



WATER PERMEABILITY THROUGH THE WALL OF BLOOD CAPILLARY

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ABSTRACT

Blood capillaries are the ends of blood vessel tissues in human bodies, which are importantly functioned as exchanging the nutrients between bloods and interstitial fluids. Most blood capillaries such as in skins, skeletal muscles, cardiac muscles and lungs etc. have the walls with massively distributed cylindrical nanopores with diameters around from 50nm to 60nm. These pores are the only channels for transporting water, oxygen, carbon dioxide and ions, while preventing all the substances with the diameters greater than 60nm or a little more from passing through. The present paper presents the analytical results for the water permeability through these pores based on the nanoscale flow equation. The important results have been calculated like the pressure drop and the critical power loss on the whole pore channel for initiating the wall slippage and the volume flow rates through one single pore as function of the power loss on the pore channel.

Keywords: Blood capillary; Nanopore; Transport; Water; Wall slippage

1. INTRODUCTION

In human bodies, blood capillaries are everywhere and the places for exchanging the nutrients between bloods and interstitial fluids. Most of blood capillaries such as in skins, skeletal muscles, cardiac muscles and lungs etc have very thin walls with the thicknesses between 100nm and 200nm composed of endothelial cells, among which are formed cylindrical nanopores with the diameters around between 50nm and 60nm for transporting the substances with small molecules such as water, ions, oxygen and carbon dioxide etc. Figure 1 shows the massively distributed nanopores on the wall of a blood capillary photographed by the electron microscope with the magnification power 36000.

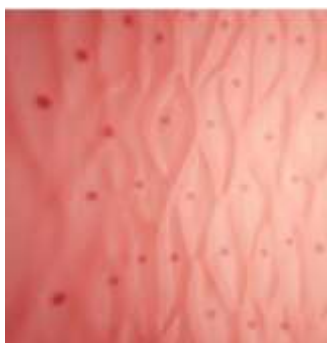


Fig. 1 Massively distributed nanopores on the very thin wall of a blood capillary.

Water transport is the critical function of these tiny pores, by which other very small substances can be fast transported through the capillary wall. To understand the mechanism of this water transport is vital to expanding the medical knowledge and finding the way for medical curing. However, it is not clear how water behaves in so small pores when it flows, as classical hydrodynamic flow theory (Lenk, 1978) might be inaccurate for this water flow because of the nanopore size effect (Bitsanis et al. 1987; Takaba et al. 2007).

When the diameter of a nanopore is only several nanometers so that across the pore radius are only distributed several water molecules, the water in the pore behaves as non-continuum and molecular dynamics simulation was popularly used in modeling this water flow (Dai et al. 2016; Khademi and Sahimi, 2011; Thomas et al. 2010; Wang et al. 2012). It was experimentally found that when the pore diameter is no more than 7nm, the water flow in the carbon nanotube is much faster than the classical continuum flow theory prediction owing to the wall slippage (Borg and Reese, 2017; Holt et al., 2006; Majumder et al., 2005). However, for the water flow through the capillary wall, molecular dynamics simulation should be much more difficult to implement because of the much larger pore diameter and the quite big wall thickness both of which result in the much more increased computational burden.

The water flow in the pore of the capillary wall may be multiscale as the flow of the adsorbed layer adjacent to the pore wall and the continuum water flow in the central region of the pore should simultaneously occur (Zhang 2020a,b). For this multiscale flow, Zhang (2020a,b) has developed the theoretical analysis by respectively considering the wall slippage or not. We have presented the theoretical results for the water transport in cellular connexon of human bodies (Wang and Zhang, 2021). The present study is the subsequent research. It is aimed to give the calculation results for the water permeability

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through the blood capillary wall by using the previously developed multiscale analysis (Zhang 2020a,b).

2. MULTISCALE WATER FLOW IN THE NANOPORE OF THE BLOOD CAPILLARY WALL

Figures 2(a) and (b) show the geometrical structure of one nanopore on the blood capillary wall. The pore axial length l i.e. the wall thickness ranges between 100nm and 200nm, depending on where the blood capillary is located. The inner radius R of the pore is normally between 50nm and 60nm. Because of the pore size, when the water flows through, there are several water molecule layers with the thickness h_{bf} annularly adsorbed to the pore wall i.e. the physical adsorbed layer on the pore wall, while in the central region of the pore is the continuum water covering the circular area with the radius R_0 . The flow in the pore is essentially multiscale owing to both the non-continuum adsorbed layer flow and the continuum water flow.

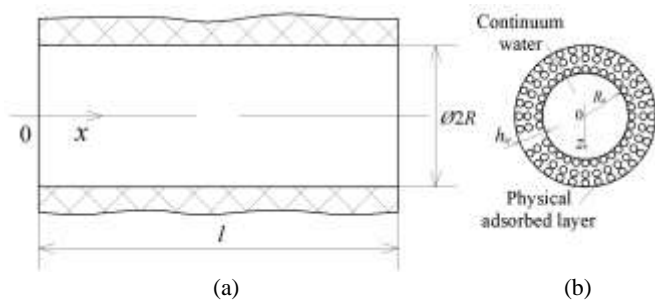


Fig. 2 A nanopore of the blood capillary wall. (a) Front view, $R=25\text{nm}\sim 30\text{nm}$, $l=100\text{nm}\sim 200\text{nm}$; (b) Multiscale flow regime in the pore.

3. ANALYSIS

For the present multiscale flow, the classical analytical approach is to model the adsorbed layer flow by molecular dynamics simulation and model the continuum water flow by the continuum rheological model (Atkas et al., 2002; Liu et al., 2007; Sun et al., 2010). This approach will bring heavy computational burden in the cost of computational time and computer storage owing to the sizes of the whole nanopore. The present study used the nanoscale flow equation (Zhang, 2015) to describe the adsorbed layer flow and simulate the continuum water flow by the Newtonian fluid model. By neglecting the pressure influences on both the viscosity and density of water and assuming the flow as isothermal and in laminar flow, Zhang (2020a,b) has derived the detailed analytical results for the multiscale flow in cylindrical nanotubes by respectively considering the wall slippage or not. Here, we just cite his results and present the corresponding calculation results.

3.1 For the case of wall slippage

The water has a relatively weak interaction with the organic pore wall. When the pressure drop on the whole pore is large enough, the magnitude of the pressure gradient-induced shear stress on the pore wall will exceed the durability i.e. shear strength of the adsorbed layer-pore wall interface and the wall slippage will consequently occur (Zhang, 2020b).

According to the analysis (Zhang, 2020b), the pressure drop on the whole pore for initiating the wall slippage is:

$$DP = \frac{l\tau_s}{R_0 + D(n-1)} \quad (1)$$

where τ_s is the shear strength of the adsorbed layer-pore wall interface, D is the water molecule diameter, and n is the number of the water molecules across the adsorbed layer thickness.

The critical power loss on the whole pore for initiating the wall slippage is (Zhang, 2020b):

$$POW_{cr} = \frac{K_{cr}l(\tau_s h_{bf})^2}{\eta} \quad (2)$$

where η is the water bulk viscosity and:

$$K_{cr} = \left[\frac{1}{2\lambda_{bf} \left(1 + \frac{D(n-1)}{R_0}\right)} \right]^2 \left\{ 2\pi \frac{R_e}{R_0} \left[\frac{4\varepsilon\lambda_{bf}^3}{C_y \left(1 + \frac{\Delta x}{D}\right)} \left[1 + \frac{1}{2\lambda_{bf}} \right. \right. \right. \right. \\ \left. \left. \left. - \frac{\Delta_{n-2}(q_0 - q_0^n)}{h_{bf}(q_0^{n-1} - q_0^n)} \right] - \frac{2F_1\lambda_{bf}^3}{3C_y} \right\} + \frac{\pi}{4} - \frac{4\pi}{C_y} \left[\frac{F_2\lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta x}{D}} \right. \right. \\ \left. \left. \cdot \left[\frac{1}{2} + \lambda_{bf} - \frac{\Delta_{n-2}(q_0 - q_0^n)}{2R_0(q_0^{n-1} - q_0^n)} \right] \right] \right\} \quad (3)$$

Here, $\lambda_{bf} = h_{bf}/(2R_0)$, R_e is an equivalent constant radius and often $R_e/R_0 = 1 + \lambda_{bf}$, Δx is the separation between the neighboring water molecules in the axial direction in the adsorbed layer, $C_y = \eta_{bf}^{eff}/\eta$,

η_{bf}^{eff} is the effective viscosity of the adsorbed layer and formulated as

$$\eta_{bf}^{eff} = Dh_{bf}/[(n-1)(D+\Delta x)(\Delta_l/\eta_{line,l})_{avr,n-1}], \quad \varepsilon = (2DI + II)/[h_{bf}(n-1)(\Delta_l/\eta_{line,l})_{avr,n-1}],$$

and $q_0 = \Delta_{j+1}/\Delta_j$ and q_0 is constant, $F_1 = \eta_{bf}^{eff}(12D^2\psi + 6D\phi)/h_{bf}^3$, and $F_2 = 6\eta_{bf}^{eff}D(n-1)(l\Delta_{l-1}/\eta_{line,l-1})_{avr,n-1}/h_{bf}^2$,

$$I = \sum_{i=1}^{n-1} i(\Delta_l/\eta_{line,l})_{avr,i}, \quad II = \sum_{i=0}^{n-2} [i(\Delta_l/\eta_{line,l})_{avr,i} + (i+1)(\Delta_l/\eta_{line,l})_{avr,i+1}]\Delta_i, \quad \psi = \sum_{i=0}^{n-1} i(l\Delta_{l-1}/\eta_{line,l-1})_{avr,i}, \quad \phi = \sum_{i=0}^{n-2} [i(l\Delta_{l-1}/\eta_{line,l-1})_{avr,i} + (i+1)(l\Delta_{l-1}/\eta_{line,l-1})_{avr,i+1}]\Delta_i,$$

$$i(\Delta_l/\eta_{line,l})_{avr,i} = \sum_{j=1}^i \Delta_{j-1}/\eta_{line,j-1},$$

$$i(l\Delta_{l-1}/\eta_{line,l-1})_{avr,i} = \sum_{j=1}^i j\Delta_{j-1}/\eta_{line,j-1}, \quad \eta_{line,j-1} \text{ and } \Delta_{j-1} \text{ are}$$

respectively the local viscosity and the separation between the j^{th} and $(j-1)^{th}$ water molecules across the adsorbed layer thickness, and j and $(j-1)$ are respectively the order numbers of the water molecules across the adsorbed layer thickness.

When the power loss POW on the whole pore is greater than POW_{cr} , the wall slippage occurs and the volume flow rate through the pore is (Zhang, 2020b):

$$q_v = \frac{C_1 POW}{\tau_s}, \quad \text{for } POW > POW_{cr} \quad (4)$$

where

$$C_1 = \frac{h_{bf}}{l} \left[1 + \frac{1}{2\lambda_{bf}} - \frac{1 + \frac{\Delta_{n-2}(q_0 - q_0^n)}{D(q_0^{n-1} - q_0^n)}}{2\lambda_{bf} \frac{R_0}{D}} \right] \quad (5)$$

3.2 For the case of no wall slippage

When $POW \leq POW_{cr}$, no wall slippage occurs, and the volume flow rate through the pore is (Zhang, 2020b):

$$q_v = C_2 h_{bf}^2 \sqrt{\frac{POW}{\eta l}}, \quad \text{for } POW \leq POW_{cr} \quad (6)$$

where

$$C_2 = \frac{\sqrt{\pi}}{4\lambda_{bf}^2} \left\{ 8 \frac{R_e}{R_0} \frac{\varepsilon\lambda_{bf}^3}{C_y \left(1 + \frac{\Delta x}{D}\right)} \left[1 + \frac{1}{2\lambda_{bf}} - \frac{\Delta_{n-2}(q_0 - q_0^n)}{h_{bf}(q_0^{n-1} - q_0^n)} \right] \right. \\ \left. - \frac{4R_e F_1 \lambda_{bf}^3}{3R_0 C_y} + \frac{1}{4} - \frac{4}{C_y} \left[\frac{F_2 \lambda_{bf}^2}{6} - \frac{\lambda_{bf}}{1 + \frac{\Delta x}{D}} \right] \right\}$$

$$\left[\frac{1}{2} + \lambda_{bf} - \frac{\Delta_{n-2}(q_0 - q_0^n)}{2R_0(q_0^{n-1} - q_0^n)} \right]^{1/2} \quad (7)$$

3.3 For the classical continuum flow theory

The classical continuum flow theory ignores the adsorbed layer on the pore wall and assumes the water as continuum in the whole pore. For the purpose of comparison, the volume flow rate through the pore was calculated from the classical continuum flow theory as:

$$q_v = \frac{R^2}{2} \sqrt{\frac{\pi POW}{\eta l}} \quad (8)$$

4. CALCULATION

In the present calculation, the following values were taken: $R = 27.5\text{nm}$, $D = 0.28\text{nm}$, $\eta = 0.001\text{pa} \cdot \text{s}$, $\tau_s = 10\text{kPa}$, $\Delta x/D = \Delta_{n-2}/D = 0.15$. It is formulated that $\eta_{line,i}/\eta_{line,i+1} = q_0^m$, where q_0 and m are respectively positive constant (Zhang 2020a,b).

The parameter C_y is expressed as (Zhang 2020a,b):

$$C_y(H_{bf}) = 0.9507 + \frac{0.0492}{H_{bf}} + \frac{1.6447E - 4}{H_{bf}^2} \quad (9)$$

where $H_{bf} = h_{bf}/h_{cr,bf}$ and $h_{cr,bf}$ is a critical thickness and here taken as 2.8nm . Equation (9) represents the weak interaction between water and the blood capillary wall which weakly influences the effective viscosity of the water molecule layer adhering to the nanopore.

The parameters ε , F_1 and F_2 are respectively expressed as (Zhang 2020a,b,c):

$$\varepsilon = (4.56E - 6)(\Delta_{n-2}/D + 31.419)(n + 133.8) \cdot (q_0 + 0.188)(m + 41.62) \quad (10)$$

$$F_1 = 0.18(\Delta_{n-2}/D - 1.905)(\ln n - 7.897) \quad (11)$$

$$F_2 = -(3.707E - 4)(\Delta_{n-2}/D - 1.99)(n + 64) \cdot (q_0 + 0.19)(m + 42.43) \quad (12)$$

For the water-pore wall interaction, it was chosen that $m=0.5$, $n=3$, and $q_0 = 1.03$. It is resulting that $h_{bf} = 0.92\text{nm}$.

5. RESULTS

Figure 3 shows the pressure drop DP on the whole pore required for initiating the wall slippage as dependent on the pore axial length l . The value of DP ranges between 38kPa and 74kPa , and is significantly linearly increased with the increase of l . If the pressure difference between the blood and the interstitial fluids is no more than DP , no wall slippage occurs when the water flows through the pore; Otherwise, the wall slippage occurs and classical hydrodynamic flow theory may thus fail to describe the water flow.

Figure 4 shows the critical power loss POW_{cr} on the whole pore for initiating the wall slippage. When the power loss on the whole pore is no more than POW_{cr} , no wall slippage occurs; Otherwise, the water flows with the wall slippage. The value of POW_{cr} ranges between $6.0E-12\text{Watt}$ and $1.2E-11\text{Watt}$, and is significantly increased with the increase of l . This critical power loss is much larger than the critical power loss on the cellular connexon for initiating the wall slippage, which ranges between $1.0E-16\text{Watt}$ and $1.0E-15\text{Watt}$ (Wang and Zhang, 2021). Nevertheless, the wall slippage inside the nanopore of the blood capillary wall is still easily generated just by putting a very small power loss on the whole pore.

Figures 5(a) and (b) show the calculated volume flow rates of the water through the nanopore as dependent on the power loss on the whole pore when the pore axial length l is respectively 100nm and 200nm . The water flow should be essentially considered as occurring with the provision of the wall slip (denoted by the legend "Wall slip" in Fig.5). The present calculated volume flow rate is compared with those calculated respectively based on the assumption of no wall slippage and

from the classical continuum flow theory. Figures 5(a) and (b) show that when $POW \leq POW_{cr}$, no wall slippage occurs and the present multiscale calculation is nearly overlaid with the classical continuum flow theory calculation; This shows the very weak multiscale effect and the adsorbed layer effect is negligible. However, when $POW > POW_{cr}$, the wall slippage occurs, the present multiscale calculated volume flow rate is normally significantly greater than that calculated based on the assumption of no wall slippage (but still with the multiscale approach). It shows the essential contribution of the wall slippage to the great enhancement of the water flow rate through the nanopore as having been discussed (Borg and Reese, 2017; Holt et al., 2006; Majumder et al., 2005). For the water flow through the blood capillary wall, owing to the wall slippage, the flow rate can be increased by 10 times. For $POW > POW_{cr}$, if the wall slippage was ignored, the present multiscale calculation is still overlaid with the classical continuum flow theory calculation for $l=100\text{nm}$, and has a more pronounced divergence from the latter for $l=200\text{nm}$. Overall, for the present water flow, the multiscale effect owing to the adsorbed layer is weak.

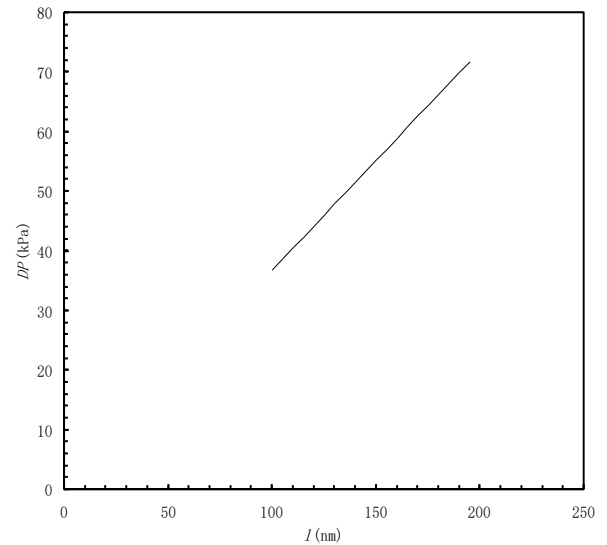


Fig. 3 Values of the pressure drop DP on the whole pore required for initiating the wall slippage.

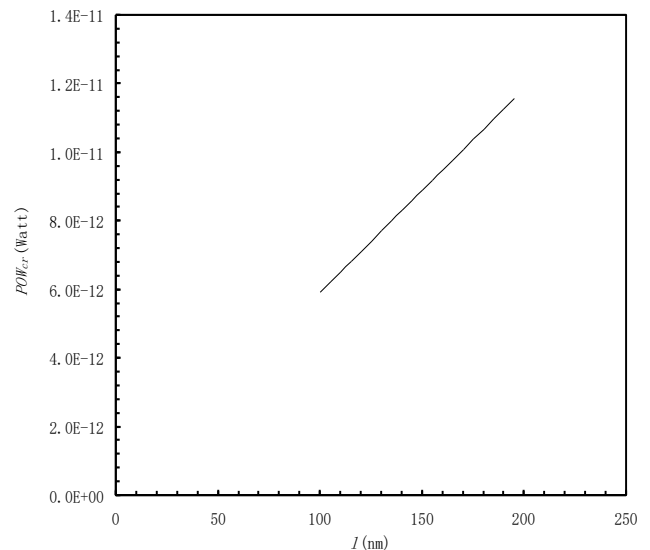
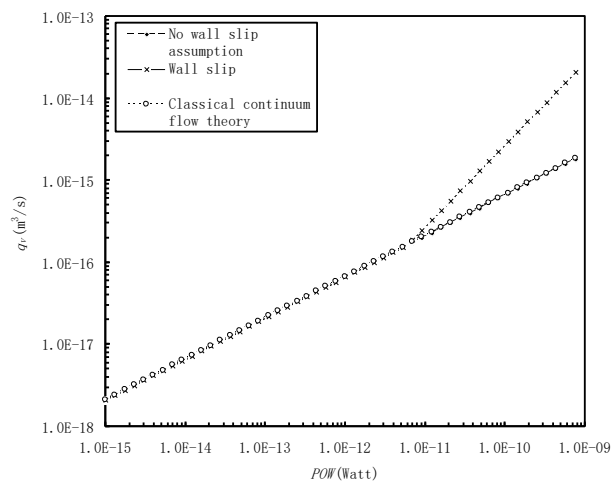
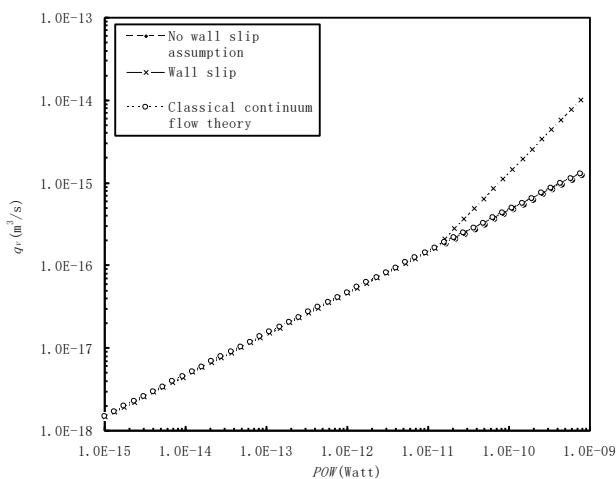


Fig. 4 Critical power loss on the whole pore for initiating the wall slippage.



(a) $l=100\text{nm}$



(b) $l=200\text{nm}$

Fig. 5 Volume flow rate of the water through the nanopore of the blood capillary wall as dependent on the power loss on the whole pore.

6. CONCLUSIONS

The multiscale calculation results are presented for the water flow through the nanopore of the blood capillary wall in human bodies the diameter of which is 55nm and representative. The pore axial length is varied from 100nm to 200nm. Both the pressure drop and the critical power loss on the whole pore for initiating the wall slippage were calculated. The former ranges between 38kPa and 74kPa, dependent on and linearly increased with the pore axial length; The latter ranges between $6.0\text{E-}12\text{Watt}$ and $1.2\text{E-}11\text{Watt}$, also linearly increased with the pore axial length. It was found that when the pressure drop on the whole pore does not reach the critical level, no wall slippage occurs when the water flows through and the power loss on each pore is fairly small (less than the critical power loss on the whole pore for initiating the wall slippage), for this case the calculated volume flow rate through the pore well matches with that calculated from the classical continuum flow theory; When the power loss on the whole pore is larger than the critical power loss on the whole pore for initiating the wall slippage, the wall slippage occurs and the calculated volume flow rate through the pore is normally significantly greater than that calculated from the classical continuum flow theory; The flow rate enhancement due to the wall slippage depends on the power loss POW on the whole pore and can be 10 times if $POW=1.0\text{E-}9\text{Watt}$.

The present study indicates that the power loss on each pore of the blood capillary wall is very small (no more than $1.0\text{E-}9\text{Watt}$ or less) when the water flows through; This is particularly true when no wall slippage occurs, even for the wall slippage case still only a very small power loss on the pore is required for greatly enhancing the flow rate through the pore. The study substantiates the vital contribution of the wall slippage to the water transport through the blood capillary wall. While for this water flow, the multiscale effect is quite weak and the effect of the adsorbed layer on the pore wall may be negligible.

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