb index, ABC index, GA index, the fourth ABC index (ABC4), fifth GA index (GA5) for certain groups of graphs [7]. These graphs provide the basis for calculating many other families of graphs for chemical compounds and computer networks [8].

In the view of study, the headway of large-scale integrated circuit innovation has empathy the development of interconnected networks which are complex in nature. Graph theory gives a key apparatus to designing and evaluating such networks. Connected networks and graph theory give a detailed comprehension of these connected themes. Chemical graph theory is a part of numerical science wherein they apply devices of graph theory to demonstrate the chemical wonder numerically. In network vertex is addresses network hub like PC, switch, switch some other gadget and an edge addresses a network path through which transmission completed. It calculates the different topological indices of eccentricity-based for binary tree up to k-level. The results of a paper can be used for computer networks and chemical networks in topological characterization [9].

The calculation of the irregularity indices of honey comb networks, hexagonal networks, oxide networks, and silicate networks is done. The results are very helpful in understanding the behavior of different computer networks and chemical networks. After understanding these formulas different

researchers can construct their own best networks in chemistry and computer also [10]. Further, study elaborates that graph theory is a field through which they calculate topological indices for finding the properties of different chemicals without performing any types of experiments on them. It also calculates topological indices for m-polynomial block shift networks which is a part of different chemical compounds with the help of division of edge [11].

It is determined that it can solved the real world problem just by modeled the behavior of the problem by applying Chemical reaction network theory (CRNT). There are lot of applications of chemical graph theory in bio chemistry and theoretical chemistry. It is derived networks used in different chemical compounds and computer science and calculate their topological indices for 1st type, 2nd type and 3rd type. They said that these results are very much effective in preparation of new drugs and helpful in to understand the properties of chemical compounds [12]. Simonraj et al. described the chemical network and is displayed numerically by topological indices in chemical graph theory. On the other hand, specific physicochemical properties are correlated with basic chemical compounds. The graph assumes an indispensable part in displaying and planning any chemical network. determined another sort of graphs, which is named a third kind of hex-inferred networks. It contemplated a recently framed third kind of hex-inferred networks. As these significant outcomes are useful from numerous chemical perspectives just as for pharmaceutical sciences, these outcomes additionally give the premise to comprehend the profound basic geographies of the above networks [13].

It is defined that chemical reaction network theory is a space of applied arithmetic that undertaking to show the direct of genuine chemical structures. Since its foundation during the 1960 s, it has drawn in a creating research local area, for the most part due to its applications in natural chemistry and hypothetical science [14]. It has also drawn in revenue from pure mathematicians in light of the captivating issues that rise up out of the numerical designs included. In this report, they process as of late described topological indices; specifically, number-crunching mathematical index, SK index, 1 SK index, and 2 SK index of the octagonal network [15]. Another study told that topological indices and network polynomials are invariants of molecular graphs. Expected properties of structures of molecules can be studied with the help of invariants. In benzene networks honeycomb networks are significantly used. In the current article, new topological portrayals of honeycomb networks are given as degree-based descriptors [16]. These indices of honeycomb networks are very much effective to understand physiochemical properties of chemicals. These realities might be valuable for individuals working in software engineering and science who experience honeycomb networks. An ideal level of a specific index can be acquired by putting a limitation on n [17].

In this study, the authors got numerous degree based topological indices for benzene ring installed in p-type-surface network and Tickysim spinnaker model (TSM) sheet. Initially, they figured m-polynomial of these graphs and later recuperated numerous degree-based topological indices applying it. Further they have shown outcomes graphically. These outcomes can assume a significant part to imagine geography of the aforesaid networks [18]. Further, calculation of topological indices of pent-heptagonal Nano-sheet are done. QSARs address prescient models got from the use of factual apparatuses relating to the organic action (counting alluring therapeutic effects and unfortunate side effects) of synthetics (drugs/poisons/ecological toxins) with descriptors illustrative of molecular structure as well as properties [19].

It is illustrated that Hand gestures recognition (HGR) is one of the primary spaces of research for Human computer interaction (HCI) applications. There are used a distinct and wide-range dataset with 31 types of gesture of different quantities of fingers with raising isolation. These are total 2170 images. A large portion of the current datasets incorporate a predetermined number of gestures, while

here they considered various disfigurements and varieties, for example, pivot in 2 and 3 dimensional space and scale additionally slight verbalization. It is assessed our technique on SBU-1 and examined our strategy against scale, revolution and clamor. It is executed our calculation for this dataset and got the mean precision up to 90%. Additionally, it is contrasted the outcomes and some datasets that mostly share the pictures of our dataset. They shouted the soundness of our calculation in presence of commotion and freedom of it to scale and turn both theoretically and tentatively. Since the GNG diagram is not special, they additionally tried the affectability of our algorithm to this graph [20].

Research theme elaborate that topological indices can be engineered by converting a chemical structure into a numerical value. These topological indices partner certain physico-substance properties like limit, security, and strain energy and so on of synthetic mixtures. Graph theory has tracked down an impressive use around here of exploration. In this paper, they stretch out this examination to interconnection networks and determine scientific shut aftereffects of General Randi c' index R (G) for various upsides of "α" for octagonal network, toroidal polyhex and summed up crystal. Interconnection networks of multiprocessor are regularly needed to interface a great many homogeneously reproduced processor-memory combines, every one of which is known as a handling hub [21].

Readings of the work shows how a high density of catalytic sites joined with specific spatial directions of those sites can cause a supra molecular catalytic machine that works deliberately to work with the development of pertinent progress states and, thusly, upgrade response rates. The surviving variety of MOF topologies proposes that these permeable systems will end up being helpful organizational structures for a rich assortment of multisite-catalyzed compound responses [22].

According to the research of this paper, physicochemical properties and topological indices are discussed and comprehensively examined in the QSAR/QSPR, for example, Randić, Zagreb, and ABC index are utilized to foresee bioactivity of the synthetic mixtures. Chart theory has discovered significant use in science, especially in demonstrating substance structures. Topological indices are planned fundamentally by changing a molecular graph into a number. In this paper they compute the randić, zagreb, and ABC index of silicate, honeycomb and hexagonal networks. Assembled recursively utilizing the hexagon decoration, honeycomb networks are broadly utilized in computer illustrations, phone base stations, picture preparing, and in science as the portrayal of benzenoid hydrocarbons [23]. This likewise gives possible assistance to researchers to consider better characteristics of networks [24].

According to readings explanation that another arising science is cheminformatics which is a combination of chemistry, mathematics and computer science. It is the main concern and constituent part is QSAR and QSPR which mainly explore physicochemical properties of chemicals and their structures. This paper tended to the OTIS traded networks and bi swapped networks and investigated on their topological indices. They decided the overall Randić, general aggregate availability, first and second Zagreb, first and second multiple Zagreb, hyper Zagreb molecule bond and mathematical indices for both the group of networks by considering the premise network as way Pn and k-customary graph Rk. They likewise gave express formulae for ABC4 and GA5 indices of these networks with the premise Rk network [25].

Another study conveyed that since computers as a tool used in the world for source of information, they increasing in numbers numerously. To such an extent, today, it is hard to track down any space, logical or other, which is not dependent on their application. Indeed, even general sets of laws have been affected by novel processing and data strategies. A considerable lot of these QSAR procedures depend on the utilization of structural boundaries, which are mathematical arrangement that classify helpful structural data and empathy relationships among structure and natural properties [26].

In every one of these cases, they can compute boundaries called TIs that mathematically depicted the connectedness designs between the nodes or entertainers in a network. Thus, TIs are helpful as contributions for QSPR models at all structural levels. Indeed, even general sets of laws might be moved toward utilizing figuring and data strategies like networks. So they can develop an unpredictable network of overall sets of laws associating laws (nodes) that manage basic organic points for example [27].

The research study told that graphs are considered as a stunning modeling apparatus which can be utilized to show and clarify diverse sort of relations between actual issues. A great deal of issues and issues can be investigated exhaustively with the assistance of graph theory. These applications are acquainted especially with stretch out graph theory and to display its objective and significance in software engineering designing. This paper is planned to benefit the under investigations of software engineering to get significance data on graph theory and its significance with different subjects like working frameworks, networks, databases, programming, etc. this paper focused in on the various uses of huge graph theory that have congruity to the field of software engineering and applications [28].

The hypothetical thoughts of graph are especially utilized by software engineering applications. Especially in research spaces of software engineering such information mining, picture division, grouping, picture catching, networking, etc Also fundamental thought of graph concealing is utilized in asset distribution, booking. Furthermore, ways, walks and circuits in graph theory are used in enormous applications say versatile deals issue, information base arrangement thoughts, resource networking. This prompts headway of new estimations and new hypotheses that can be used in gigantic applications. It has been parceled into two regions. First region gives certain establishment of graph theory and a couple of uses in booking. Second region underlines how graph theory is utilized in various PC applications [29].

Research material of this paper explains that the field of math accepts vital part in various fields. The most important piece of the science is graph theory and it is utilized in structure modeling. Regardless, for each portrayed theory they show the fields wherein it is used. It tends to be utilized in multiprocessor frameworks, in factual information bases, in information mining field, in web looking and arranging, in acknowledgment of examples and in PC vision too. This paper gives a diagram of the uses of graph theory in heterogeneous fields to some degree yet basically software engineering is the field around which graph theory thoughts work [30]. Thusly, these fields have fortified the improvement of various new chart hypothetical thoughts and incited many testing graph theory issues. They can expect that the proceeded with exchange between graph theory and various spaces of use will provoke critical new developments. The critical occupation of graph theory in PC applications is the improvement of graph estimations [31].

Topological invariants empower us to accumulate information about algebraic structures and gives us a mathematical technique to guess the hidden properties of different structures [32]. Numerous methods are present history to check the quality of a topological index [33]. There are two main clashes of topological indices, first one is the degree based topological and the second class is known as distance based topological indices. There are hundreds of such invariants are present in history [34,35]. K-banhatti sombor index and its reduced version has good potential of prediction in the field of computer science, mathematics, chemistry, pharmacy, informatics and biology in context with physical and chemical structures and networks [36].

3 Research Methodology

Methodology is based on quantitative inquiry. The purpose of this research is to explore and develop understanding about critical concerns error free, failure free, best performance and having advance capabilities computer networks, interconnection network of processors, and power interconnection networks.

3.1 Objectives

The main objective of this study is to investigate the topological invariants of bridge computer networks. The study finds out the intensity of seriousness of topological indices in certain networks like computer networks, interconnection network of processors, power interconnection networks and chemical structures etc. In this paper study explains the k-banhatti sombor indices, its reduced form and their benefits. This article explains and aware recent use of k-banhatti sombor indices. Its prime objective is to develop formulas, so that it can check the topology, performance of certain networks without doing/performing experiments and also before manufacturing of them. The work deduced some results which are used in the modeling of certain computer networks, interconnection networks, power interconnection networks and chemical structures [37].

3.2 Significance

The study is very significant in these days because it creates awareness about Topological invariants of certain networks like computer networks, interconnection network of processors, power interconnection networks and chemical structures etc. It is also discovering new and significant solutions or formulas for modeling certain networks because no any adequate solution has been found till now due to incremental and fast nature [38].

3.3 Hypothesis

The study considers the following hypothesis for the development of certain networks like computer networks, interconnection network of processors, power interconnection networks and chemical structures because venders and manufacturer need to understand the complexity and intensity of performance and failure free products. Analysis provides the strength to developed error free, failure free and best performer computer networks, interconnection network of processors, and power interconnection networks.

3.4 Method

In this study it will take an existing bridge network, associate it with graph and solve topology of graph with the help of k-banhatti Sombor indices and its reduced form. The concerning results in the form of formulas will compare with existing results.

These deduced results will be applicable on many other networks in the fields of computer networks, processor interconnection networks, memory interconnection networks, power interconnection networks, and image processing afterward. This model is very much concerning as it solved the topology of a bridge network in numeric and graphical form and give accurate results. After analysis a simulation tool maple is used for the verification and validation of results. Fig. 1 shows the flow of systematic study of this article in which take existing bridge networks associate it with graph theory, solve the topology of graph by k-banhatti sombor indices and compare the results, and deduced results will be used for modeling of certain networks.

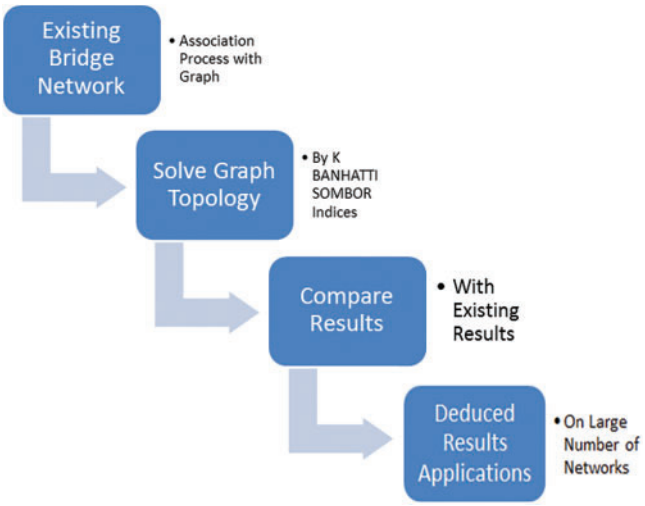

Figure 1: Research methodology

4 Experimental Results

A bridge graph is a graph obtain from number of graphs $G_1, G_2, G_3, \ldots G_m$ by associate the vertices v_i and v_{i+1} by an edge$\forall$, $i = 1, 2, \ldots, m-1$ [2]. K-Banhatti sombor indices have two variants, k-banhatti sombor index and its reduced version [4].

$$KBSO(G) = \sum_{ue} \sqrt{d_u^{\,2} + d_Q^{\,2}} \tag{1}$$

Eq. (1) shows the k-banhatti sombor index which will be used for the solution of bridge networks mentioned in the Figs. 2, 4, and 6.

$$KBSO_{rQ}\,\underline{d}\,(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + (d_Q - 1)^2} \tag{2}$$

Eq. (2) shows the k-banhatti sombor reduced index which will be used for the solution of bridge networks mention in the Figs. 2, 4, and 6.

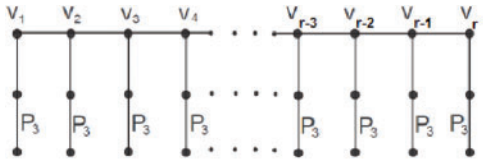

Figure 2: G_r (P_s, v) over P_s for bridge network

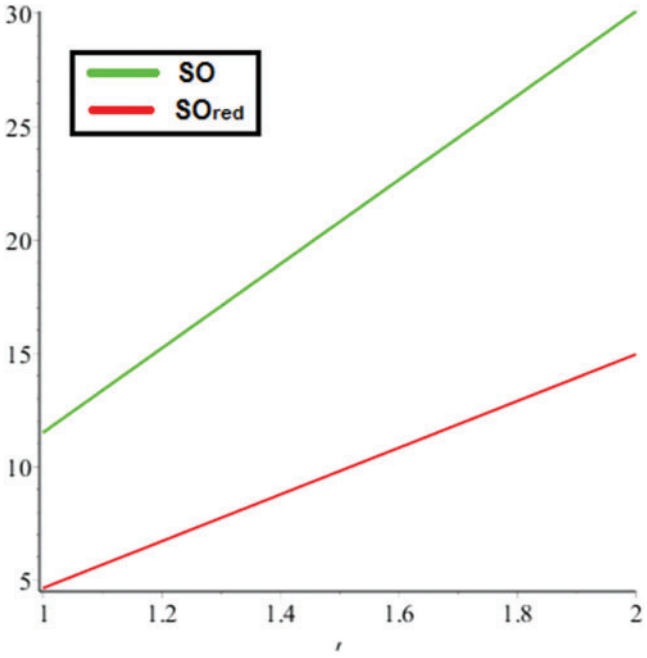

Figure 3: KBSO and $KBSO_{red}$ for G_r (K_s, v) over K_s

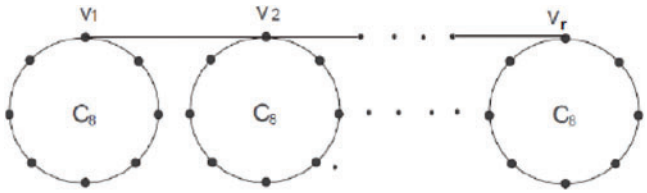

Figure 4: G_r (C_s, v) over C_s for bridge network

$$de = du + dv - 2$$

Tab. 1 describes the edge partitions of graph G_r ($P_{s,\ v}$) over P_s of bridge graph given in Fig. 2.

Table 1: Edge partition of G_r ($P_{s,v}$) over P_s

E	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	ε(1 , 2)	1	ε(1 , 1)	R
ε_2	ε(2 , 2)	2	ε(2 , 2)	3r + 2
ε_3	ε(2 , 3)	3	ε(2 , 3)	R
ε_4	ε(3 , 3)	4	ε(3 , 4)	r – 3

4.1 Main Results

Fig. 2 shows bridge networks in which bus networks and star networks bridge in a tree like structure.

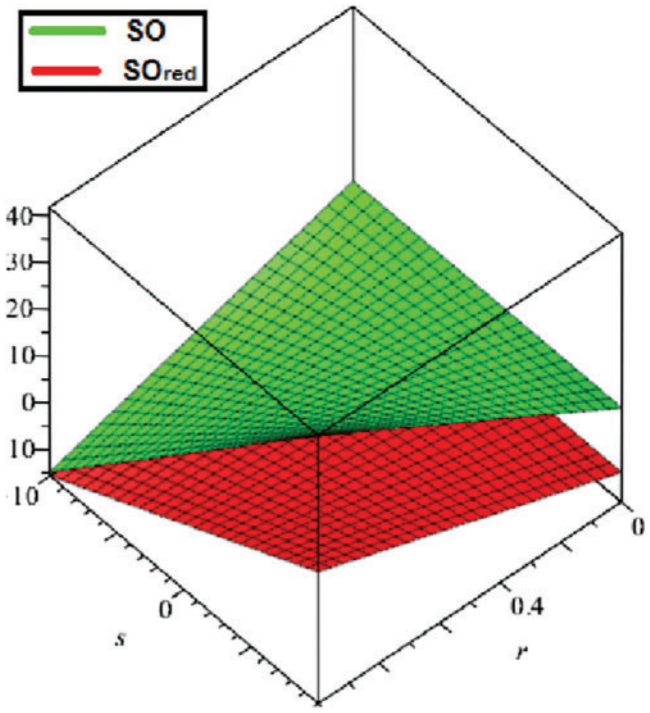

Figure 5: KBSO and $KBSO_{red}$ for G_r (K_s, v) over K_s

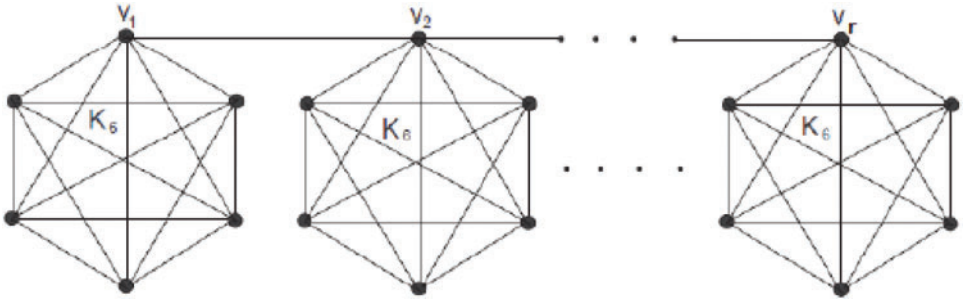

Figure 6: G_r (K_s, v) over K_s

4.1.1 Bridge Graph G_r (P_s, v) Over Path

If the vertex set is V then by the observation of Fig. 2, it can order this vertex set into four subsets $V1$, $V2$, $V3$ and $V4$, Such that $V = V1 + V2 + V3 + V4$. If E represents the edge set. The Fig. 2 shows that there are four distinct kinds of edges present in the graph of bridge graph G_r (P_s, v) over path of hybrid networks. The Tab. 1, explain in detail the edges partition.

4.1.2 Theorem 1

Let G be a graph of G_r (P_s, v) over P_s, then, KBSO and $KBSO_{red}$ indices are

$$KBSO(G) = 7\sqrt{2}\ (r) + 4\sqrt{2} + \sqrt{13}\, r - 5r - 15 \tag{3}$$

$$KBSO_{red}(G) = 3\sqrt{2}\ (r) + 2\sqrt{2} + \sqrt{5}r + \sqrt{13}\ r - 3\sqrt{13} \quad (4)$$

Eqs. (3) and (4) represents the proved results of graph of G_r (P_s, v) over P_s mentioned in the Fig. 2.

4.1.3 Investigation of Bridge Graphs by K-Banhatti Sombor Indices

Proof

$$KBSO(G) = \sum_{ue} \sqrt{{d_u}^2 + {d_Q}^2}$$

$$KBSO(G) = \sqrt{1^2 + 1^2}\ (r) + \sqrt{2^2 + 2^2}\ (3r + 2) + \sqrt{2^2 + 3^2}\ (r) - 5\sqrt{3^2 + 4^2}\ (r - 3)$$

$$KBSO(G) = \sqrt{2}\ (r) + 6\sqrt{2}r + 4\sqrt{2} + \sqrt{13}\ r - 5r - 15$$

$$KBSO(G) = 7\sqrt{2}\ (r) + 4\sqrt{2} + \sqrt{13}\ r - 5r - 15$$

$$KBSO_{rQ\underline{d}}(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + (d_Q - 1)^2}$$

$$KBSO_{red}(G) = \sqrt{(1-1)^2 + (1-1)^2}\ r + 5\sqrt{(2-1)^2 + (2-1)^2}\ (3r + 2) + 4\sqrt{(2-1)^2 + (3-1)^2}\ r + \sqrt{(3-1)^2 + (4-1)^2}\ (r - 3)$$

$$KBSO_{red}(G) = 0 + 3\sqrt{2}\ (r) + 2\sqrt{2} + \sqrt{5}r + \sqrt{13}\ r - 3\sqrt{13}$$

$$KBSO_{red}(G) = 3\sqrt{2}\ (r) + 2\sqrt{2} + \sqrt{5}r + \sqrt{13}\ r - 3\sqrt{13}$$

Fig. 3 shows the results (Eqs. (3) and (4)) of k-banhatti sombor indices and its reduced form in green and red lines respectively.

Tab. 2 describes the edge partitions of graph G_r ($K_{s,\ v}$) Over K_s of bridge graph given in Fig. 4 with frequencies.

Table 2: Edge partition of G_r (K_s, v) over K_s

ε	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	$\varepsilon_{(2,2)}$	2	$\varepsilon_{(2,2)}$	rs – 2r
ε_2	$\varepsilon_{(2,3)}$	3	$\varepsilon_{(2,3)}$	4
ε_3	$\varepsilon_{(2,4)}$	4	$\varepsilon_{(2,4)}$	2r – 4
ε_4	$\varepsilon_{(3,4)}$	5	$\varepsilon_{(3,5)}$	2
ε_5	$\varepsilon_{(4,4)}$	6	$\varepsilon_{(4,6)}$	r–3

4.2 Main Results

Fig. 4 shows the bridge networks in which bus networks and ring networks bridge together.

4.2.1 Bridge Graph $G_r\ (C_s, v)$ Over Cycle

Assuming V is the arrangement of vertices saw in Fig. 4, this arrangement of vertices can be parted into four subsets *V1, V2, V3,* and *V4* to *V=V1+V2+V3+V4*. When $\varepsilon\ (D2\ (m))$ addresses an edge set. Fig. 4 shows a half and half network cycle with five distinct kinds of edges in the network graph of the bridge graph *Gr (Cs, v)*. Tab. 2 provides a detailed description of the edge set.

4.2.2 Theorem 2

Let G be a graph of G_r (C_s, v) over C_s Then *KBSO* and $KBSO_{red}$ are

$$KBSO\,(G) = 2\sqrt{2}\ (rs) - 4\sqrt{2} - 2\sqrt{13} + 4\sqrt{5}\ r - 8\sqrt{5} - 2\sqrt{34} + 2\sqrt{13}\ r. \tag{5}$$

$$KBSO_{red}\,(G) = \sqrt{2}\ (rs) + 2\sqrt{10}r - 4\sqrt{10} - 2\sqrt{2}\ r + 8\sqrt{5} - 3\sqrt{34} + \sqrt{34}\ r. \tag{6}$$

Eqs. (5) and (6) represent the proved results of graph of G_r (C_s, v) over Cycle mentioned in the Fig. 4.

4.2.3 Investigation of Bridge Graphs by K-Banhatti Sombor Indices

Proof

$$KBSO\,(G) = \sum_{ue} \sqrt{{d_u}^2 + {d_Q}^2}$$

$$KBSO\,(G) = \sqrt{2^2 + 2^2}\,(rs - 2r) + \sqrt{2^2 + 3^2}\,(4) + \sqrt{2^2 + 4^2}\,(2r - 4) - \sqrt{3^2 + 5^2}\,(2) + \sqrt{4^2 + 6^2}\,(r - 3)$$

$$KBSO\,(G) = 2\sqrt{2}\ (rs) - 4\sqrt{2} - 2\sqrt{13} + 4\sqrt{5}\ r - 8\sqrt{5} - 2\sqrt{34} + 2\sqrt{13}\ r.$$

$$KBSO_{rQ\underline{d}}\,(G) = \sum_{ue} \sqrt{(d_u - 1)^2 + \left(d_Q - 1\right)^2}$$

$$KBSO_{red}(G) = \sqrt{(2-1)^2 + (2-1)^2}\,(rs - 2r) + \sqrt{(2-1)^2 + (3-1)^2}\ \ (4)$$

$$+\sqrt{(2-1)^2 + (4-1)^2}\,(2r - 4) + \sqrt{(3-1)^2 + (5-1)^2}\,(2) + \sqrt{(4-1)^2 + (6-1)^2}\,(r - 3)$$

$$KBSO_{red}\,(G) = \sqrt{2}\ (rs) + 2\sqrt{10}r - 4\sqrt{10} - 2\sqrt{2}\ r + 8\sqrt{5} - 3\sqrt{34} + \sqrt{34}\ r.$$

Fig. 5 shows the results (Eqs. (5) and (6)) of k-banhatti sombor indices and its reduced form in green and red color respectively in 3D version.

Tab. 3 describes the edge partitions of graph G_r ($C_{s,\,v}$) Over C_s of bridge graph given in Fig. 6 with number of occurrences.

4.3 Main Results

Fig. 6 shows the bridge networks in which bus networks and fully connected networks bridge together.

Table 3: Edge partition of G_r ($C_{s,v}$) over C_s

ε	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	$\varepsilon_{(2,2)}$	2	$\varepsilon_{(2,2)}$	rs – 2r
ε_2	$\varepsilon_{(2,3)}$	3	$\varepsilon_{(2,3)}$	4
ε_3	$\varepsilon_{(2,4)}$	4	$\varepsilon_{(2,4)}$	2r – 4
ε_4	$\varepsilon_{(3,4)}$	5	$\varepsilon_{(3,5)}$	2
ε_5	$\varepsilon_{(4,4)}$	6	$\varepsilon_{(4,6)}$	r–3

4.3.1 Bridge Graph G_r (K_s, v) Over Complete Graph

Assuming that vertices set is V, understanding Fig. 6 allows us to sort this set of vertices into three subsets V1, V2, and V3 so that V=V1+V2+V3. If E shows the edge set, Fig. 6 shows the bridge graph Gr (Ks, v) of the complete graph of the hybrid network. Bridge graph of the network graph has five different edges. Tab. 3 provides a detailed description of the edge set.

4.3.2 Theorem 3

Let G be a graph of G_r (K_s, v) over K_s. Then, KBSO and $KBSO_{red}$ indices are

$$\begin{aligned} KBSO\,(G) \;=\; & \\ & 2\sqrt{65}+2\sqrt{s^2+2s+17}+\sqrt{89}\,(r)-2\sqrt{89}+\sqrt{s^2+4s+29}\,(r) \\ & -2\sqrt{s^2+4s+29}+\left(\frac{1}{2}\right)\sqrt{5s^2+18s+17}\left(r^2s-rs-2r-2\right) \end{aligned} \tag{7}$$

$$\begin{aligned} KBSO_{red}(G) = {} & 6\sqrt{5}+2\sqrt{s^2+9}+\sqrt{65}\,(r)-2\sqrt{65}+\sqrt{s^2-2s+17}\,(r) \\ & -2\sqrt{s^2-2s+17}+\left(\frac{1}{2}\right)\sqrt{5s^2-24s+29}\left(r^2s-rs-2r-2\right). \end{aligned} \tag{8}$$

Eqs. (7) and (8) represent the proved results of graph of G_r (K_s, v) over complete graph mentioned in the Fig. 6.

4.3.3 Investigation of Bridge Graphs by K-Banhatti Sombor Indices

Proof

$$KBSO\,(G)=\sum_{ue}\sqrt{{d_u}^2+{d_Q}^2}$$

$$\begin{aligned} KBSO\,(G) \;=\; & \sqrt{4^2+7^2}\,(2)+\sqrt{4^2+(s+1)^2}\,(2)+\sqrt{5^2+8^2}\,(r-2) \\ & -\sqrt{5^2+(s+2)^2}\,(r-2)+\sqrt{(s-1)^2+(2s-4)^2}\,(rs(r-1)-2\,(r+1))/2. \end{aligned}$$

$$\begin{aligned} KBSO\,(G) \;=\; & 2\sqrt{65}+2\sqrt{s^2+2s+17}+\sqrt{89}\,(r)-2\sqrt{89}+\sqrt{s^2+4s+29}\,(r) \\ & -2\sqrt{s^2+4s+29}+\left(\frac{1}{2}\right)\sqrt{5s^2+18s+17}\left(r^2s-rs-2r-2\right) \end{aligned}$$

$$KBSO_{rQ}\,\underline{d}\,(G)=\sum_{ue}\sqrt{(d_u-1)^2+\left(d_Q-1\right)^2}$$

$$KBSO_{red}(G) = \sqrt{(4-1)^2+(7-1)^2}\,(2) + \sqrt{(4-1)^2+(s+1-1)^2}\;\;(2) + \sqrt{(5-1)^2+(8-1)^2}\,(r-2)$$
$$+ \sqrt{(5-1)^2+(s+2-1)^2}\,(r-2)$$
$$+ \sqrt{(s-1-1)^2+(2s-4-1)^2}\,(rs\,(r-1)-2\,(r+1))\,/2.$$

$$KBSO_{red}(G) = \sqrt{45}\,(2) + \sqrt{9+s^2}\;\;(2) + \sqrt{65}\,(r-2) + \sqrt{16+(s+1)^2}$$
$$(r-2) + \sqrt{(s-2)^2+(2s-5)^2}\,(rs\,(r-1)-2\,(r+1))\,/2.$$
$$KBSO_{red}(G) = 6\sqrt{5} + 2\sqrt{s^2+9} + \sqrt{65}\,(r) - 2\sqrt{65} + \sqrt{s^2-2s+17}\,(r)$$
$$-2\sqrt{s^2-2s+17} + \left(\frac{1}{2}\right)\sqrt{5s^2-24s+29}\,\left(r^2s-rs-2r-2\right).$$

Fig. 7 shows the results (Eqs. (7) and (8)) of k-banhatti sombor indices and its reduced form in green and red color respectively in 3D version.

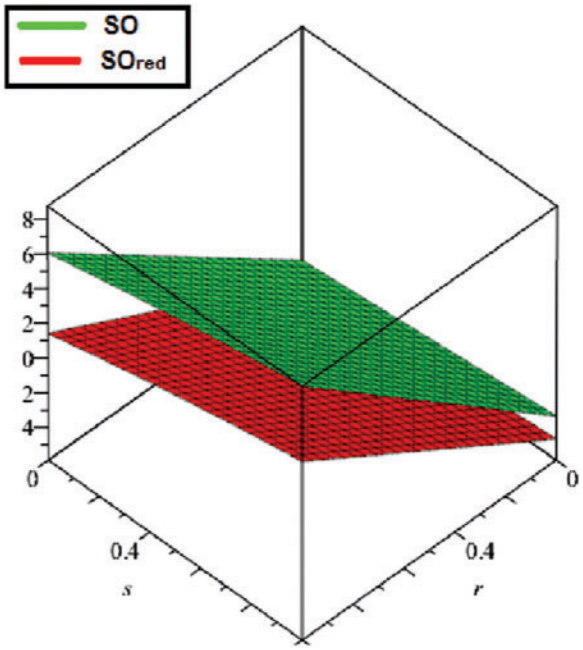

Figure 7: KBSO and $KBSO_{red}$ for G_r (K_s, v) over K_s

5 Conclusion

TIs have lots of uses and implementations in many fields of computer science, chemistry, biology, informatics, arithmetic, material sciences, and many more. But the utmost significant application is in the non-exact QSPR and QSAR. TIs are associated with the structure of networks, backbone of internet, local area networks and chemical structure. But in present article, it discusses the k-banhatti sombor invariants which are freshly presented and have numerous prediction quality for different variants of bridge graphs or networks, i.e., G_r (P_s, v), G_r (C_s, v) and G_r (K_s, v). Figs. 3, 5 and 7 gives

the graphical representation of sombor indices for above mentioned bridge graphs of networks. These deduced results will be used for the modeling of computer networks (like LAN, MAN, WAN, and backbone of internet), interconnection networks, power generation networks, chemical structures, image processing, bio-informatics, and memory interconnection networks etc.

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References

[1] T. Mansour and M. Schork, "The vertex PI index and szeged index of bridge graphs," *Discrete Applied Mathematics*, vol. 157, no. 7, pp. 1600–1606, 2009.
[2] A. J. M. Khalaf, M. Farhan, M. Siddiqui and M. Farahani, "On degree based topological indices of bridge graphs," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 23, no. 1, pp. 1139–1156, 2020.
[3] I. Gutman, "Some basic properties of sombor indices," *Open Journal of Discrete Applied Mathematics*, vol. 4, no. 1, pp. 1–3, 2021.
[4] B. F. Rasulev, N. D. Abdullaev, V. N. Syrov and J. Leszczynski, "A quantitative structure-activity relationship study of the antioxidant activity of flavonoids," *Quantitative Structure-Activity Relationship (QSAR) & Combinatorial Science*, vol. 24, no. 9, pp. 1056–1065, 2021.
[5] R. Kiralj and M. Ferreira, "Basic validation procedures for regression models in QSAR and QSPR studies: Theory and application," *Journal of the Brazilian Chemical Society*, vol. 20, no. 4, pp. 770–787, 2021.
[6] S. Hayat and M. Imran, "Computation of topological indices of certain networks," *Applied Mathematics and Computation*, vol. 240, no. 1, pp. 213–228, 2014.
[7] S. Sultan, W. Gharibi and A. Ahmad, "Computing the topological indices for certain families of graphs," *Science International*, vol. 27, no. 6, pp. 4957–4961, 2015.
[8] K. C. Das, F. M. Bhatti, G. Lee and I. Gutman, "Spectral properties of the he matrix of hexagonal systems," *Match Communications in Mathematical and in Computer Chemistry*, vol. 65, no. 1, pp. 753–774, 2011.
[9] M. Imran, M. A. Iqbal, Y. Liu, A. Q. Baig, W. Khalid *et al.,* "Computing eccentricity-based topological indices of 2-power interconnection networks," *Journal of Chemistry*, vol. 1, no. 1, pp. 1–7, 2020.
[10] J. Liu, L. Cai, W. Akhtar, S. A. Maitla and Y. Wei, "Computation of irregularity indices of certain computer networks," *Mathematical Problems in Engineering*, vol. 1, no. 1, pp. 1–17, 2020.
[11] M. Cancan, I. Ahmad and S. Ahmad, "Study of topology of block shift networks via topological indices," *Proyecciones (Antofagasta)*, vol. 39, no. 4, pp. 887–902, 2020.
[12] M. S. Ahmad, W. Nazeer, S. M. Kang, M. Imran and W. Gao, "Calculating degree-based topological indices of dominating david derived networks," *Open Physics*, vol. 15, no. 1, pp. 1015–1021, 2017.
[13] C. C. Wei, H. Ali, M. A. Binyamin, M. N. Naeem and J. B. Liu, "Computing degree based topological properties of third type of hex-derived networks," *Mathematics*, vol. 7, no. 4, pp. 368–376, 2019.
[14] M. S. Ahmad, D. Afzal, W. Nazeer and S. M. Kang, "On topological indices of octagonal network," *Journal of Optoelectronics and Advanced Materials*, vol. 18, no. 1, pp. 884–892, 2017.
[15] M. Naeem, M. K. Siddiqui, J. L. G. Guirao and W. Gao, "New and modified eccentric indices of octagonal grid omn," *Applied Mathematics and Nonlinear Sciences*, vol. 3, no. 1, pp. 209–228, 2018.
[16] Z. Hussain, M. Munir, S. Rafique and S. M. Kang, "Topological characterizations and index-analysis of new degree-based descriptors of honeycomb networks," *Symmetry*, vol. 10, no. 10, pp. 478–485, 2018.

[17] M. Riaz, W. Gao and A. Q. Baig, "M-Polynomials and degree-based topological indices of some families of convex polytopes," *Open Journal of Mathematical Sciences*, vol. 2, no. 1, pp. 18–28, 2018.
[18] S. Mondal, D. E. Nilanjan and P. A. L. Anita, "Topological properties of networks using m-polynomial approach," *Konuralp Journal of Mathematics*, vol. 8, no. 1, pp. 97–105, 2020.
[19] F. Deng, X. Zhang, M. Alaeiyan, A. Mehboob and M. R. Farahani, "Topological indices of the pent-heptagonal nanosheets VC5C7 and HC5C7," *Advances in Materials Science and Engineering*, vol. 1, no. 1, pp. 1–12, 2019.
[20] N. Mirehi, M. Tahmasbi and A. T. Targhi, "Hand gesture recognition using topological features," *Multimedia Tools and Applications*, vol. 78, no. 10, pp. 13361–13386, 2019.
[21] M. K. Siddiqui, M. Naeem, N. A. Rahman and M. Imran, "Computing topological indices of certain networks," *Journal of Optoelectronics and Advanced Materials*, vol. 18, no. 1, pp. 884–892, 2016.
[22] P. Deria, D. A. G. Gualdrón, I. Hod, R. Q. Snurr, J. T. Hupp *et al.,* "Framework-topology-dependent catalytic activity of zirconium-based (porphinato) zinc (II) MOFs," *Journal of the American Chemical Society*, vol. 138, no. 43, pp. 14449–14457, 2016.
[23] A. W. B. Rajan, C. Grigorious and S. Stephen, "On certain topological indices of silicate, honeycomb and hexagonal networks," *Journal of Computer and Mathematical Sciences*, vol. 3, no. 5, pp. 498–556, 2012.
[24] J. B. Liu, T. Zhang and S. Hayat, "The calculations of topological indices on certain networks," *Journal of Mathematics*, vol. 2021, no. 1, pp. 1–12, 2021.
[25] A. Aslam, S. Ahmad, M. A. Binyamin and W. Gao, "Calculating topological indices of certain OTIS interconnection networks," *Open Chemistry*, vol. 17, no. 1, pp. 220–228, 2019.
[26] S. Bordoloi and B. Kalita, "Designing graph database models from existing relational databases," *International Journal of Computer Applications*, vol. 74, no. 1, pp. 25–31, 2013.
[27] A. D. Sanchez, G. Patlewicz and H. G. Diaz, "Network topological indices from chem-bioinformatics to legal sciences and back," *Current Bioinformatics*, vol. 6, no. 1, pp. 53–70, 2011.
[28] R. P. Singh, "Application of graph theory in computer science and engineering," *International Journal of Computer Applications*, vol. 104, no. 1, pp. 1–10, 2014.
[29] J. Nazir, M. W. Iqbal, T. Alyas, M. Hamid, M. Saleem *et al.,* "Load balancing framework for cross-region tasks in cloud computing," *Computers, Materials & Continua (CMC)*, vol. 70, no. 1, pp. 1479–1490, 2022.
[30] B. Tosuni, "Graph theory in computer science–an overview," *International Journal of Academic Research and Reflection*, vol. 3, no. 4, pp. 55–62, 2015.
[31] E. Arif, S. K. Shahzad, R. Mustafa, M. A. Jaffar and M. W. Iqbal, "Deep neural networks for gun detection in public surveillance," *Intelligent Automation & Soft Computing (IASC)*, vol. 32, no. 2, pp. 909–922, 2022.
[32] M. Munir, W. Nazeer, Z. Shahzadi and S. M. Kang, "Some invariants of circulant graphs," *Symmetry*, vol. 8, no. 11, pp. 1–8, 2016.
[33] A. R. Virk, M. A. Rehman and W. Nazeer, "New definition of atomic bond connectivity index to overcome deficiency of structure sensitivity and abruptness in existing definition," *Scientific Inquiry and Review*, vol. 3, no. 4, pp. 1–20, 2019.
[34] M. Hashmi, F. Chaudhry, A. J. M. Khalaf and M. Farahani, "Investigation of dendrimer structures by means of reverse atomic bond connectivity index," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 24, no. 1, pp. 473–485, 2021.
[35] Y. C. Kwun, A. R. Virk, W. Nazeer, M. A. Rehman and S. M. Kang, "On the multiplicative degree-based topological indices of silicon-carbon si2c3-I [p, q] and si2c3-II [p, q]," *Symmetry*, vol. 10, no. 8, pp. 1–11, 2018.

[36] A. R. Virk, "Multiplicative shingali and kanabour indices for bismuth tri-iodide," *Journal of Prime Research in Mathematics*, vol. 16, no. 2, pp. 80–88, 2020.
[37] X. R. Zhang, W. F. Zhang, W. Sun, X. M. Sun and S. K. Jha, "A robust 3-D medical watermarking based on wavelet transform for data protection," *Computer Systems Science & Engineering*, vol. 41, no. 3, pp. 1043–1056, 2022.
[38] X. R. Zhang, X. Sun, X. M. Sun, W. Sun and S. K. Jha, "Robust reversible audio watermarking scheme for telemedicine and privacy protection," *Computers, Materials & Continua*, vol. 71, no. 2, pp. 3035–3050, 2022.