



# EFFECT OF DIFFUSION-THERMO ON MHD FLOW OF MAXWELL FLUID WITH HEAT AND MASS TRANSFER

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## ABSTRACT

A magnetohydrodynamics (MHD) flow of fractional Maxwell fluid past an exponentially accelerated vertical plate is considered. In addition, other factors such as heat generation and chemical reaction are used in the problem. The flow model is solved using Caputo fractional derivative. Initially, the governing equations are made non-dimensional and then solved by Laplace transform. The influence of different parameters like diffusion thermo, fractional parameter, Magnetic field, chemical reaction, Prandtl number and Maxwell parameter are discussed through numerous graphs. From figures, it is observed that fluid motion decreases with increasing values of Schmidt number and chemical reaction, whereas velocity field decreases with decreasing values of diffusion thermo and heat generation.

**Keywords:** Free convection, heat generation, Maxwell fluid, chemical reaction, magnetic field, Dufour effect, Caputo fractional derivative.

## 1. INTRODUCTION

Convection flow in the presence of porosity has numerous important applications such as flows in soils, solar power collectors, heat transfer correlated with geothermal systems, heat source in the field of agricultural storage system, heat transfer in nuclear reactors, heat transfer in aerobic and anaerobic reactions, heat evacuation from nuclear fuel detritus, and heat exchangers for porous material. Convection flow of MHD fluid has many implementations in meteorology, distillation of gasoline, boundary layer control, energy generators, geophysics, accelerators, petroleum industry, astrophysics, polymer technology, aerodynamics, and in material processing such as metal forming, glass fiber drawing, extrusion, and casting wire. Shah *et al.* (2019) analyzed the influence of magnetic field on double convection problem of fractional viscous fluid over an exponentially moving vertical plate. The combined effect of heat and mass diffusion on fluid flow through a plate has been observed by Chaudhary and Jain (2007). The analytical solution for magnetohydrodynamics flow through a perpendicular plate in the existence of porosity is obtained by Sivaiah *et al.* (2009). Das and Jana (2010) discussed the solution for MHD flow through a plate in the presence of porous media.

Now a days, magnetohydrodynamic (MHD) has been extended into wide areas of basic and applied research in sciences and engineering. The study of non-Newtonian fluid becomes very interested due to variety of technological applications like making of plastic sheets, lubricant's performance and motion of biological fluid. Numerous non-Newtonian fluid models have been presented to demonstrate the distinction between Newtonian and non-Newtonian fluids. Shah *et al.* (2016) discussed the effects of the fractional order and magnetic field on the blood flow in cylindrical domains. Kai-Long Hsiao (2017) worked on MHD heat transfer thermal extrusion system using non-Newtonian Maxwell fluid with radiative and viscous dissipation effects. A comparative study and

analysis of natural convection flow of MHD non-Newtonian fluid in the presence of heat source and first order chemical reaction was studied by Ahmad *et al.* (2019). During the last decade, different generalized fractional derivatives have appeared in the literature that are derivatives of Caputo, Caputo-Fabrizio, constant proportional Caputo by Atangana *et al.* (2020) and Baleanu *et al.* (2020). Soret and radiation effects on MHD free convection flow over an inclined porous plate with heat and mass flux was studied by Kumar *et al.* (2016). Sandeep *et al.* (2016) analyzed the heat and mass transfer in nano fluid over an inclined stretching sheet with volume fraction of dust and nanoparticles. Ahammad *et al.* (2017) analyzed the radiation effect with Eckert number and forchimer number on heat and mass transfer over an inclined plate in the influence of suction/injection flow. Ramzan *et al.* (2021) analyzed the unsteady free convective magnetohydrodynamics flow of a Casson fluid through a channel with double diffusion and ramp temperature and concentration Khan *et al.* (2018) discussed the multiples solutions of non-Newtonian Carreau fluid flow over an inclined shrinking sheet.

Some mathematical models of second grade fluids are industrial oils, slurry streams, and dilute polymer solutions with different geometry and boundary conditions. Fetecau *et al.* (2005) analyzed the solution of unsteady unidirectional flows of second grade fluid at plate with the assistance of the Fourier sine transformation. Ahmed *et al.* (2015) has analyzed MHD boundary layer convective heat transfer with flow pressure gradient. Convective mixed MHD flow studied by Narayana (2015), while Nadeem *et al.* (2014) discussed the thermo-diffusion effects on MHD oblique stagnation point flow of viscoelastic fluid. Because of its rising significance, engineering needs to incorporate non-Newtonian fluid. Shafique *et al.* (2022) presented a fractional flow of the Jeffrey fluid on a vertical plate driven by temperature as well as concentration gradients. Khan *et al.* (2014) discussed magnetohydrodynamic flow in the existence of permeable media through plate. Tran *et al.* (2020) worked on mandatory stability of fractional

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derivatives for fractional calculus equations, and the mathematical model used for transference of COVID-19 with Caputo fractional derivatives also discussed by Tuan *et al.* (2020). Seth *et al.* (2015) discussed the MHD convection flow over a vertical plate with ramped temperature. Razaq *et al.* (2019) discussed the nanofluid with Newtonian heating over a plate. Ramzan *et al.* (2022) discussed the viscous fluid over an inclined plate. Khan *et al.* (2018) investigated the Brinkman fluid effect between two side walls. Liu *et al.* (2017) presented the flow of Maxwell fluid with slip using a heat generation. Ali *et al.* (2013) discussed on the magnetohydrodynamics fluid with heat transport. Ramzan *et al.* (2021) reported the behavior of heat absorption/generation on the MHD flow of Brinkman fluid. Ali *et al.* (2018) studied the magnetohydrodynamics oscillating and rotating flows of Maxwell electrically conducting fluid in a porous plane.

In this problem, an unsteady MHD flow of Maxwell fluid over a vertical plate is considered with Dufour effect. The impact of chemical reaction and heat absorption/generation is added into account. Firstly, the governing equations have been made non-dimensional and then solved semi analytically. The results for velocity profile, temperature profile, and concentration profile are obtained and then analyzed graphically. From figures, it is observed that fluid motion decreases with increasing values of Schmidt number and chemical reaction, whereas velocity field increases with increasing values of diffusion thermo and heat generation. The comparison between ordinary and fractionalized fluid is drawn graphically and found that Caputo fractional derivative is the best choice for controlled fluid velocity.

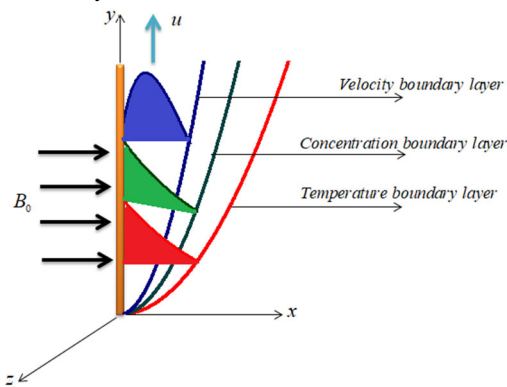


Fig 1: Flow diagram

## 2. MATHEMATICAL MODEL

The flow of Maxwell fluid with heat and mass transfer over a vertical plate is considered. The flow is taken along the  $y^*$ -axis and the  $x^*$ -axis is perpendicular to the plate. The fluid and plate have temperature  $T_\infty^*$  and concentration  $C_\infty^*$  at time  $t_1^* = 0$  with zero velocity. But for  $0 < t_1^*$ , the plate begins to move with velocity  $U_1 f(t_1^*)$ . The concentration and temperature of the plate is raised to  $C_\infty^* + (C_w^* - C_\infty^*)t_1^*$  and  $T_\infty^* + (T_w^* - T_\infty^*)t_1^*$  with time  $t_1^* > 0$ . A uniform transverse magnetic field  $B_0$  is applied in  $x$  direction. The flow diagram is shown in Fig. The flow governing equations are being taken into account as (see [Ali *et al.* (2018)])

$$\text{div}U = 0. \quad (1)$$

$$\rho \left[ \frac{\partial U}{\partial t_1^*} + (U \cdot \nabla)U \right] = \text{div}S - \sigma \beta_0^1 U + g \beta_{T^*} (T^*(x^*, t_1^*) - T_\infty^*) + g \beta_{C^*} (C^*(x^*, t_1^*) - C_\infty^*), \quad (2)$$

The extra stress tensor  $S$  for Maxwell fluid satisfies

$$S + \lambda_0 \left[ \frac{\partial S}{\partial t_1^*} + (U \cdot \nabla)S - LS - SL t_1^* \right] = \mu A_1, \quad (3)$$

where  $L = \nabla U$ ,  $\mu$  indicate dynamic viscosity,  $\Lambda_0$  indicate the relaxation time and  $A_1$  represents first Rivlin-Ericken tensor given as  $A_1 = L + L^T$ .

Using Boussinesq's approximation and these assumption, the linear momentum Eq. of Maxwell fluid with diffusion thermo effects over a plate is (see [Shah *et al.* (2019), Sheikh *et al.* (2020), Ahmad *et al.* (2020)])

$$\left( 1 + \lambda_0 \frac{\partial}{\partial t_1^*} \right) \frac{\partial u_1(x^*, t_1^*)}{\partial t_1^*} = \nu \frac{\partial \tau(x^*, t_1^*)}{\partial x^*} + \left( 1 + \lambda_0 \frac{\partial}{\partial t_1^*} \right) g \beta_{T^*} (T^*(x^*, t_1^*) - T_\infty^*) + \left( 1 + \lambda_0 \frac{\partial}{\partial t_1^*} \right) g \beta_{C^*} (C^*(x^*, t_1^*) - C_\infty^*) - \left( 1 + \lambda_0 \frac{\partial}{\partial t_1^*} \right) \frac{\sigma \beta_0^2}{\rho} u_1(x^*, t_1^*), \quad (4)$$

shear stress  $\tau$  is

$$\tau = \mu \frac{\partial u_1(x^*, t_1^*)}{\partial x^*}. \quad (5)$$

thermal Eq. is

$$\rho C_p \frac{\partial T^*(x^*, t_1^*)}{\partial t_1^*} = - \frac{\partial q(x^*, t_1^*)}{\partial x^*} + Q_1 (T^*(x^*, t_1^*) - T_\infty^*) - \frac{\rho K_T D_m}{D C_s} \frac{\partial J(x^*, t_1^*)}{\partial x^*}. \quad (6)$$

According to Fourier's Law,  $q(x^*, t_1^*)$  is given by

$$q(x^*, t_1^*) = -k \frac{\partial T^*(x^*, t_1^*)}{\partial x^*}. \quad (7)$$

Diffusion Eq. is

$$\frac{\partial C^*(x^*, t_1^*)}{\partial t_1^*} = - \frac{\partial J(x^*, t_1^*)}{\partial x^*} - K_r (C^*(x^*, t_1^*) - C_\infty^*). \quad (8)$$

According to Fourier's Law,  $J(x^*, t_1^*)$  is given by

$$J(x^*, t_1^*) = -D \frac{\partial C^*(x^*, t_1^*)}{\partial x^*}. \quad (9)$$

with boundary condition

$$u_1(x^*, t_1^*) = 0, \quad T^*(x^*, t_1^*) = T_\infty^*, \quad C^*(x^*, t_1^*) = C_\infty^*, \quad y^* > 0, \quad t_1^* > 0, \quad (10)$$

$$u_1(x^*, t_1^*) = U_1 f(t_1^*), \quad T^*(x^*, t_1^*) = T_\infty^* + (T_w^* - T_\infty^*)t_1^*,$$

$$C^*(x^*, t_1^*) = C_\infty^* + (C_w^* - C_\infty^*)t_1^*, \quad t_1^* > 0, \quad x^* > 0, \quad (11)$$

$$u_1(x^*, t_1^*) \rightarrow 0, \quad T^*(x^*, t_1^*) \rightarrow 0, \quad C^*(x^*, t_1^*) \rightarrow 0, \quad x^* \rightarrow \infty, \quad t_1^* > 0. \quad (12)$$

To write the flow model in dimensionless form, we used the following dimensionless variables

$$x^* = \frac{U x^*}{\nu}, \quad t = \frac{U t_1^*}{\nu}, \quad T = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad Pr = \frac{\rho \nu C_p}{k}, \quad u = \frac{u_1}{U},$$

$$Gr = \frac{\rho \nu \beta_{T^*} (T_w^* - T_\infty^*)}{U^3}, \quad M = \frac{\beta_0^2 \sigma}{\rho U}, \quad Q = \frac{Q_1 \nu}{U^2 \rho C_p}, \quad R = \frac{R_1 \nu}{U^2},$$

$$C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, \quad Gm = \frac{\rho \nu \beta_{C^*} (C_w^* - C_\infty^*)}{U^3}, \quad J = \frac{J^*}{J_0}, \quad q = \frac{q^*}{q_0}, \quad \tau = \frac{\tau^*}{\tau_0}, \quad Sc = \frac{\nu}{D}. \quad (13)$$

Using non-dimensional variables from Eq. (10) into the Eqs. (1-9), we have

$$\left[ 1 + \lambda \partial_t \right] \frac{\partial u(x, t)}{\partial t} = n_1 \frac{\partial^2 \bar{u}(x, t)}{\partial x^2} + \left[ 1 + \lambda \partial_t \right] Gr T(x, t) + \left[ 1 + \lambda \partial_t \right] Gm C(x, t) - \left[ 1 + \lambda \partial_t \right] Mu(x, t), \quad (14)$$

$$\tau = K_1 \frac{\partial u(x, t)}{\partial x}, \quad (15)$$

$$\frac{\partial T(x,t)}{\partial t} = -n_2 \frac{\partial q}{\partial x} + QT(x,t) + Du \frac{\partial J(x,t)}{\partial t}, \quad (16)$$

$$q = -p_1 \frac{\partial T(x,t)}{\partial x}, \quad (17)$$

$$\frac{\partial C(x,t)}{\partial t} = -n_3 \frac{\partial J(x,t)}{\partial x} - RC(x,t), \quad (18)$$

$$J = -m_1 \frac{\partial C(x,t)}{\partial x}, \quad (19)$$

where

$$n_1 = \frac{\tau_0}{U\rho}, \quad K_1 = \frac{U^2\mu}{\tau_0\nu}, \quad n_2 = \frac{q_0}{U\rho c_p(T_w - T_\infty)}, \quad p_1 = \frac{(T_w - T_\infty)KU}{q_0\nu},$$

$$n_3 = \frac{J_0}{(C_w - C_\infty)U}, \quad m_1 = \frac{UD(C_w - C_\infty)U}{J_0\nu},$$

Gr, Sc, Q, M, Pr, Gm, and  $u$  represents the Grashof number for heat transfer, Schmidt number, non-dimensional heat source, magnetic field, Prandtl number, mass Grashof number, and velocity of fluid respectively.

### 3. GENERALIZED MODEL

Eq. (15) is fractionally generalized by Blair and Caffyn (see [Blair *et al.* (1955)])

$$\tau = K_{1-\beta} D_t^{1-\beta} \frac{\partial u}{\partial x}, \quad 1 \geq \beta > 0, \quad (20)$$

Put Eq. (20) into Eq. (14), we have

$$[1 + \lambda \partial_t] \frac{\partial u(x,t)}{\partial t} = n_1 \frac{\partial}{\partial x} [L_{1-\beta} D_t^{1-\beta} \frac{\partial u(x,t)}{\partial x}] + [1 + \lambda \partial_t] GrT(x,t) + [1 + \lambda \partial_t] GmC(x,t) - [1 + \lambda \partial_t] Mu(x,t). \quad (21)$$

Eq. (17) is generalized by using Fourier's Law defined by Povstenko and Hristov (see [Povstenko *et al.* (2015), Hristov *et al.* (2017)])

$$q = -p_{1-\gamma} D_t^{1-\gamma} \frac{\partial T}{\partial x}, \quad 1 \geq \gamma > 0, \quad (22)$$

Put Eq. (22) into Eq. (16), we have

$$\frac{\partial T(x,t)}{\partial t} = -n_2 \frac{\partial}{\partial x} [l_{1-\gamma} D_t^{1-\gamma} \frac{\partial T}{\partial x}] + QT(x,t) + Du D_t^{1-\alpha} \frac{\partial^2 C(x,t)}{\partial x^2}, \quad (23)$$

Eq. (19) is generalized by using Fick's Law defined by

$$J = -m_{1-\alpha} D_t^{1-\alpha} \frac{\partial C(x,t)}{\partial x}, \quad 1 \geq \alpha > 0. \quad (24)$$

Put Eq. (24) into Eq. (18), we have

$$\frac{\partial C(x,t)}{\partial t} = n_3 \frac{\partial}{\partial x} [n_{1-\alpha} D_t^{1-\alpha} \frac{\partial C(x,t)}{\partial x}] - RC(x,t), \quad (25)$$

Taking inversion left operator on Eqs. (21), (23), and (25), we obtain

$$[1 + \lambda \partial_t] I_t^{1-\beta} \frac{\partial u(x,t)}{\partial t} = L_1 \frac{\partial^2 u(x,t)}{\partial x^2} + [1 + \lambda \partial_t] GrI_t^{1-\beta} T(x,t) + [1 + \lambda \partial_t] GmI_t^{1-\beta} C(x,t) - [1 + \lambda \partial_t] MI_t^{1-\beta} u(x,t), \quad (26)$$

$$I_t^{1-\gamma} \frac{\partial T(x,t)}{\partial t} = L_2 \frac{\partial^2 T(x,t)}{\partial x^2} + QI_t^{1-\gamma} T(x,t) + Du \frac{\partial^2 C(x,t)}{\partial x^2}, \quad (27)$$

$$I_t^{1-\alpha} \frac{\partial C(x,t)}{\partial t} = L_3 \frac{\partial^2 C(x,t)}{\partial x^2} - RI_t^{1-\alpha} C(x,t). \quad (28)$$

or equivalently

$$[1 + \lambda \partial_t] D_t^\beta u(x,t) = L_1 \frac{\partial^2 u(x,t)}{\partial x^2} + [1 + \lambda \partial_t] GrI_t^{1-\beta} T(x,t) + [1 + \lambda \partial_t] GmI_t^{1-\beta} C(x,t) - [1 + \lambda \partial_t] MI_t^{1-\beta} u(x,t), \quad (29)$$

for  $\alpha = \gamma$

$$D_t^\gamma T(x,t) = L_2 \frac{\partial^2 T(x,t)}{\partial x^2} + QI_t^{1-\gamma} T(x,t) + Du \frac{\partial^2 C(x,t)}{\partial x^2}, \quad (30)$$

$$D_t^\alpha C(x,t) = L_3 \frac{\partial^2 C(x,t)}{\partial x^2} - RI_t^{1-\alpha} C(x,t). \quad (31)$$

where  $L_1 = n_1 K_{1-\beta} = 1$  when  $\beta \rightarrow 1$ ,  $L_2 = n_2 p_{1-\gamma} = 1/Pr$  when  $\gamma \rightarrow 1$ ,  $L_3 = n_3 m_{1-\alpha} = 1/Sc$  when  $\alpha \rightarrow 1$ .

The initial and boundary conditions are

$$u(x,t) = T(x,t) = C(x,t) = 0, \quad x > 0, \quad t = 0, \quad (32)$$

$$u(0,t) = f(t), \quad T(0,t) = C(0,t) = t, \quad t > 0, \quad (33)$$

$$u(x,t) \rightarrow 0, \quad T(x,t) \rightarrow 0, \quad C(x,t) \rightarrow 0, \quad x \rightarrow \infty, \quad t > 0, \quad (34)$$

where  $D_t^\beta u(x,t)$  is the Caputo derivative of  $u(x,t)$  defined as

$$D_t^\beta u(x,t) = \begin{cases} \frac{1}{\Gamma(1-\beta)} \int_0^t (t-s)^{-\beta} \frac{\partial u(x,s)}{\partial s} ds, & 0 \leq \beta < 1; \\ \frac{\partial u(x,t)}{\partial t}, & \beta = 1. \end{cases} \quad (35)$$

### 4. SOLUTION OF PROBLEM

Eqs. (29-34) has been solved semi analytically (see [Tzou *et al.* (1997), Stehfest *et al.* (1970)])

#### 4.1 Calculation of Concentration

By taking Laplace transform on Eq. (31), we have

$$s^\alpha \bar{C}(x,s) = L_3 \frac{\partial^2 \bar{C}(x,s)}{\partial x^2} - \frac{R}{s^{1-\alpha}} \bar{C}(x,s). \quad (36)$$

Boundary conditions satisfying Eq. (36) are

$$\bar{C}(0,s) = \frac{1}{s^2}, \quad \bar{C}(x,s) \rightarrow 0, \quad x \rightarrow \infty. \quad (37)$$

Substitute Eq. (37) into Eq. (36), we have

$$\bar{C}(x,s) = s^{-2} e^{-x \sqrt{\frac{s+R}{L_3 s^{1-\alpha}}}}. \quad (38)$$

#### 4.2 Calculation of Temperature

By taking Laplace transform on Eq. (30), we have

$$s^\gamma \bar{T}(x,s) = L_2 \frac{\partial^2 \bar{T}(x,s)}{\partial x^2} + R s^{\gamma-1} \bar{T}(x,s) + Du \frac{\partial^2 \bar{C}(x,s)}{\partial x^2}. \quad (39)$$

Boundary conditions satisfying Eq. (39) are

$$\bar{T}(0,s) = \frac{1}{s^2}, \quad \bar{T}(x,s) \rightarrow 0, \quad x \rightarrow \infty. \quad (40)$$

Substitute Eq. (40) into Eq. (39), we have

$$\bar{T}(x, s) = \left(\frac{1}{s^2}\right) e^{-x\sqrt{\frac{(s-Q)}{s^2-\gamma}}} + \frac{Du(s+R)\left(\frac{1}{L_2L_3s^2}\right)}{\left(\frac{s+R}{L_3}\right)\left(\frac{s-Q}{L_2}\right)} \left[ e^{-x\sqrt{\frac{(s-Q)}{L_2s^2-\gamma}}} - e^{-x\sqrt{\frac{(s+R)}{L_3s^2-\alpha}}}\right]. \quad (41)$$

Suitable form of Eq. (41) for  $\alpha = \gamma$  is

$$\bar{T}(x, s) = \left(\frac{1}{s^2}\right) e^{-x\sqrt{\frac{(s-Q)}{L_2s^2-\alpha}}} + \frac{Du(s+R)\left(\frac{1}{L_2L_3s^2}\right)}{\left(\frac{s+R}{L_3}\right)\left(\frac{s-Q}{L_2}\right)} \left[ e^{-x\sqrt{\frac{(s-Q)}{L_2s^2-\gamma}}} - e^{-x\sqrt{\frac{(s+R)}{L_3s^2-\alpha}}}\right]. \quad (42)$$

### 4.3 Calculation of Velocity

By taking Laplace transform on Eq. (29), we have

$$[1 + \lambda s]s^\beta \bar{u}(x, s) = L_1 \frac{\partial^2 \bar{u}(x, s)}{\partial x^2} + [1 + \lambda s] \frac{1}{s^{1-\beta}} Gr \bar{T}(x, s) [1 + \lambda s] \frac{1}{s^{1-\beta}} Gm \bar{C}(x, s) - [1 + \lambda s] \frac{1}{s^{1-\beta}} M \bar{u}(x, s), \quad (43)$$

Boundary conditions satisfying Eq. (43) are

$$\bar{u}(0, s) = \frac{s}{\omega^2 + s^2}, \quad \bar{u}(x, s) \rightarrow 0, \quad x \rightarrow \infty. \quad (44)$$

Substitute Eq. (44) into Eq. (43), we have

$$\begin{aligned} \bar{u}(x, s) = & \frac{s}{\omega^2 + s^2} e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\beta}}}} + \frac{[1+\lambda s]s^{\beta-1} \left[ \frac{Gr}{s^2} + \frac{GrDu(s+R)L_3^{-1}L_2^{-1}s^{-2}}{L_3^1(s+R)L_2^{-1}(s-Q)} \right]}{L_1L_2^{-1}(s-Q)s^{\gamma-1} - (1+\lambda s)(s+M)s^{\beta-1}} \\ & \left[ e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\alpha}}}} - e^{-x\sqrt{\frac{(s-Q)}{L_2s^2-\gamma}}} \right] + \frac{[1+\lambda s]s^{\beta-1} \left[ \frac{Gm}{s^2} + \frac{GrDu(s+R)s^{-2}L_3^1L_2^{-1}}{L_3^1(s+R)L_2^{-1}(s-Q)} \right]}{L_1L_3^{-1}(s+R)s^{\alpha-1} - (1+\lambda s)(s+M)s^{\beta-1}} \\ & \left[ e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\beta}}}} - e^{-x\sqrt{\frac{(s+R)}{L_3s^2-\alpha}}} \right]. \end{aligned} \quad (45)$$

By taking  $\alpha = \beta = \gamma$ , suitable form of Eq. (45) is

$$\begin{aligned} \bar{u}(x, s) = & \frac{s}{\omega^2 + s^2} e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\alpha}}}} + \frac{[1+\lambda s] \left[ \frac{Gr}{s^2} + \frac{GrDuL_3^{-1}L_2^{-1}(s+R)s^{-2}}{L_3^1(s+R)L_2^{-1}(s-Q)} \right]}{L_1L_2^{-1}(s-Q) - (1+\lambda s)(s+M)} \\ & \left[ e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\alpha}}}} - e^{-x\sqrt{\frac{(s-Q)}{L_2s^2-\alpha}}} \right] + \frac{[1+\lambda s] \left[ \frac{Gm}{s^2} + \frac{GrDuL_3^{-1}L_2^{-1}(s+R)s^{-2}}{L_3^1(s+R)L_2^{-1}(s-Q)} \right]}{L_1L_3^{-1}(s+R) - (1+\lambda s)(s+M)} \\ & \left[ e^{-x\sqrt{\frac{(1+\lambda s)(s+M)}{L_1s^{1-\alpha}}}} - e^{-x\sqrt{\frac{(s+R)}{L_3s^2-\alpha}}} \right]. \end{aligned}$$

## 5. RESULT AND DISCUSSION

The solution for the impact of diffusion thermo, magnetic field, and heat generation on flow of Maxwell's fluid past over a vertical plate are developed by using Laplace transform technique. The effect of numerous parameters used in the governing equations of velocity fields have been analyzed in Figures.

Fig. 2 represent the result of Gr on fluid velocity  $u(x,t)$ . The fluid velocity increases with increasing values of Gr, and it represents the impact of thermal buoyancy force to viscous force. Therefore, maximizing the values of Gr exceed the temperature gradient due to which velocity field rises. The impact of Gm on fluid velocity  $u(x,t)$  is illustrate in Fig. 3. It is highlighted that fluid motion increases with increasing values of Gm. Physically higher the values of Gm increase the concentration gradients which make the buoyancy force significant and hence it is examined that velocity field is raising.

Fig. 4 highlights the effect of Du on  $u(x,t)$ . Graph shows that the  $u(x,t)$  is reduced with reducing values of Du. Physically, mass buoyancy force is dominant with raising values of Du which speed up the  $u(x,t)$ . Fig. 5 shows the influence of  $\alpha = \beta = \gamma$  on  $u(x,t)$ . The behavior of Graph indicates that for accelerating values of fractional parameters, fluid velocity is increased. Fig. 6 shows the influence of  $\lambda$  on  $u(x,t)$ . Graph

shows that  $u(x,t)$  increases with decreasing values of Maxwell parameter. The impact of M on  $u(x,t)$  is reported in Fig. 7. Graph shows that fluid speed  $u(x,t)$  is reduced with accelerating values of parameter M. Resistivity becomes dominant with raising M which reduced the speed of fluid. An increasing value of R decreases the  $u(x,t)$  as appeared in Fig. 8. The impact of Sc on  $u(x,t)$  is indicate in Fig. 9. It is highlighted that maximizing the values of Sc slow down the fluid motion due to decay of molecular diffusion.

The behavior of fractional parameter on  $T(x,t)$  is discussed in Fig. 10. Fig. 11 indicates the effect of Du on  $T(x,t)$ . Temperature  $T(x,t)$  increases with increasing values of Du as shown in graph. The influence of Pr on  $T(x,t)$  is reported in Fig. 12. As we increased the value of Pr, heat diffusion is reduced which slow down the fluid motion. The influence of heat Q on  $T(x,t)$  is reported in Fig. 13. This Fig. shows that temperature  $T(x,t)$  is accelerated with increasing values of Q. Physically, thermal conductivity is larger for increasing values of heat generation

Fig. 14 represents the effect of fractional parameter on  $C(x,t)$ . Fig. 15 reports the influence of R on  $C(x,t)$ . The influence of Sc on  $C(x,t)$  is shown in Fig. 16. The  $C(x,t)$  is decreases with increasing values of Sc as depicted in graph. The comparison of present work with Ahmad *et. al* (2020) is shown in Fig. 17. If we put  $\gamma = \alpha = \beta = 1$ ,  $Q=R=\omega = Gm=Du=0$ , Maxwell parameter  $\rightarrow 0$ , and  $t \rightarrow 1$ , the velocity profiles show the validity of present work as depicted in Fig 18. The velocity profiles overlap which shows the authenticity of inversion algorithms as presented in Fig. 19. Fig. 20 and 21 represents the authenticity of inversion algorithms for  $T(x,t)$  and  $C(x,t)$ .

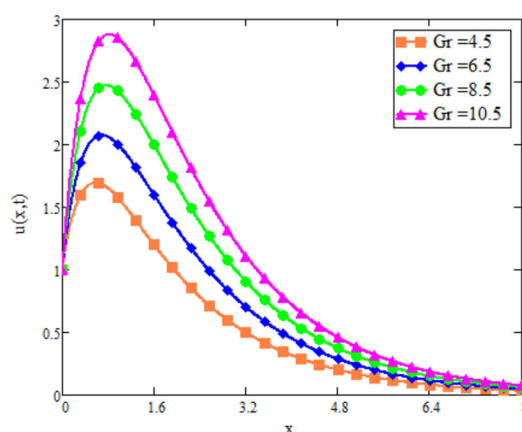


Fig. 2: Velocity profile  $u(x,t)$  different values of Gr at  $Pr=0.5$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $\lambda=0.6$ ,  $Gm=8$ ,  $Du=0.2$ ,  $Sc=1.5$ .

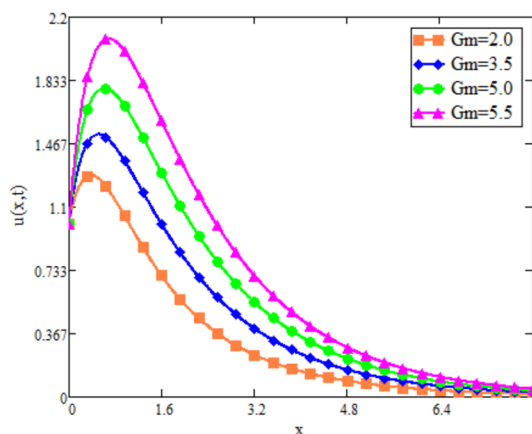
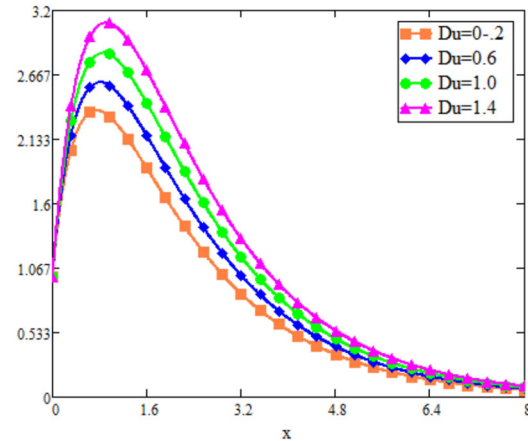
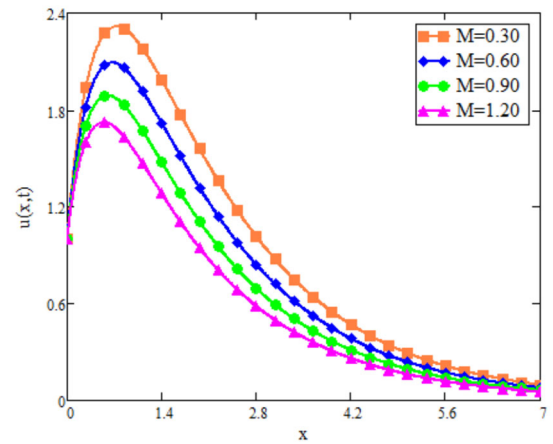


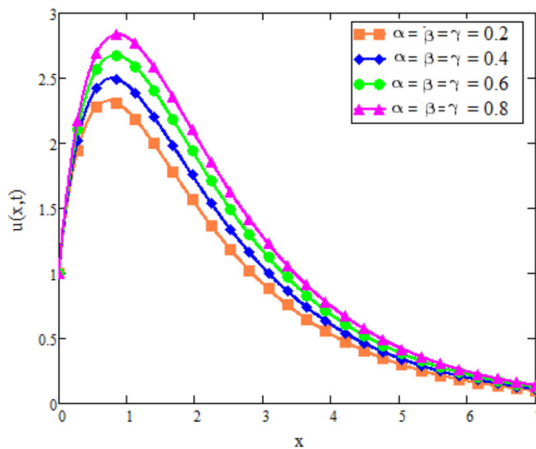
Fig. 3: Velocity profile  $u(x,t)$  different values of Gm at  $Pr=0.5$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $\lambda=0.6$ ,  $Gr=8$ ,  $Du=0.2$ ,  $Sc=1.5$ .



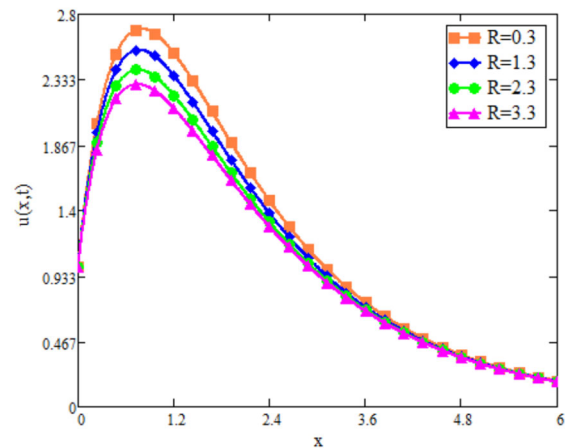
**Fig. 4:** Velocity profile  $u(x,t)$  for different values of  $Du$  at  $Pr=0.5$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $\lambda=0.6$ ,  $Gr=8$ ,  $Gm=8$ ,  $Sc=1.5$ .



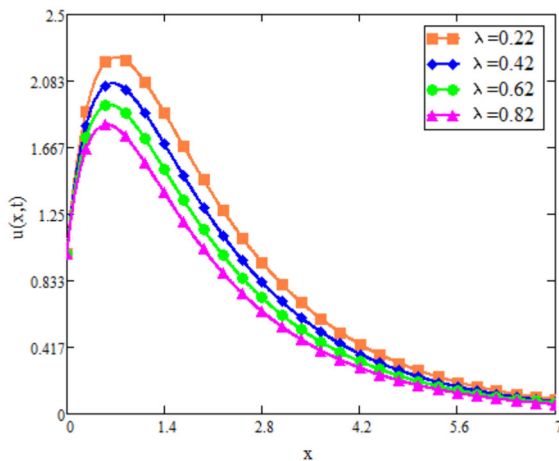
**Fig. 7:** Velocity profile  $u(x,t)$  for different values of  $M$  at  $R=1.5$ ,  $K=0.3$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Gr=9$ ,  $Gm=14$ ,  $Sr=0.4$ ,  $Sc=2.2$ ,  $Pr=0.9$ ,  $Q=0.5$ .



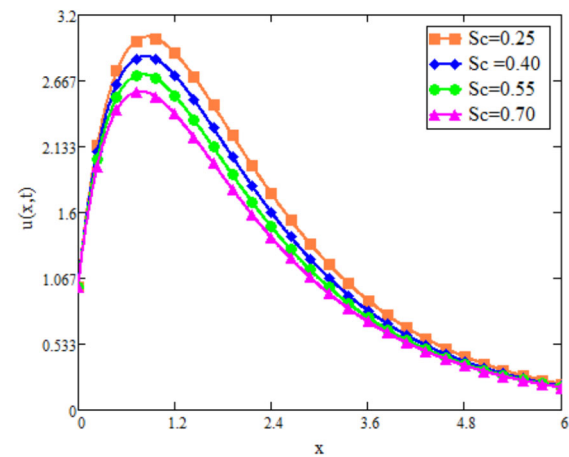
**Fig. 5:** Velocity profile  $u(x,t)$  for fractional different values of  $\alpha=\beta=\gamma$  at  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $\lambda=0.6$ ,  $Gr=8$ ,  $Du=0.2$ ,  $Sc=1.5$ ,  $Pr=0.5$ .



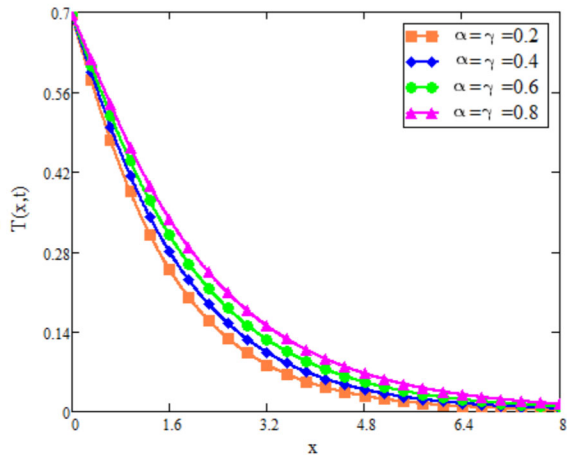
**Fig. 8:** Velocity profile  $u(x,t)$  for Chemical reaction different values of  $R$  at  $Q=4$ ,  $Du=0.4$ ,  $Gr=14$ ,  $Gm=8$ ,  $M=4$ ,  $Sc=2.5$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Pr=6$ ,  $K=2$ .



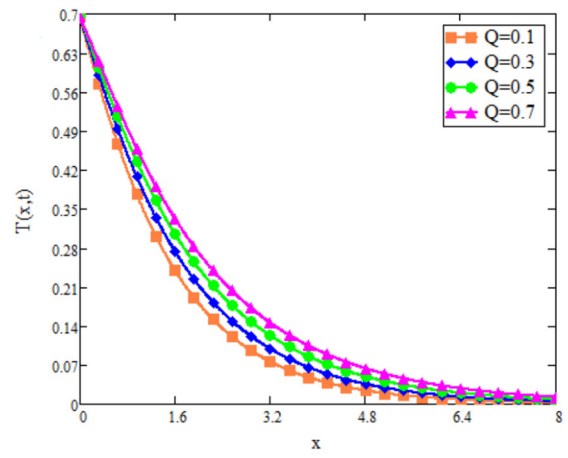
**Fig. 6:** Velocity profile  $u(x,t)$  for different values of  $\lambda$  at  $Gm=8$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $Gr=8$ ,  $Du=0.2$ ,  $Sc=1.5$ ,  $Pr=0.5$ .



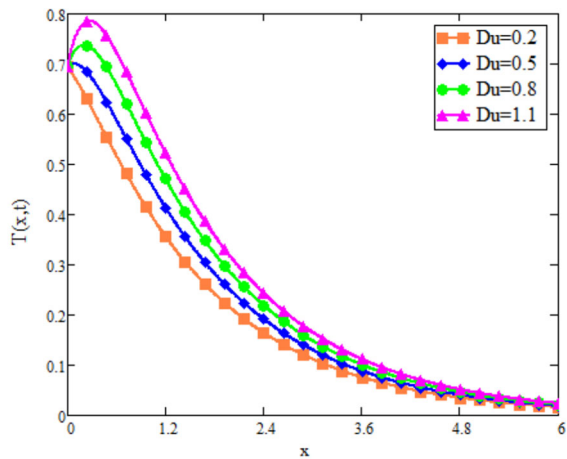
**Fig. 9:** Velocity profile  $u(x,t)$  for different values of  $Sc$  at  $Pr=0.5$ ,  $\alpha=\beta=\gamma=0.5$ ,  $Q=0.34$ ,  $\lambda=0.6$ ,  $Gr=8$ ,  $Gm=8$ ,  $Sc=1.5$ .



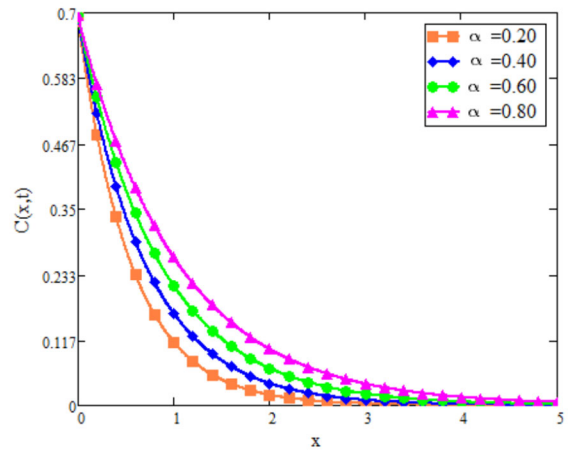
**Fig. 10:** Temperature profile  $T(x,t)$  for different values of  $\alpha=\gamma$  at  $Du=0.2$ ,  $Pr=0.5$ ,  $Sc=1.5$ ,  $R=2.6$ ,  $Q=0.34$ ,  $t=0.7$ .



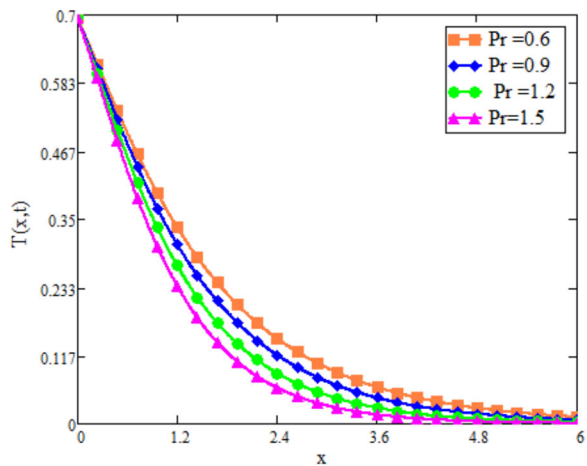
**Fig. 13:** Temperature profile  $T(x,t)$  for different values of  $Q$  at  $Du=0.2$ ,  $\alpha=\gamma=0.5$ ,  $Sc=1.5$ ,  $R=2.6$ ,  $Pr=0.5$ ,  $t=0.7$ .



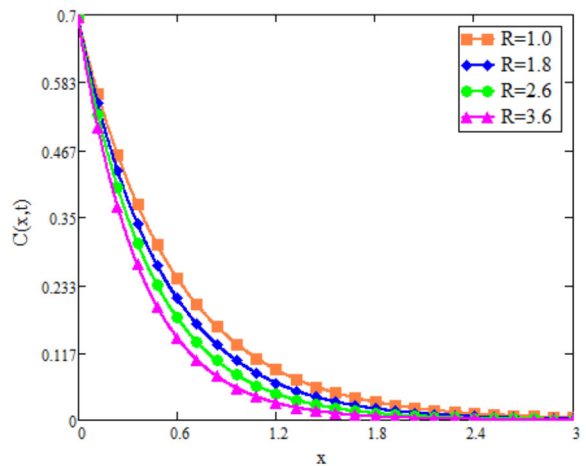
**Fig. 11:** Temperature profile  $T(x,t)$  for different values of  $Du$  at  $\alpha=\gamma=0.5$ ,  $Pr=0.5$ ,  $Sc=1.5$ ,  $R=2.6$ ,  $Q=0.34$ ,  $t=0.7$ .



**Fig. 14:** Concentration profile  $C(x,t)$  for different values of  $\alpha$  at  $t=0.7$ .



**Fig. 12:** Temperature profile  $T(x,t)$  for different values of  $Pr$  at  $Du=0.2$ ,  $\alpha=\gamma=0.5$ ,  $Sc=1.5$ ,  $R=2.6$ ,  $Q=0.34$ ,  $t=0.7$ .



**Fig. 15:** Concentration profile  $C(x,t)$  for different values of  $R$  at  $t=0.7$ .

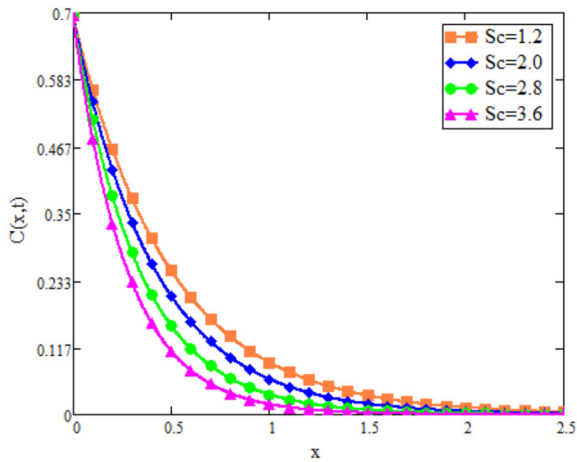


Fig. 16: Concentration profile  $C(x,t)$  for different values of  $Sc$  at  $t=0.7$ .

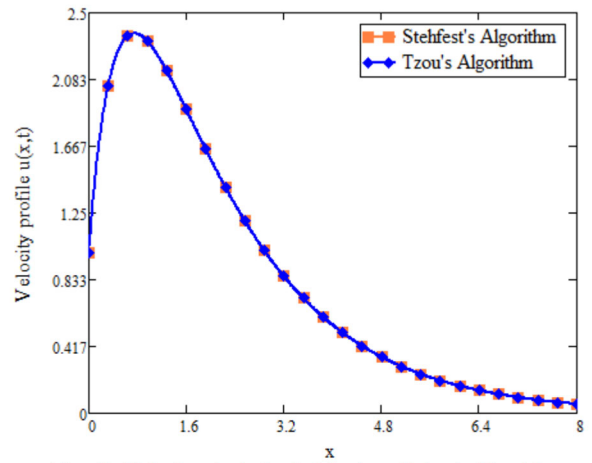


Fig. 19: Velocity obtain by Stehfest's and Tzou's Algorithms.

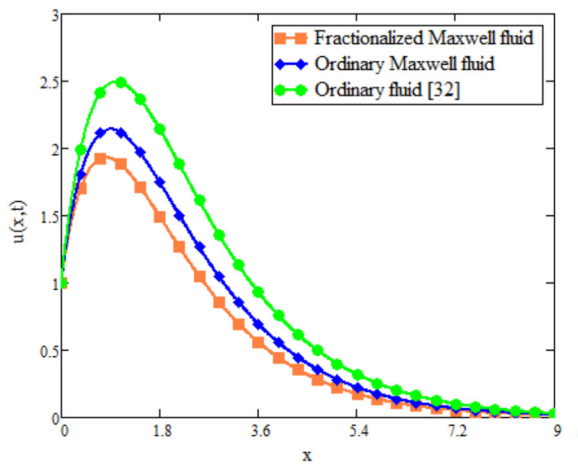


Fig. 17: Velocity distribution  $u(x,t)$  for comparison of fluids

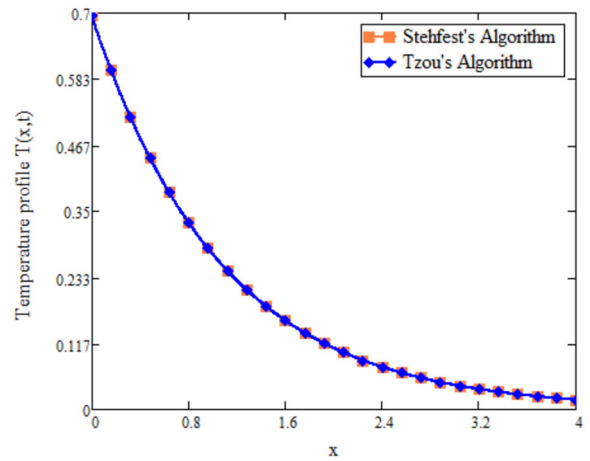


Fig. 20: Temperature obtain by Stehfest's and Tzou's Algorithms.

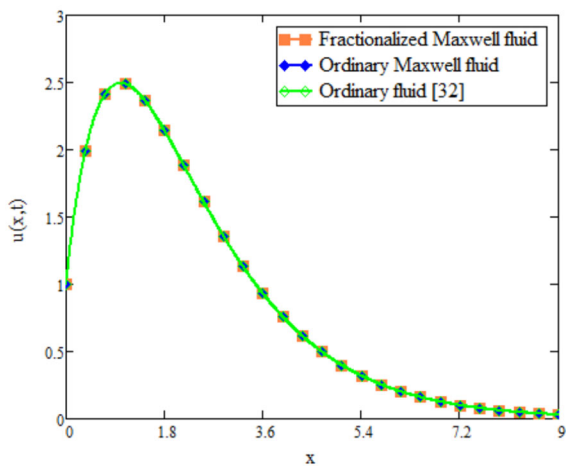


Fig. 18: Velocity distribution  $u(x,t)$  for comparison of fluids

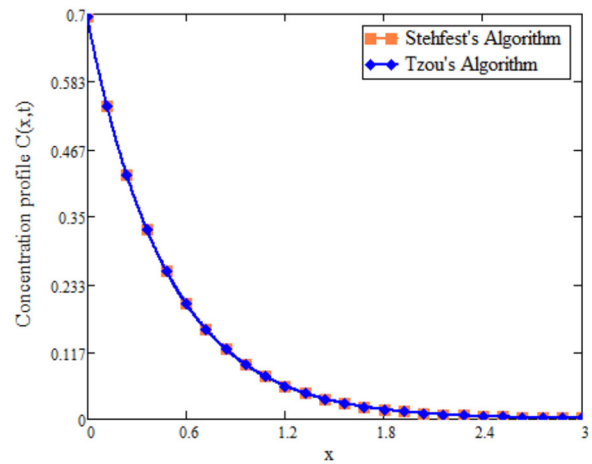


Fig. 21: Concentration obtain by Stehfest's and Tzou's Algorithms.

## 6. CONCLUSIONS

A magnetohydrodynamics flow of Maxwell's fractional fluid model has been taken and solved using Laplace transform with solution. The conditions of flow problem are satisfied by the results. Different graphs have been plotted for flow parameters and then discussed. The key points of this flow model are:

- Fluid velocity is an increasing function of fractional parameter.
- Thermal buoyancy forces accelerate the fluid velocity.
- The velocity of fluid decreases as values of magnetic parameter, Schmidt number, and chemical reaction parameter increases.
- The fluid velocity decreases with an increasing values of Maxwell fluid.
- The temperature profile increases with decreasing values of Prandtl number.
- The concentration profile decreases with increasing values of Schmidt number.
- The concentration profile decreases with an increasing value of chemical reaction.
- The concentration level is an increasing function of  $\alpha$ .

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