



MHD FLOW OF JEFFREY FLUID WITH HEAT ABSORPTION AND THERMO-DIFFUSION

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ABSTRACT

Unsteady flow of fractionalized Jeffrey fluid over a plate is considered. In addition, thermo diffusion and slip effects are also used in the problem. The flow model is solved using Constant proportional Caputo fractional derivative. Initially, the governing equations are made non-dimensional and then solved by Laplace transform. From the Figs., it is observed that Prandtl and Smith numbers have decreasing effect on fluid motion, whereas thermo-diffusion have increasing effect on fluid motion. Moreover, comparison among fractionalized and ordinary velocity fields is also drawn.

Keywords: Free convection, Jeffrey fluid, Slip effect, Sorret effect, CPC fractional derivative.

1. INTRODUCTION

Now a days, magnetohydrodynamic (MHD) has been extended into wide areas of basic and applied research in sciences and engineering. The study of non-Newtonian fluid becomes very interested due to variety of technological applications like making of plastic sheets, lubricant's performance and motion of biological fluid. Non-Newtonian fluid models have extensive use in industrial and engineering processes such as production of paper, polymer processing, ink printing, paint suspension, and biological flows. Thus, the analysis of such fluids is of substantial research interest and significant importance. Typical characteristics of the flow of non-Newtonian fluids have become a crucial area of research for engineers, mathematician, scientists, and researchers. Strain rate and stress are a combination of linear and nonlinear relations characterized by Newtonian fluid and non-Newtonian fluids, respectively. With the relationship between strain rate and stress, non-Newtonian fluids, polymer solutions, slurries, and pastes, to mention just a few, are difficult for developing mathematical modeling in terms of differential equations. Due to this reason, non-Newtonian fluids give rise to an abundance of rheological mathematical models of fluids. We classify such fluids models by century: 18th century, from 1867 to 1893 (Barus and Maxwell model), and 19th century, from 1922 to 1995 (Blatter model, Ellis model, Giesekus model, Phan–Thien–Tanner model, Johnson–Tevaarwerk model, Carreau–Yasuda model, Carreau model, Cross model, Rivlin–Eriksen model, Oldroyd-8 constants model, Oldroyd-B model, Rivlin model, generalized Burgers, Eyring, and Williamson fluid model), among others. Kai-Long Hsiao (2017) worked on combined effects of electrical MHD heat transfer thermal extrusion system using non-Newtonian Maxwell fluid with radiative and viscous dissipation effects. Ramzan et al. [2022] discussed the effect of diffusion thermo on MHD flow of Maxwell fluid with heat and mass transfer. The model on Jeffrey fluid be the simplest and most popular, and it has attracted the interest of researchers in the field. Some of the work on Jeffrey fluid are of Das [2012] and Qasim [2013].

A comparative study and analysis of natural convection flow of MHD non-Newtonian fluid in the presence of heat source and first order chemical reaction was studied by Ahmad et al. (2019). During the last decade, different generalized fractional derivatives have appeared in the

literature that are derivatives of Caputo, Caputo-Fabrizio, constant proportional Caputo by Atangana et al. (2020) and Baleanu et al. (2020). Soret and radiation effects on MHD free convection flow over an inclined porous plate with heat and mass flux was studied by Kumar et al. (2016). Sandeep et al. (2016) analyzed the heat and mass transfer in nano fluid over an inclined stretching sheet with volume fraction of dust and nanoparticles. Ahammad et al. (2017) studied the radiation effect with Eckert number and Forchheimer number on heat and mass transfer over an inclined plate in the influence of suction/injection flow. Ali et al. (2013) studied the conjugate effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium. Sayeda et al. (2011) analyses the effect of viscosity and thermal conductivity on MHD flow. Shafique et al. (2022) studied the unsteady magnetohydrodynamic flow of second grade fluid. Khan et al. (2018) discussed the multiples solutions of Carreau fluid flow over an inclined shrinking sheet.

Some mathematical models of second grade fluids are industrial oils, slurry streams, and dilute polymer solutions with different geometry and boundary conditions. Fetecau et al. (2005) analyzed the solution of unidirectional flows of second grade fluid at plate with the assistance of the Fourier sine transformation. Ahmed et al. (2015) has analyzed the linearization method to MHD flow with heat and mass transfer boundary layer convective heat transfer. Effects of variable permeability and radiation absorption on MHD mixed convective flow in a vertical wavy channel studied by Narayana (2015), Nadeem et al. (2014) discussed the thermo-diffusion effects on MHD oblique stagnation point flow of viscoelastic fluid. Because of its rising significance, engineering needs to incorporate non-Newtonian fluid. Khan et al. (2017) discussed the Atangana Baleanu and Caputo Fabrizio fractional derivative for heat and mass transfer of second grade fluid. Khan et al. (2014) discussed the effects of wall shear stress MHD conjugate flow in the existence of permeable media over an inclined plate. Seth et al. (2015) discussed the MHD natural convection flow over an exponentially accelerated vertical plate with heat absorption. Tran et al. (2020) worked on stability of fractional derivatives for fractional calculus equations, Tuan et al. (2020) studied the mathematical model used for transference of COVID-19 with Caputo fractional derivatives. Shateyi et al. (2011) discussed the

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unsteady magnetohydrodynamic convective heat and mass transfer past an infinite vertical plate in a porous medium. Ramzan et al. (2021) analyzed the unsteady free convective magnetohydrodynamics flow of a Casson fluid through a channel with double diffusion and ramp temperature and concentration Khan et al. (2018) investigated unsteady flow of the Brinkman fluid between two side walls. Ali et al. (2013) discussed the heat and mass transfer with free convection MHD flow past a vertical plate embedded in a porous medium. Ramzan et al. (2021) discussed the behavior of heat absorption/generation on the MHD flow of Brinkman fluid. Khalid et al. (2015) studied the exact solution for unsteady free convective flows of Casson fluid over an oscillating vertical plate with constant wall temperature. Sheikh et al. (2020) studied the new model of fractional Casson fluid based on generalized Fick's laws and Fourier's laws together with heat and mass transfer. Shah et al. (2019) analyzed the Influence of magnetic field on double convection of fractional viscous fluid over an exponentially moving vertical plate. New trends of Caputo time fractional derivative model.

The goal of recent work is to examine the effect of incompressible Jeffrey fluid flow over an infinite vertical plate with slip and sorret effects. The MHD flow together with heat and mass transfer is considered. Initially, the dimensional equations have been made non-dimensional and then solved these equations via Laplace transform. All velocity, temperature, and concentration distribution results have been obtained and evaluated graphically. The comparison among fractionalized and ordinary fluids are distinguished graphically.

2. MATHEMATICAL MODEL

The flow of fractionalized Jeffrey fluid over a plate is studied in the presence of thermo-diffusion and slip effect. The fluid is flowing vertically upward along y' -axis and the x' -axis is normal to the plate. The fluid and plate have concentration C_∞ and temperature T_∞ at time $t' = 0$ with zero velocity. But for $t' > 0$, the plate starts to move in the plane with uniform velocity $U_1 f(t')$. The concentration and temperature of the plate is increased to C_w and $T' = T_w(1 - ce^{-dt'})$ with time t' . In view of above assumption, the convection flow of viscous fluid with Sorret effect over a plate [28,31], linear momentum Eq. is

$$\frac{\partial u_1(x',t')}{\partial t'} = \frac{(1+\lambda_3)\frac{\partial}{\partial t'}}{1+\lambda_2} \frac{\partial \tau(x',t')}{\partial x'} + g\beta_T(T' - T_\infty) + g\beta_C(C' - C_\infty) - \frac{\sigma\beta_0^2 u_1(x',t_1)}{\rho} - \frac{\mu u_1(x',t_1)}{\rho k_2}, \quad (1)$$

shear stress τ is

$$\tau = \frac{\partial u_1(x',t_1)}{\partial x'}. \quad (2)$$

thermal Eq. is

$$\frac{\partial T(x',t')}{\partial t'} = -\frac{\partial q(x',t')}{\rho c_p \partial x'} - R_1(T' - T_\infty). \quad (3)$$

According to Fourier's Law, $q_1(x', t_1)$ is given by

$$q_1(x', t_1) = -\alpha_0 \frac{\partial T(x', t_1)}{\partial x'}. \quad (4)$$

Diffusion Eq. is

$$\frac{\partial C(x',t')}{\partial t'} = -\frac{\partial J(x',t')}{\partial x'} - \frac{D_{KT}}{T_m} \frac{\partial q(x',t')}{\partial x'}. \quad (5)$$

According to Fick's Law, $J_1(x', t_1)$ is given by

$$J_1(x', t_1) = -D_m \frac{\partial C(x', t_1)}{\partial x'}. \quad (6)$$

The conditions for the model are

$$u_1(x', t') = 0, \quad T'(x', t') = T_\infty, \quad C'(y', t') = C_\infty, \quad y' > 0, \quad t' = 0, \quad (7)$$

$$u_1(0, t') - S_1 \frac{\partial u_1}{\partial x'} = U_1 f(t'), \quad T'(0, t') = T_\infty + T_w(1 - ce^{-dt'}), \quad C'(0, t') = C_w, \quad t' > 0, \quad (8)$$

$$u_1(x', t') \rightarrow 0, \quad T'(x', t') \rightarrow 0, \quad C'(x', t') \rightarrow 0, \quad x' \rightarrow \infty, \quad t' > 0. \quad (9)$$

3. GENERALIZED MODEL

Dimensionless form of the variables are

$$x^* = \frac{Ux'}{v}, \quad t^* = \frac{U^2 t'}{v}, \quad T^* = \frac{T' - T_\infty}{T_w - T_\infty}, \quad u^* = \frac{u_1}{U},$$

$$Gr^* = \frac{\nu \beta_T (T_w - T_\infty)}{U^3}, \quad C^* = \frac{C' - C_\infty}{C_w - C_\infty}, \quad Gm^* = \frac{\nu \beta_C (C_w - C_\infty)}{U^3}. \quad (10)$$

Eq. (1) is generalized fractionally by [32]

$$\tau = L_\beta D_t^\beta \frac{\partial u(x,t)}{\partial x}, \quad 1 \geq \beta > 0, \quad (11)$$

where $L_\beta = n_1 K_\beta = 1$ when $\beta \rightarrow 1$. Put Eq. (11) into Eq. (1) and using non-dimensional parameters from Eq. (10), we have

$$\frac{\partial u(x,t)}{\partial t} = L_\beta \frac{1+\lambda_1 s}{1+\lambda} D_t^\beta \frac{\partial^2 \bar{u}(x,t)}{\partial x^2} - (K^{-1} + M)u(x,t) + GrT(x,t) + GmC(x,t), \quad (12)$$

Eq. (2) is generalized fractionally by [33,34]

$$q = -A_\gamma D_t^\gamma \frac{\partial T(x,t)}{\partial x}, \quad 1 \geq \gamma > 0, \quad (13)$$

where thermal conductivity has generalized coefficient A_γ . Put Eq. (13) into Eq. (2) and making non-dimensional results, we have

$$\frac{\partial T(x,t)}{\partial t} = \frac{1}{Pr} D_t^\gamma \frac{\partial^2 T}{\partial x^2} - RT, \quad (14)$$

where $Pr = \frac{\rho \nu c_p}{A_\gamma}$.

Eq. (3) is generalized by using Fick's Law defined by

$$J = -B_\alpha D_t^\alpha \frac{\partial C(x,t)}{\partial x}, \quad 1 \geq \alpha > 0. \quad (15)$$

where molecular diffusion has generalized coefficient B_α . Put Eq. (15) into Eq. (3) and making non-dimensional results, we have

$$\frac{\partial C(x,t)}{\partial t} = \frac{1}{Sc} D_t^\alpha \frac{\partial^2 C(x,t)}{\partial x^2} + Sr D_t^\gamma \frac{\partial^2 T(x,t)}{\partial x^2}, \quad (16)$$

where $Sc = \frac{\nu}{B_\alpha}$ is the generalized Schimdt number.

Initial and boundary conditions are

$$u(x, t) = T(x, t) = C(x, t) = 0, \quad t = 0, \quad (17)$$

$$u(0, t) - S \frac{\partial u}{\partial x} = f(t), \quad T(0, t) = 1 - ce^{-dt}, \quad C(0, t) = 1, \quad t > 0, \quad (18)$$

$$u(x, t) \rightarrow 0, \quad T(x, t) \rightarrow 0, \quad C(x, t) \rightarrow 0, \quad x \rightarrow \infty, \quad t > 0, \quad (19)$$

where Gm , \S , M , R , and u represents the mass Grashof number, slip parameter, magnetic field, non-dimensional heat absorption parameter, mass Grashof number, and motion of fluid respectively and $D_t^\beta u(x, t)$ is the CPC derivative of $u(x, t)$ given by

$$D_t^\beta u(x, t) = \frac{1}{\Gamma(1-\beta)} \int_0^t [K_1(\beta)u(x, \tau) + K_0(\beta)u'(x, \tau)](t-\tau)^{-\beta} d\tau \quad (20)$$

4. SOLUTION OF PROBLEM

Eqs. (12,14,16) with conditions have been solved analytically.

4.1 Temperature profile

From Eq. (14), we have

$$s\bar{T}(x, s) = \frac{1}{Pr} \left(\frac{K_1(\gamma)}{s} + K_0(\gamma) \right) s^\gamma \frac{\partial^2 \bar{T}(x, s)}{\partial x^2} - RT(x, s). \quad (21)$$

Eq. (21) is satisfied by

$$\bar{T}(0, s) = \frac{1}{s} - \frac{c}{s+d}, \quad \bar{T}(x, s) \rightarrow 0, \quad x \rightarrow \infty. \quad (22)$$

Put Eq. (22) in Eq. (21)

$$\bar{T}(x, s) = \left(\frac{1}{s} - \frac{a}{s+b} \right) e^{-x \sqrt{\frac{Pr(s+R)}{K_1(\gamma) + K_0(\gamma)} s^\gamma}}, \quad (23)$$

4.2 Calculation of Concentration

Solution of Eq. (16) with conditions

$$s\bar{C}(x, s) = \frac{1}{Sc} \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha \frac{\partial^2 \bar{C}(x, s)}{\partial x^2} + Sr \left[\frac{K_1(\gamma)}{s} + K_0(\gamma) \right] s^\gamma \frac{\partial^2 \bar{T}(x, s)}{\partial x^2} \quad (24)$$

$$\bar{C}(0, s) = s^{-1}, \quad \bar{C}(x, s) \rightarrow 0, \quad x \rightarrow \infty. \quad (25)$$

Using Eq. (25) in Eq. (24) for $\alpha = \gamma$,

$$\bar{C}(x, s) = \left[s^{-1} + \frac{SrScPr(s+R) \left(\frac{1}{s} - \frac{c}{s+d} \right)}{(s+R)Pr-sSc} \right] e^{-x \sqrt{\frac{Scs}{K_1(\alpha) + K_0(\alpha)} s^\alpha}} - \frac{SrScPr(s+R) \left(\frac{1}{s} - \frac{c}{s+d} \right)}{(s+R)Pr-sSc} e^{-x \sqrt{\frac{Pr(R+s)}{K_1(\alpha) + K_0(\alpha)} s^\alpha}}. \quad (26)$$

4.3 Calculation of Velocity

Solution of Eq. (12) with conditions

$$s\bar{u}(x, s) = L_\beta \left[\frac{K_1(\beta)}{s} + K_0(\beta) \right] s^\beta \left[\frac{1+\lambda_1 s}{1+\lambda} \right] \frac{\partial^2 \bar{u}(x, s)}{\partial x^2} - (K^{-1} + M)\bar{u}(x, s) + Gr\bar{T}(x, s) + Gm\bar{C}(x, s), \quad (27)$$

$$\bar{u}(0, s) - S \frac{\partial \bar{u}(0, s)}{\partial x} = s^{-2}, \quad \bar{u}(x, s) \rightarrow 0, \quad x \rightarrow \infty. \quad (28)$$

Putting Eq. (28) in Eq. (27) for $\alpha = \beta = \gamma$

$$\bar{u}(x, s) = \frac{1}{s^2} \times \left[\frac{1+s \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}}}{1+s \sqrt{\frac{Pr(R+s)}{K_1(\alpha) + K_0(\alpha)} s^\alpha}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} + \left[\frac{Gr \left[\frac{1}{s} - \frac{c}{s+d} \right] - \left[\frac{1}{s} - \frac{c}{s+d} \right] PrScGmSr(R+s)}{(1+\lambda)Pr(R+s) - (s+k^{-1}+M)} \right] \times \left[\frac{1+s \sqrt{\frac{Pr(R+s)}{K_1(\alpha) + K_0(\alpha)} s^\alpha}}{1+s \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} - e^{-x \sqrt{\frac{Pr(R+s)}{K_1(\alpha) + K_0(\alpha)} s^\alpha}} \right] + \left[\frac{Gm + \left[\frac{1}{s} - \frac{c}{s+d} \right] PrScGmSr(R+s)}{(1+\lambda)Sc - (s+k^{-1}+M)} \right] \times \right]$$

$$\left[\frac{1+s \sqrt{\frac{sSc}{\left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}}}{1+s \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} e^{-x \sqrt{\frac{(s+K^{-1}+M)(1+\lambda)}{(1+\lambda_1 s)L_\beta \left[\frac{K_1(\alpha)}{s} + K_0(\alpha) \right] s^\alpha}} - e^{-x \sqrt{\frac{Pr(R+s)}{K_1(\alpha) + K_0(\alpha)} s^\alpha}} \right]. \quad (29)$$

5. RESULT AND DISCUSSION

The solution for the impact of thermo-diffusion, magnetic field, and heat consumption on flow of Jeffrey fluid past over a vertical plate are developed by using Laplace transform technique. The effect of numerous parameters used in the governing equations of velocity fields have been analyzed in Figures.

The impact of Gm on fluid velocity $u(x, t)$ without slippage is illustrate in Fig. 1(a). It is highlighted that fluid motion raises as values of Gm increasing. Physically higher the values of Gm increase the concentration gradients which make the buoyancy force significant and hence it is examined that velocity field is raising. The impact of Gm on $u(x, t)$ with slip effect is reported in Fig. 1(b). Fig. 2(a) represent the result of Gr on $u(x, t)$. The fluid motion rises up with maximizing the values of Gr, and it represents the impact of thermal buoyancy force to viscous force. Therefore, maximizing the values of Gr exceed the temperature gradient due to which velocity field rises. Fig. 2(b) represent the result of Gr on $u(x, t)$ with slippage. The effect of Sr on $u(x, t)$ without slippage is depicted in Fig.3(a). The $u(x, t)$ increases with the effect of Sr. Physically, mass buoyancy force is significant with raising effect of Sr which raises the fluid motion. Fig. 3(b) represents the effect of Sr on $u(x, t)$ with slippage. Figs. 4(a) and 4(b) represents the negative values of Sr on $u(x, t)$ without slippage and with the effect of slip. The behavior of λ_1 on $u(x, t)$ with non-slippage is reported in Fig. 5(a). It is highlighted that fluid motion raises as values of λ_1 increases. Fig. 5(b) display the λ_1 on $u(x, t)$ with slippage. The behavior of λ on $u(x, t)$ with non-slippage is reported in Fig. 6(a). It is highlighted that fluid motion decays as values of λ_1 increases. Fig. 6(b) display the λ on $u(x, t)$ with slippage. The impact of M on $u(x, t)$ without slip effect is reported in Fig. 7(a). Graph shows that fluid speed $u(x, t)$ is reduced with accelerating values of parameter M. Resistivity becomes dominant with raising M which reduced the speed of fluid. The impact of M on $u(x, t)$ slip effect is reported in Fig. 7(b).

Fig. 8(a) shows the importance of t on $u(x, t)$ without slip effect. Fig. 8(b) shows the importance of Sc on $u(x, t)$ without slip effect. It is observed that maximizing the values of Sc slow down the fluid motion due to decay of molecular diffusion. Fig. 9(a) indicates the impact of Pr on $T(x, t)$. Fig. 9(b) indicates the effect of R on $T(x, t)$. The behavior of Sc on $C(x, t)$ are shown in Fig. 10(a). The concentration level is higher with increasing effects of Sr as display in Fig. 10(b). Fig. 11(a) shows the influence of heat consumption on $C(x, t)$. The $C(x, t)$ increases with decreasing values of heat consumption. Figs. 11(b)-12(a) shows the comparison of present work with Asma et. al [29]. If we put $\beta = \gamma = \alpha \rightarrow 1$, $Gm = Sr = Sc = R = S = \lambda_1 = 0$, the both fluids are identical. The comparison of fractional derivatives is shown in Figs 12(b)-13(a). Figs. 13(b)-14(b) are drawn for authenticity of inverse algorithms.

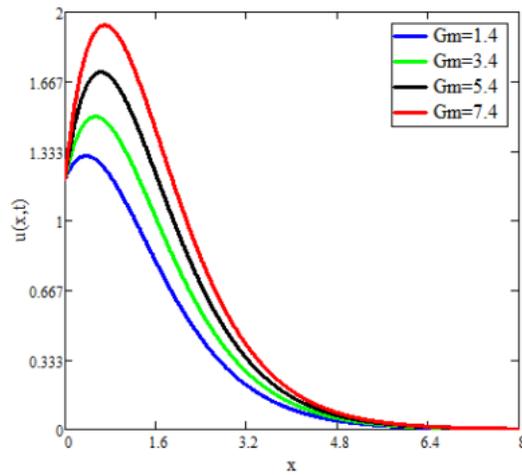


Fig. 1(a) Velocity profile $u(x,t)$ for various values of Gm at $\lambda=0.4, Sr=0.4, M=0.2, S=0.0, K=3, t=1.2, Gr=6, Pr=2.5$.

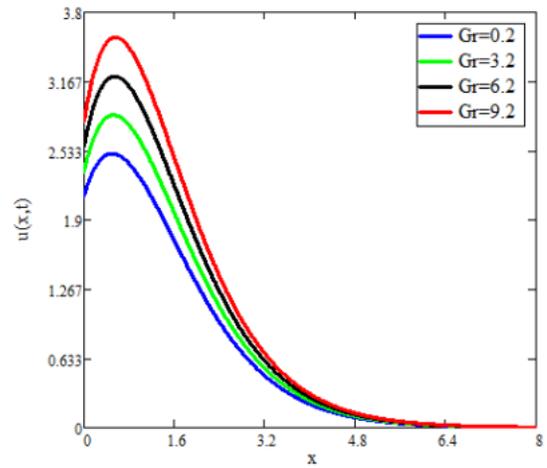


Fig. 2(b) Velocity profile $u(x,t)$ for various values of Gr at $Sc=5.5, \lambda=0.4, Sr=0.4, M=0.2, S=0.5, K=3, t=1.2, Gm=12, Pr=2.5$.

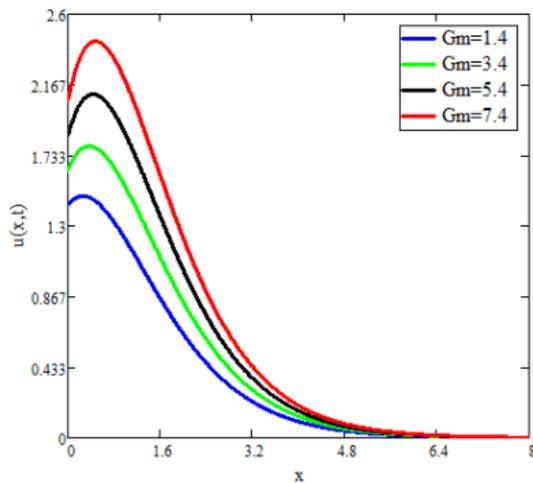


Fig. 1(b) Velocity profile $u(x,t)$ for various values of Gm at $\lambda=0.4, Sr=0.4, M=0.2, S=0.5, K=3, t=1.2, Gr=6, Pr=2.5$.

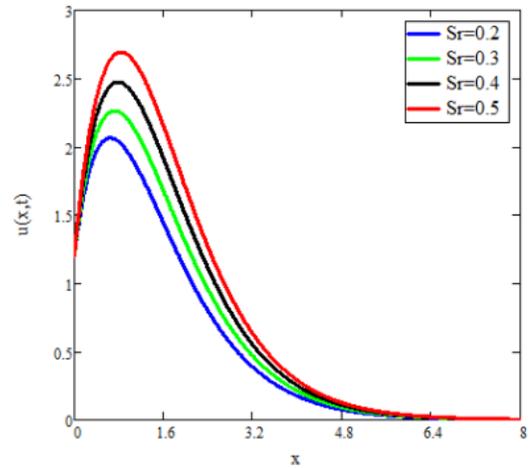


Fig. 3(a) Velocity profile $u(x,t)$ for various values of Sr at $Sc=5.5, \lambda=0.4, Gm=12, M=0.2, S=0.0, K=3, t=1.2, Gr=6, Pr=2$.

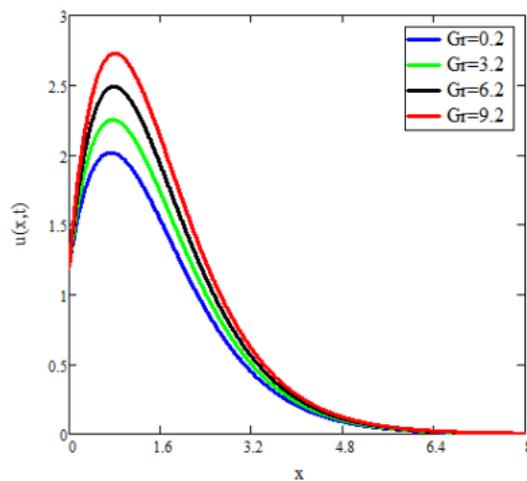


Fig. 2(a) Velocity profile $u(x,t)$ for various values of Gr at $Sc=5.5, \lambda=0.4, Sr=0.4, M=0.2, S=0.0, K=3, t=1.2, Gm=12, Pr=2.5$.

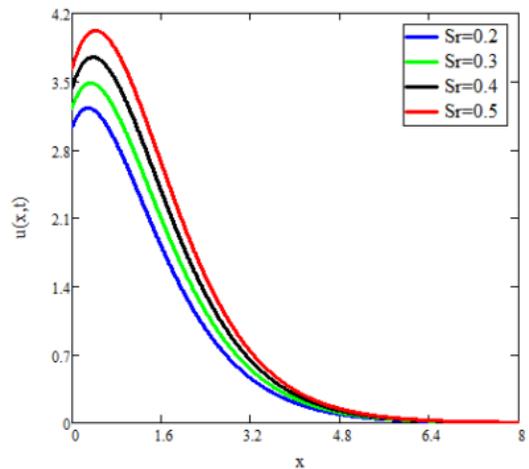


Fig. 3(b) Velocity profile $u(x,t)$ for various values of Sr at $Sc=5.5, \lambda=0.4, Gm=12, M=0.2, S=0.5, K=3, t=1.2, Gr=6, Pr=2.5$.

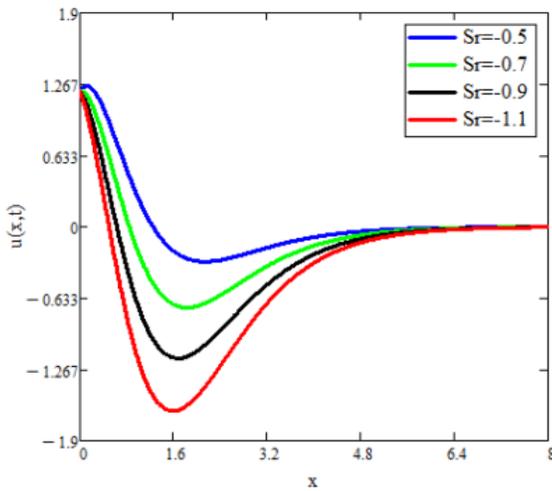


Fig. 4(a) Velocity profile $u(x,t)$ for various values of Sr at $Sc=5.5$, $\lambda=0.4$, $Gm=12$, $M=0.2$, $S=0.0$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

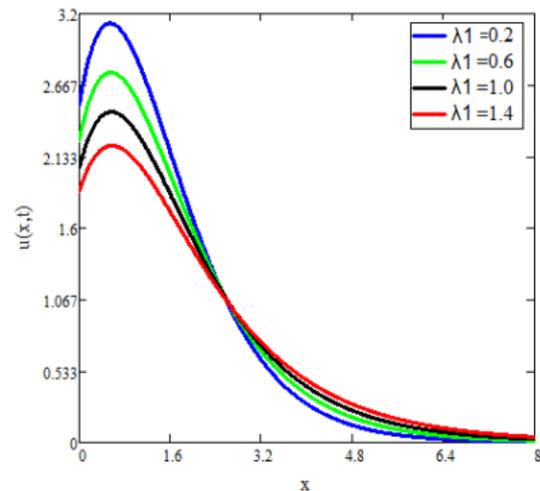


Fig. 5(b) Velocity profile $u(x,t)$ for various values of λ_1 at $Gm=12$, $\lambda=0.4$, $Sr=0.4$, $M=0.2$, $S=0.5$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

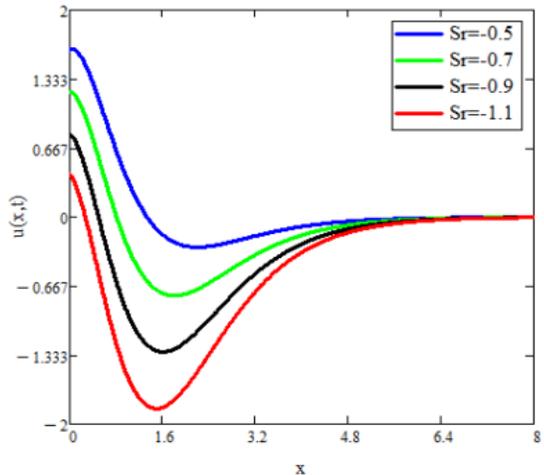


Fig. 4(b) Velocity profile $u(x,t)$ for various values of Sr at $Sc=5.5$, $\lambda=0.4$, $Gm=12$, $M=0.2$, $S=5.0$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

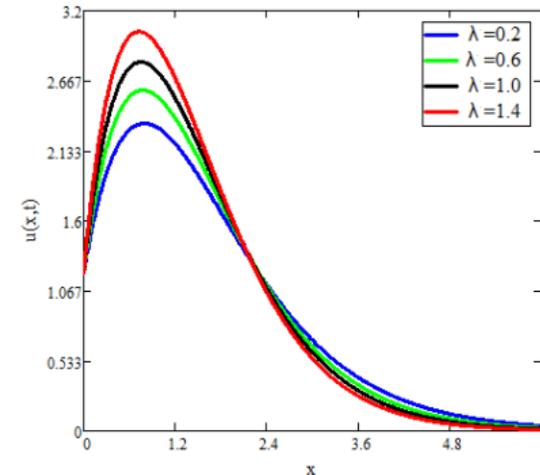


Fig. 6(a) Velocity profile $u(x,t)$ for various values of λ at $Gm=12$, $Sc=5.5$, $Sr=0.4$, $M=0.2$, $S=0.0$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

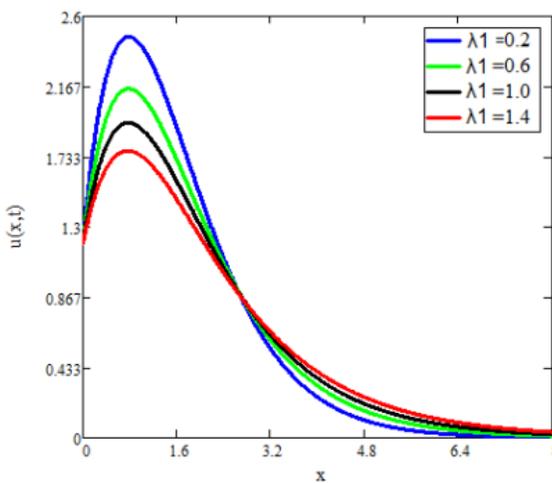


Fig. 5(a) Velocity profile $u(x,t)$ for various values of λ_1 at $Gm=12$, $\lambda=0.4$, $Sr=0.4$, $M=0.2$, $S=0.0$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

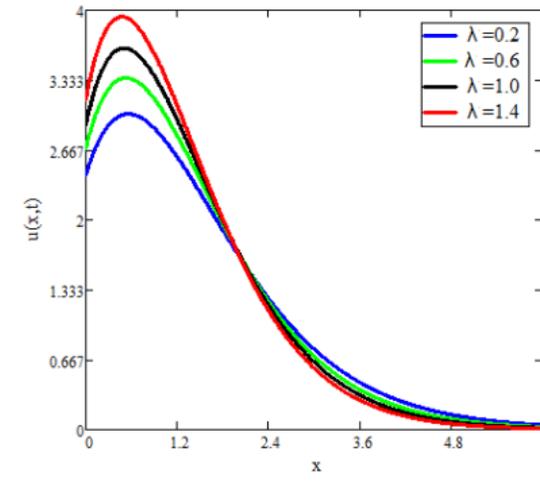


Fig. 6(b) Velocity profile $u(x,t)$ for various values of λ at $Gm=12$, $Sc=5.5$, $Sr=0.4$, $M=0.2$, $S=0.5$, $K=3$, $t=1.2$, $Gr=6$, $Pr=2.5$.

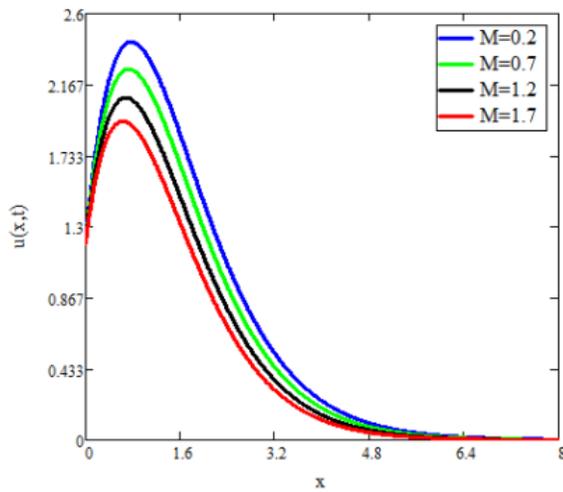


Fig. 7(a) Velocity profile $u(x,t)$ for various values of M at $Gm=12$
 $\lambda=0.4, Sr=0.4, Sc=5.5, S=0.0, K=3, t=1.2, Gr=6, Pr=2.5$.

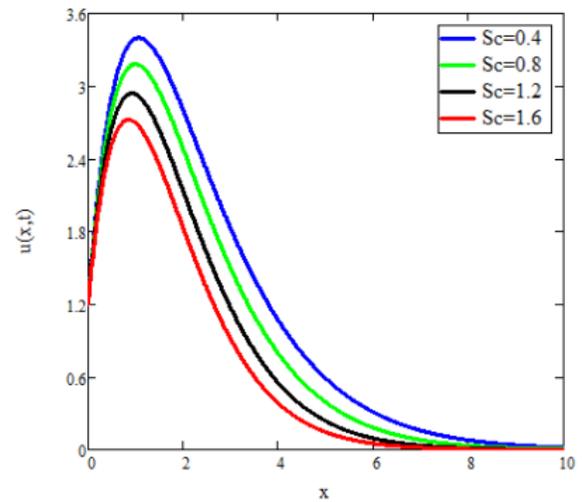


Fig. 8(b) Velocity profile $u(x,t)$ for various values of Sc at $Gm=12$
 $\lambda=0.4, Sr=0.4, M=0.2, S=0.0, K=3, t=1.2, Gr=6, Pr=2.5$.

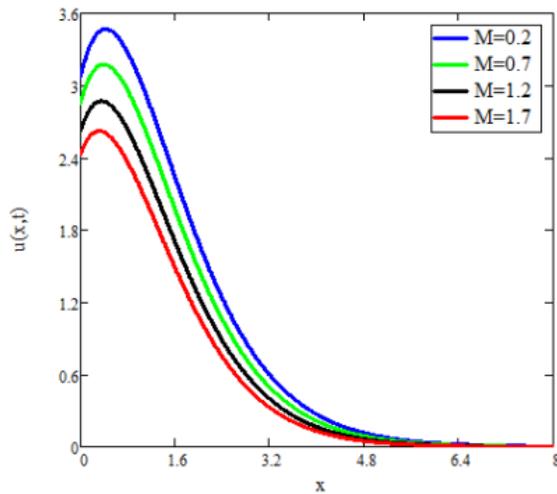


Fig. 7(b) Velocity profile $u(x,t)$ for various values of M at $Gm=12$
 $\lambda=0.4, Sr=0.4, Sc=5.5, S=0.5, K=3, t=1.2, Gr=6, Pr=2.5$.

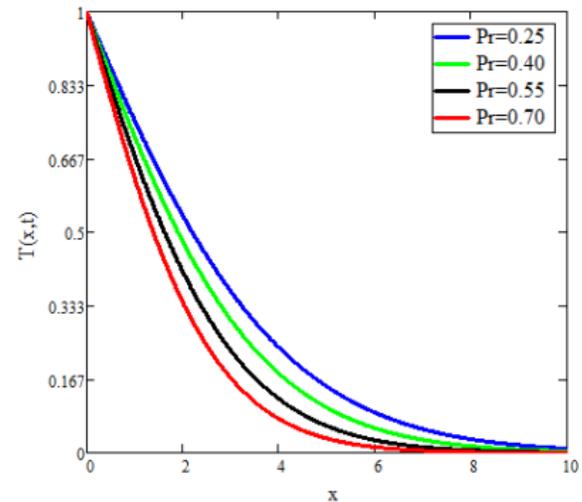


Fig. 9(a) Temperature profile $T(x,t)$ for various values of Pr .

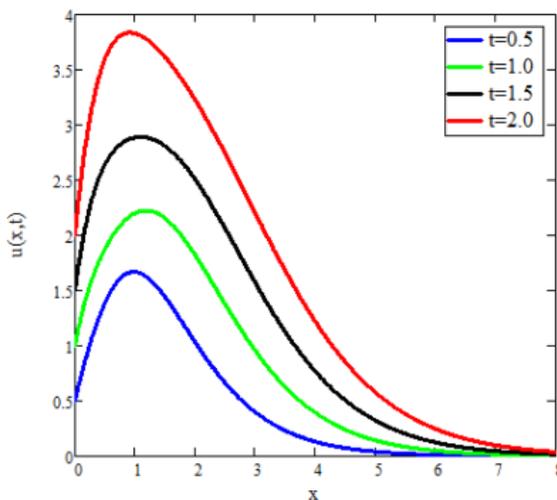


Fig. 8(a) Velocity profile $u(x,t)$ for various values of t at $Gm=12$
 $\lambda=0.4, Sr=0.4, M=0.2, S=0.0, K=3, Sc=5.5, Gr=6, Pr=2.5$.

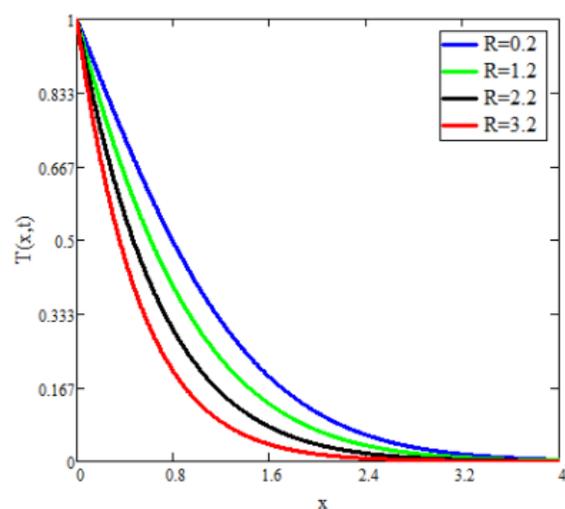


Fig. 9(b) Temperature profile $T(x,t)$ for various values of R

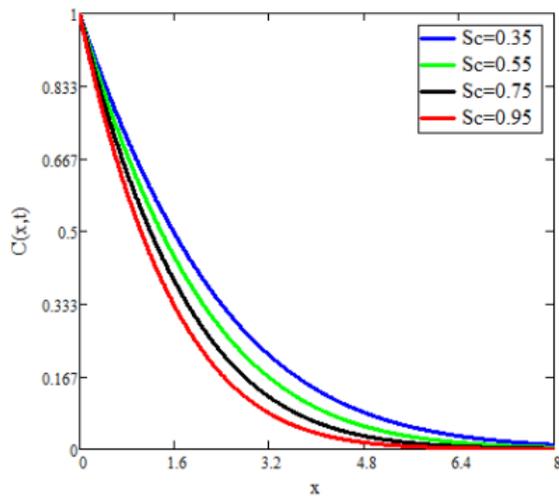


Fig. 10(a) Concentration profile $C(x,t)$ for various values of Sc .

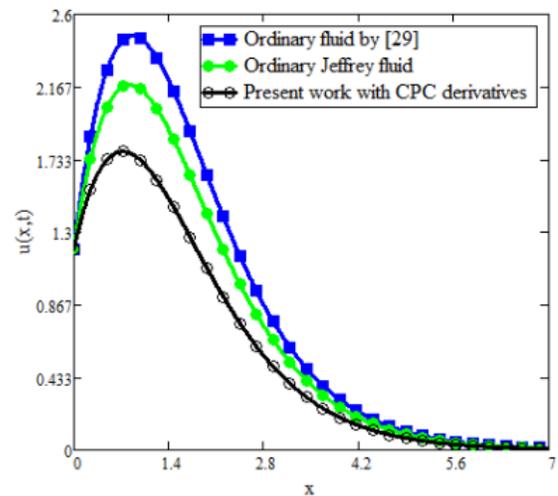


Fig. 11(b) Comparison of present work with Asma et al. [29].

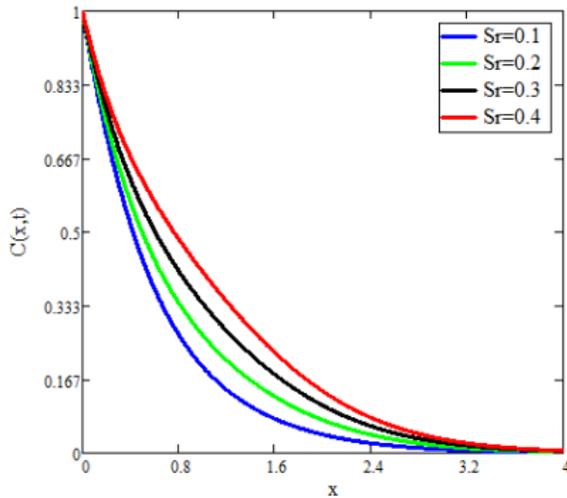


Fig. 10(b) Concentration profile $C(x,t)$ for various values of Sr .

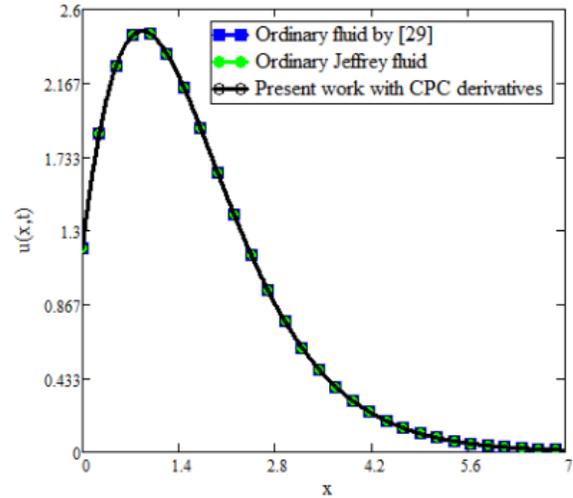


Fig. 12(a) Comparison of present work with Asma et al. [29].

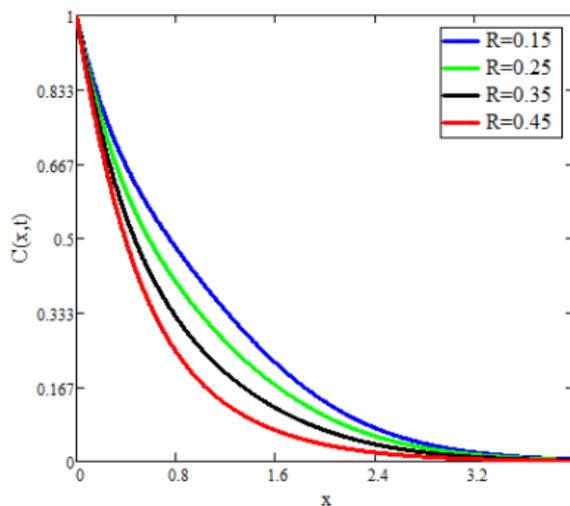


Fig. 11(a) Concentration profile $C(x,t)$ for various values of R .

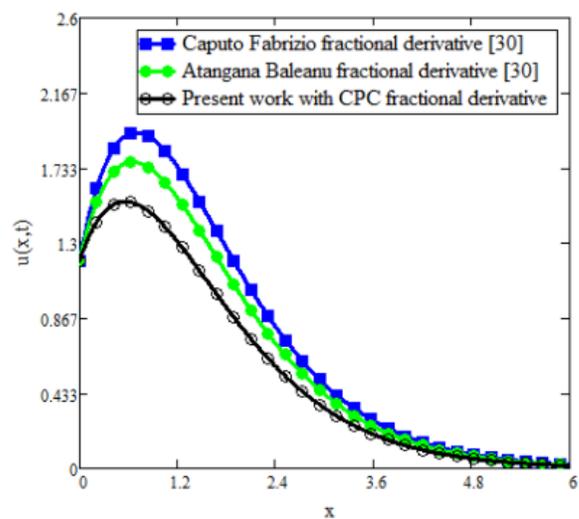


Fig. 12(b) Comparison of present work with fractional derivatives [30].

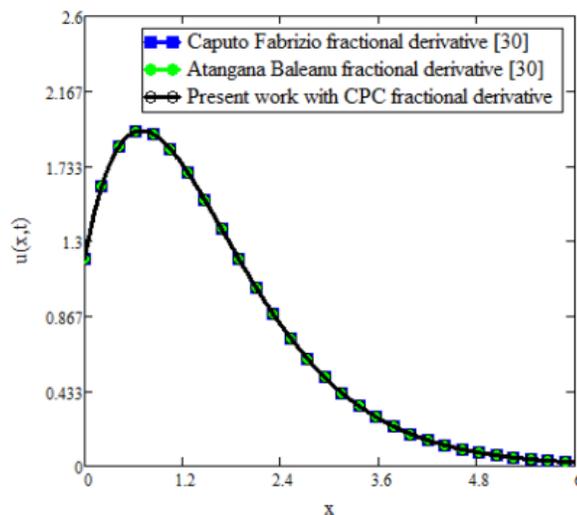


Fig. 13(a) Comparison of present work with fractional derivatives [30].

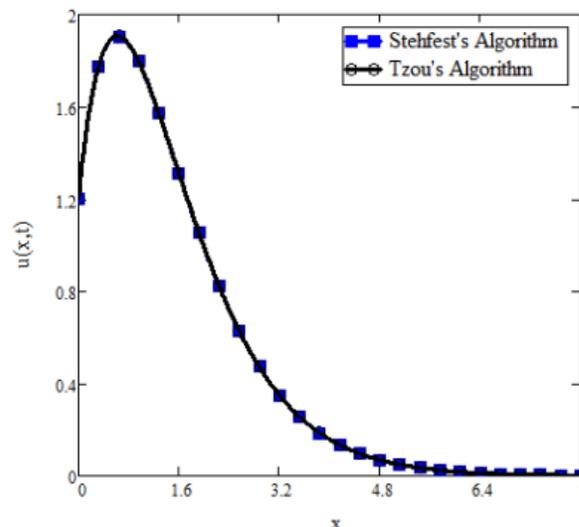


Fig. 14(b) Velocity obtain by Stehfest's and Tzou's Algorithms.

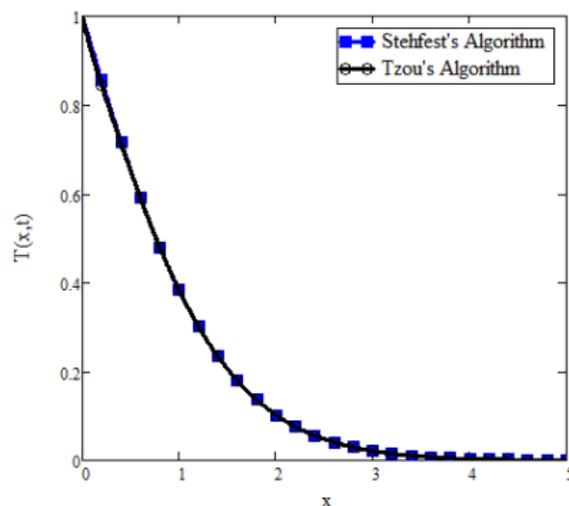


Fig. 13(b) Temperature obtain by Stehfest's and Tzou's Algorithms.

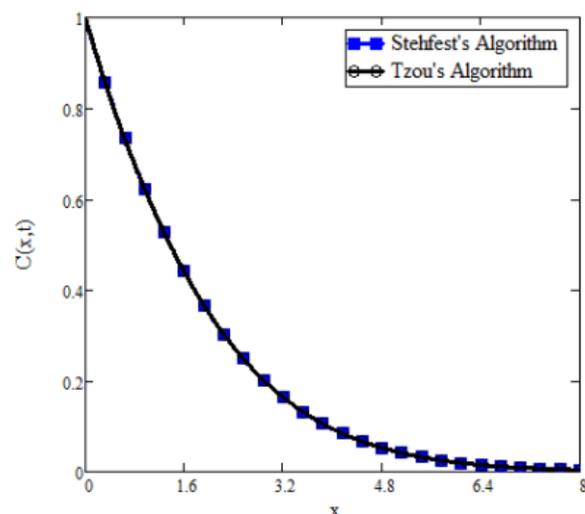


Fig. 14(a) Concentration obtain by Stehfest's and Tzou's Algorithms.

6. CONCLUSIONS

The flow of fractional Jeffrey fluid model has been taken and solved using Laplace transform with solution. The conditions of flow problem are satisfied by the results. Different graphs have been plotted for flow parameters and then discussed.

The key points of this flow model are:

- With higher Magnetic values, the velocity distribution slows down.
- Thermal buoyancy forces accelerate fluid velocity.
- The fluid velocity increased for higher values of Sorret effect.
- The Temperature of fluid decays down for larger values of R.
- The concentration of fluid is an increasing function of thermo-diffusion.
- The concentration level is a decreasing function of Schmidt number.

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