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INFLUENCE OF CRITICAL PARAMETERS ON LIQUID THIN FILM FLOW OF CASSON NANO FLUID OVER ELONGATED SHEET UNDER THERMOPHOROSIS AND BROWNIAN MOTION

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ABSTRACT

Present investigation aims at scrutinizing the properties of heat and mass transfer phenomena of liquid thin film of Casson Nano fluid over elongated sheet under the influence of thermophorosis and Brownian motion. Casson Nano particle effect on thermophorotic force and on Brownian force is studied. Variables of similarity were induced to transmute partial differential equations into dimensionless equations and are resolved numerically by elegant method bvp 4c. Thin film thickness is calculated using MATHEMATICA for different values of critical parameters. Velocity profiles diminishes for higher values of Casson parameter and magnetic field parameter. The temperature escalates for higher values of Magnetic, Casson, thermal radiation, Brownian motion and thermophorosis parameters, whereas contrary effect with other parameters is observed. Impact of Skin friction, Sherwood and Nusselt numbers on the flow configurations for diverse critical parameters are exposed realistically via graphs. Arithmetical results that obtained in the current exploration are confirmed with previously explored values in marginal way.

Key words: Similarity variable, Magnetic field, thermal radiation, film thickness, unsteadiness parameter

1. INTRODUCTION

The technology behind liquid thin film is explored by inquisitive Mathematicians in short span of time in view of their enormous applications in engineering, technology, and biology. Human eye is shielded by an aqueous tear thin film and out most layers of tear thin film is layered by lipids. Therefore structure of thin film is necessary to understand its function under different normal and uncontrolled conditions. Lukasz Cwiklik (2016) discussed structure of molecular levels in lipid layer tear by using molecular dynamic forces. Effective investigation is required to understand the mechanism of stability of surfactant lining the walls of the alveoli in the lungs. Eline Hermans *et al.* (2015) proposed effective design in synthetic surfactant replacements to treat infant and adult respiratory disorders through his investigations on thin films. For most of the thin film devices it is very important to meet the practical requirements like defect free uniform thickness, and coatings. Temparature maintenance, concentration ingredients, surfactants and evaporation plays crucial role in spreading process of liquid thin films. Wang (1990) studied about liquid film on an unsteady stretching surface. Chunxi Li *et al*. (2020) applied numerical simulation to investigate the liquid film spreading in the presence of surface acoustic waves (SAWs). Zahir shah *et al.* (2018) examined the flow of Williamson liquid film fluid with heat transmission having the impact of thermal radiation, embedded in a permeable medium over a time dependent stretching surface. Study of thin films helps to build miniature devices for tissue generation, implants and transport of drug delivery in human body. However, the interaction of materials with cells and tissue must first be investigated in detail to ensure safe and long-time handling when implanted within the human body. Nazarpour Soroush (2013) in their e - text book clearly enlightened bio compatibility of thin film and bio functional coatings and also they explored electrophoretic manipulation of oxide nanoparticles through experiments as well as theoretical computations for being deposited as bioactive thin films on substrates of various conductivities introducing some phenomena arising in biological systems.`

The viscosity of the fluids like paints, greases, lubricant oils coal tar, jellies, and paste are not fixed and they depends upon the factors like shear in fluid, pressure and temperature. These fluids are non-Newtonian in nature. Casson fluid is non-Newtonian and first proposed by enthusiastic Mathematician Casson (1959) while doing his experiments on letter press toners. The special properties of Casson fluid carried significant applications in science as well as in polymer processing and in biomechanics. Blood of human beings can be considered as Casson fluid because it contains many proteins and hormones. The Casson fluid model is found to describe accurately the flow curves of suspensions of betonies in water by Tammamasi (1968).The unsteady magnetohydrodynamic flow of a Casson fluid bounded by two parallel non-conducting porous plates is studied with heat transfer considering the Hall Effect by Attia *et al.* (2010). Vijaya *et al.* (2016&2018a) in their papers studied impact of magnetic field on Casson thin film on an unsteady stretching surface and also explored thermo physical properties of Casson fluid through an oscillating vertical wall under the influence of transverse magnetic field. Mohammad mehdi rashidi *et al*. (2016)

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investigated MHD blood flow of Casson fluid model due to peristaltic waves under different physical parameters. Asmaa *et al*. (2020) studied bacterial growth in heart valve by considering antibiotics, as commonly dispersed nano particles in blood.

Transport of momentum in Casson Nano fluid thin film flows induced by stretching surfaces have been extensively analyzed owing to their important practical applications in various industries. In manufacturing of iron, rubber and plastic sheets the primary objective is to maintain surface quality of the sheets. This quality depends upon nanoparticle volume size, magnetic field, radiation effects and the rate of cooling of filaments or sheets. Nano technology alludes to be upcoming field of science that incorporates synthesis and adjacent of different Nano materials. Nano particles are very small and their diameter varies from 1 nm to 100 nm. Nano particles made from copper, zinc, magnesium dissolved in base fluids like engine oil, water, kerosene and have tremendous heat absorption qualities which are imperative essential of all types of industries like nuclear fuel complexes, medical treatment, motor industries so many. Sulochana *et al.* (2018) analyzed the Magnetohydrodynamic film flow of kerosene based Nano fluid at various flow directions. Muhammad Jawad *et al*. (2018) investigated thermal conductivity of non Newtonian fluid with unlimited thermal conductivity of nano particles using HAM method. Zaheer shah *et al.* (2020) scrutinized the entropy optimization in electrically conduction Casson nano fluid over nonlinear stretchable surface. Vijaya *et al.* (2018b) explored the influence of magnetic field, heat radiation on nano fluid flowing over a vertical circular cylinder. Heat and mass transfer mechanism can be observed in heat exchanger, petrology, geosciences. In this mechanism heat transfer quantity may vary due to concentration gradients, mass transfer coefficient may vary due to thermal gradient. Soret and Dufour effects plays very important role in in separation of isotopes. Partha *et al.* (2009) studied extensively about Soret & Dufour effects and decomposition of thermophoresis particles in non-Darcy porous medium. MHD flow of Carreau Nano fluid explored using CNT over a nonlinear stretching sheet is studied by Nagalakshmi *et al*. (2020). Hymavathi *et al.* (2018&2019) explored influence of chemical reaction on Casson fluid flow using Keller box method. Random motion of particles suspended in a fluid is known Brownian motion of nano particles. This physical phenomena at the molecular and nano scale level plays key role in thermal behaviour of nanoparticle in nano fluids. From the literature it was found that Robert Brown (1872 –Scotland)) discovered particles trapped in cavities inside pollen grains in water. Albert Einstein presented Brownian motion equations, later Einstein and Cowper (1956) reported their investigations on Brownian movement. Khairy Zaimi *et al.* (2014) explored the effects of thermophorosis and Brownian motion parameters on boundary layer of Nano fluid. Samule O. Adesanya *et al.* (2020&2019) investigated temperature dependent fluid properties of the thin film and also they discussed Gravity-Driven Hydro magnetic couple stress on thin films. Sudipta Saha *et al.* (2020) developed multidimensional Mathematical model to describe dual mode cooling conditions by considering thin liquid water film.

There is a shortage of study on Brownian motion and thermophorosis on Casson nanofluid and their combined effects. The main goal of this study is to explore Brownian and thermophorotic motion effects on Casson nanofluid fluid. Film thick ness is also calculated under under different conditions. The powerful Mathematics soft wares MATHEMATICA and MATLAB are used to find thin film thickness and portrayal of graphs.

2. PHYSICAL MODEL

Chemically responsive Non Newtonian Casson nanofluid liquid thin film having $h(t)$ units of thickness over a heated stretching pane is emerging from a slim slit which is assumed as the source of the reference frame is shown in the Fig 1.The motion of the fluid within the film is generated by stretching the sheet along x- axis with preliminary velocity U(x, t), with 'a', 'b' as constants as follows.

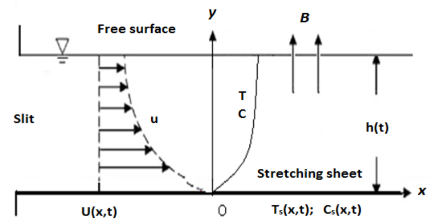

Fig. 1 Geometrical outline of the flow

The elastic sheet's temperature T_s and concentration C_s are assumed to vary with the distance 'x' from the slit as follows

$$T_s\,(x,t) = T_0 - T_{ref}\left[\frac{ax^2}{2v}\right](1-bt)^{-3/2} \tag{2}$$

$$C_s\,(x,t) = C_0 - C_{ref}\left[\frac{ax^2}{2v}\right](1-bt)^{-3/2} \tag{3}$$

In the above equations temperature and concentration at the entrance of the slit are T_0 and C_0. T_{ref} is constant reference temperature and C_{ref} is constant reference concentration. Where

$$\begin{cases} 0 \le T_{ref} \le T_0, for\ t < \frac{1}{b} \\ 0 \le C_{ref} \le C_0, for\ t < \frac{1}{b} \end{cases} \tag{4}$$

A sloping magnetic field $B = B_0(1-bt)^{-1/2}$ is reinforced on the liquid thin film. In this study influence of thermal radiation is considered. Casson fluid is Non Newtonian in nature and its constitutive equations (Eldabe and Salwa, (1995)) which are written as

$$\text{for } \pi > \pi_c\,,\ \tau_{ij} = 2\left(\mu_B + P_y(2\pi)^{\frac{-1}{2}}\right)e_{ij}$$

$$\text{for } \pi < \pi_c\,,\ \tau_{ij} = 2\left(\mu_B + P_y(2\pi_C)^{\frac{-1}{2}}\right)e_{ij} \tag{5}$$

In the above constitutive equations τ_{ij} is the (i, j)th stress tensor component, μ_B is the plastic dynamic viscosity of the fluid which is non-Newtonian. When $P_y > P_s$ fluid acts as a solid, and when $P_y < P_s$ fluid demonstrates flow characteristics, where P_y is yield stress, P_s is shear stress. π_c is the critical value of $\pi = e_{ij}e_{ij}$ (e_{ij} is the (i, j)th component of deformation rate) which depends upon non-Newtonian model. The governing equations of the flow with $\beta\ (=\ \mu_B\sqrt{2\pi_c}\,/P_y)$ as the Casson parameter and $\alpha = \frac{k}{\rho C_p}$ as thermal diffusivity are prearranged here under with terms mentioned in nomenclature.

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{6}$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma B^2}{\rho}\bar{u} \tag{7}$$

$$\frac{\partial T}{\partial t} + \bar{u}\frac{\partial T}{\partial x} + \bar{v}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_0^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2} + \frac{\rho^* c_p^*}{\rho c_p}\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) \tag{8}$$

$$\frac{\partial C}{\partial t} + \bar{u}\frac{\partial C}{\partial x} + \bar{v}\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

The flow pattern in the absence of penetration and slip can be designated as follows.

$$\bar{u} = U, \bar{v} = 0, T = T_s, C = C_s\ at\ y = 0, \tag{10}$$

$$\frac{\partial \bar{u}}{\partial y} = 0,\ \frac{\partial T}{\partial y} = 0, \frac{\partial C}{\partial y} = 0, \bar{v} = \frac{dh}{dt}\ at\ y = h(t) \tag{11}$$

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Following likeness variables are introduced for conversion

$$\eta = \left[\frac{a}{\nu(1-bt)}\right]^{\frac{1}{2}} y\,, \quad (12)$$

$$\psi = x\left[\frac{\nu a}{1-bt}\right]^{\frac{1}{2}} f(\eta), \quad (13)$$

$$T = T_0 - T_{ref}\left[\frac{ax^2}{2\nu(1-bt)^{\frac{3}{2}}}\right]\theta(\eta) \quad (14)$$

$$\theta(\eta) = \frac{T-T_0}{T_S-T_0} \quad (15)$$

$$C = C_0 - C_{ref}\left[\frac{ax^2}{2\nu(1-bt)^{3/2}}\right]\phi(\eta) \quad (16)$$

$$\phi(\eta) = \frac{C-C_0}{C_S-C_0} \quad (17)$$

$\psi(x,y)$ is the stream function which satisfies the conservation equation (6)

$$\bar{u} = \frac{\partial\psi}{\partial y} = \frac{ax}{1-bt}f'(\eta) \qquad \bar{v} = -\frac{\partial\psi}{\partial x} = -\left(\frac{\nu a}{1-bt}\right)^{1/2} f(\eta) \quad (18)$$

$f'(\eta)$ denotes differentiation with respect to η.

3. METHOD OF SOLUTION

Similarity variables were introduced in equations (6) – (9) to obtain dimensionless equations in the particular range $0-\gamma$ as follows with boundary conditions

$$\left(1+\frac{1}{\beta}\right)f''' + \left(ff'' - S(f' + \frac{\eta}{2}f'') - f'^2 - Mf'\right) = 0 \quad (19)$$

$$\left(1+\frac{4}{3}Nr\right)\theta'' + Pr\left(f\theta' - 2f'\theta - \frac{S}{2}(\eta\theta' + 3\theta) + Nb\,\theta'\phi' + Nt\,\theta'^2\right) = 0 \quad (20)$$

$$\phi'' + Le\,Pr\left(f\phi' - 2f'\phi - \frac{S}{2}(\eta\phi' + 3\phi)\right) + \frac{Nt}{Nb}\theta'' = 0 \quad (21)$$

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \quad (22)$$

$$f(\gamma) = \frac{1}{2}S\gamma, f''(\gamma) = 0, \theta'(\gamma) = 0, \phi'(\gamma) = 0 \quad (23)$$

Here $S(=b/a)$ the unsteadiness parameter, $M(=\sigma B_0^2/\rho a)$ the Magnetic field parameter, Pr $(=\nu/b)$ the Prandtl number, $Nr(=4\sigma^* T_0^3/kk^*)$ the thermal radiation parameter, Nb$\left(=\frac{\rho^* c_p^* D_B(C_w-C_0)}{\rho c_p \nu}\right)$the Brownian motion parameter, Nt$\left(=\frac{\rho^* c_p^* D_T(T_w-T_0)}{\rho c_p T_\infty \nu}\right)$ the thermophoretic parameter and

$Le\left(=\frac{b}{D_B}\right)$ is the Lewis number Let γdenotes the value of the similarity variable η at the free surface which is a constant to be determined from the problem and from equation (12) we have

$$\gamma = \left(\frac{a}{\nu(1-bt)}\right)^{1/2} h(t) \quad (22)$$

Since γ is an unknown constant, which should be determined, as a whole, from the set of the present boundary-value problem, the rate of change of the film thickness can be obtained as follows

$$\frac{dh}{dt} = -\frac{b\gamma}{2}\left(\frac{\nu}{a(1-bt)}\right)^{1/2} \quad (23)$$

Thus, the kinematic constraint at $y = h(t)$ given by equation (11) transforms to the free surface condition (22). Equations (19) – (21) together with the applicable boundary conditions equations (22) and equations (23) are solved mathematically by the well-organized fourth order R-K method as follows.

$$\frac{df_0}{d\eta} = f_1 \quad (24)$$

$$\frac{df_1}{d\eta} = f_2 \quad (25)$$

$$\left(1+\frac{1}{\beta}\right)\frac{df_2}{d\eta} = S\left(f_1 + \frac{\eta}{2}f_2\right) + f_1^2 - f_0 f_2 + M \quad (26)$$

$$\frac{d\theta_0}{d\eta} = \theta_1 \quad (27)$$

$$\left(1+\frac{4}{3}Nr\right)\frac{d\theta_1}{d\eta} = Pr\left(\frac{S}{2}(3\theta_0 + \eta\theta_1) + 2\theta_0 f_1 - \theta_1 f_0 - Nb\,\theta_1\phi_1 - Nt\,\theta_1^2\right) \quad (28)$$

$$\frac{d\phi_0}{d\eta} = \phi_1 \quad (29)$$

$$\frac{d\phi_1}{d\eta} = Sc\left(\frac{S}{2}(3\phi_0 + \eta\phi_1) + 2\phi_0 f_1 - \phi_1 f_0\right) - \frac{Nt}{Nb}\frac{d\theta_1}{d\eta} \quad (30)$$

The associated boundary conditions are converted as follows

$$f_0(0) = 0,\ f_1(0) = 1, \theta_0(0) = 1,\ \phi_0(0) = 1 \quad (31)$$

$$f_0(\gamma) = \frac{1}{2}S\gamma, f_2(\gamma) = 0,\ \theta_1(\gamma) = 0, \phi_1(\gamma) = 0. \quad (32)$$

Here $f_0(\eta) = f(\eta)$, $\theta_0(\eta) = \theta(\eta)$ & $\phi_0(\eta) = \phi(\eta)$. This requires the preliminary values $f_2(0)$, $\theta_1(0)$ and $\phi_1(0)$. To solve this suitable values are approximated and then integration is made. A step size of $\Delta\eta = 0.01$ is preferred in this study. The value of γ is obtained in such a way that the boundary condition $f_0(\gamma) = \frac{S\gamma}{2}$ is satisfied with an error of tolerance of 10^{-6}.

4. PHYSICAL QUANTITIES

The significant engineering physical quantities in this problem are C_f(=skin friction coefficient), Nu_x(=local Nusselt number), Sh_x (=local Sherwood number) respectively are defined below with $Re_x = \frac{Ux}{\nu}$ as local Reynolds number.

$$C_f {Re_x}^{1/2} = -2\left(1+\frac{1}{\beta}\right)f''(0) \quad (33)$$

$$Nu_x {Re_x}^{-1/2} = \theta'(0) \quad (34)$$

$$Sh_x {Re_x}^{-\frac{1}{2}} = \phi'(0) \quad (35)$$

5. RESULTS AND DISCUSSION

5. a) Graphical *Analysis*

In this investigation the main motivation is to scrutinize the influence of distinct critical parameters S, M, Pr, Nr, Nb, Nt, Le on velocity f′(η), temperature $\theta(\eta)$, and concentration $\phi(\eta)$. The variation of film thickness γ is calculated in each case. Variations in skin friction coefficient, Nusselt number, and Sherwood number are portrayed pictorially.

i) Influence of governing parameters on velocity profiles

The influence of non-Newtonian nature of the fluid through Casson parameter β on velocity is illustrated Fig. 2. It is observed that velocity in the vicinity of the boundary becomes a constant function of the β . However, a significant reduction in the velocity in the film is observed away from the boundary for increasing values of β. Reduction in the velocity is due to non Newtonian nature of the fluid ie., increase in β corresponds to an increase in the plastic dynamic viscosity of the fluid. Film thickness γ also gets reduced for higher values of β. In Fig. 3 it is witnessed that in the nonappearance of magnetic field M velocity progressively declines. When M increases then there is a quick drop of velocity in the area of the boundary due to the influence of Lorentz force which opposes the motion of the fluid. As the S assumes greater values fluid velocity enhanced and thickness of the film decreases. When S=1.2 film thickness is to reduce by 1.75 times to that of the film corresponding to 0.8.

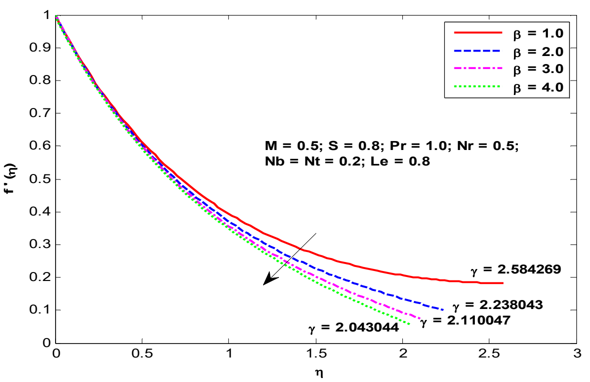

Fig. 2 Dominance of β on $f'(\eta)$

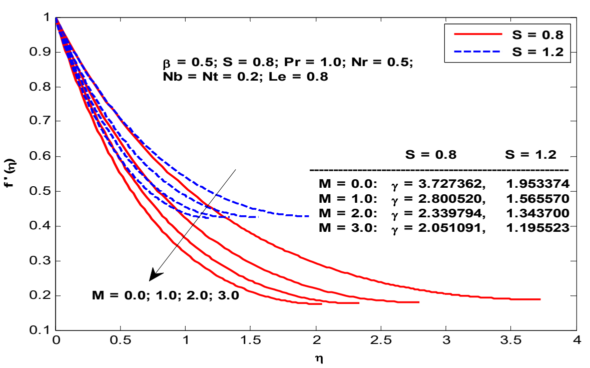

Fig.3 Dominance of M on $f'(\eta)$, for S=0.8 &1.2

ii) Influence of governing parameters on temperature profiles

Figures (4 - 9) are the plots of temperatures for diverse values of governing parameters including Casson nanoparticle size depicting variation of Brownian motion parameter Nb and thermophorosis parameter Nt. Higher values of β reduces γ, as a result more heat is generated in the fluid flow, this physical phenomenon can be clearly observed in Fig. 4. Fig. 5 reveals that smaller value of Pr yield thicker boundary layer with higher temperature across the boundary layer. Thermal boundary layer is generated within the lower part of the liquid film at higher Pr, as a result temperature gradient vanishes adjacent to the free surface. The larger values of Nr facilitates the release of greater thermal energy which heats up the fluid and leads to higher temperature as illustrated in Fig. 6. Behavior of nano particles in Casson nanofluid for higher values of Nt is portrayed in Fig.7.The thermophoretic force is proportional to the temperature gradient, so that the wider the temperature gradient, the larger the thermophoretic force. The thermophoretic force causes nanoparticle migration across the fluid in the opposite direction of the temperature. Fig. 8 illustrates the temperature distribution under Nb. Brownian forces take the particles in the opposite direction of the particle concentration gradient, trying to make the particles more homogeneous. This irregularity causes collision between fluid nano particles and generates more heat. . Temperature shows steady enhancement for growing values of the M this is attributed from the Fig. 9.

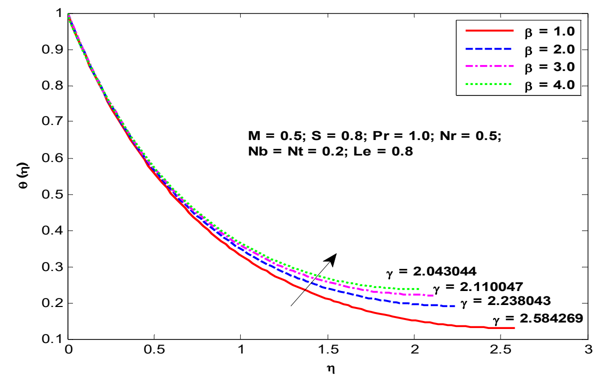

Fig. 4 Dominance of β on $\theta(\eta)$

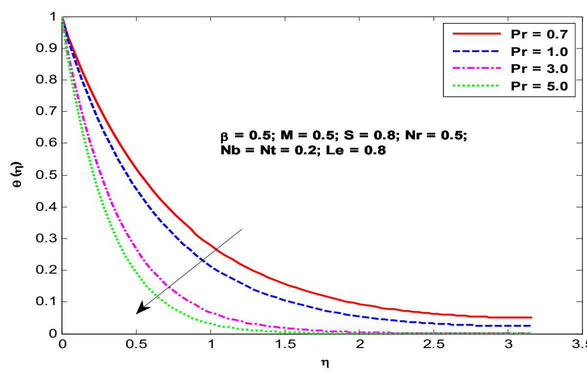

Fig. 5 Dominance of Pr on $\theta(\eta)$

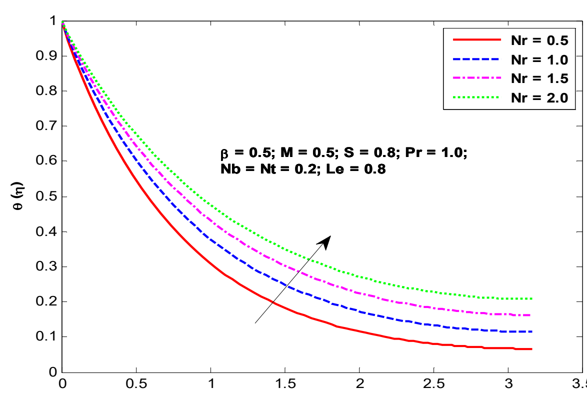

Fig. 6 Dominance of Nr on $\theta(\eta)$

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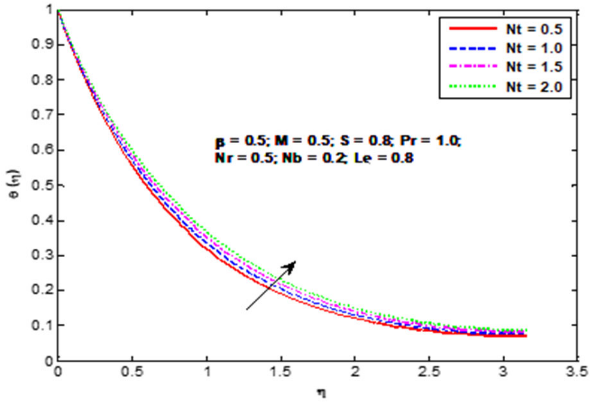

Fig. 7 Dominance of Nt on $\theta(\eta)$

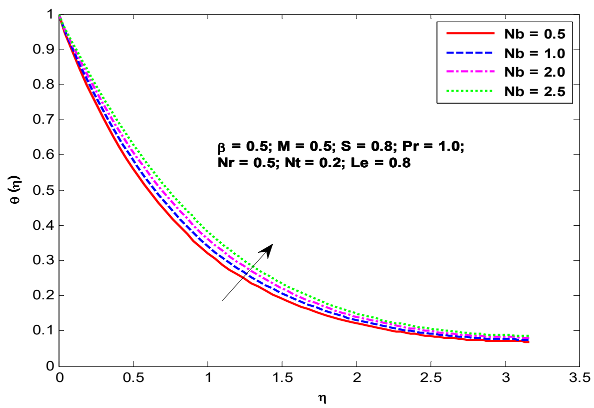

Fig. 8 Dominance of Nb on $\theta(\eta)$

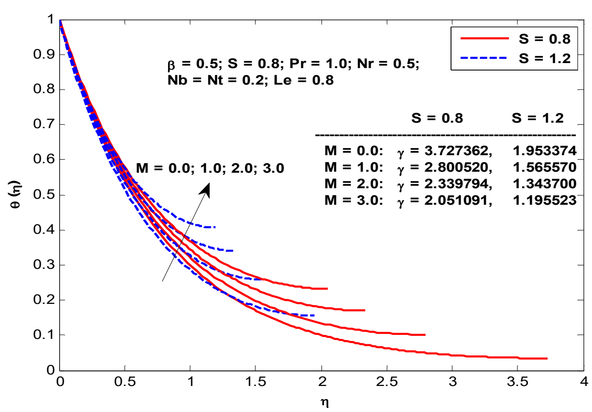

Fig. 9 Dominance of M on $\theta(\eta)$ for S=0.8 & 1.2

iii) Influence of governing parameters on concentration profiles

Impact of foremost parameters on concentration profiles is depicted in figures (10 -14). Increase in M accelerates both thermal and solutal boundary layers and accelerates concentration for both S=0.8 & 1.2 as depicted in Fig.10. At higher values of Nt kinetic energy gained by nano particles increases and rises concentration as illustrated Fig. 11. The effect of Nb on $\phi(\eta)$ can be seen in Fig. 12. Increasing value of Nb diminishes boundary layer thickness as a result concentration of the fluid decreases. Influence of Le can be seen from the Fig. 13.Concentration is inversely proportional to Lewis number.

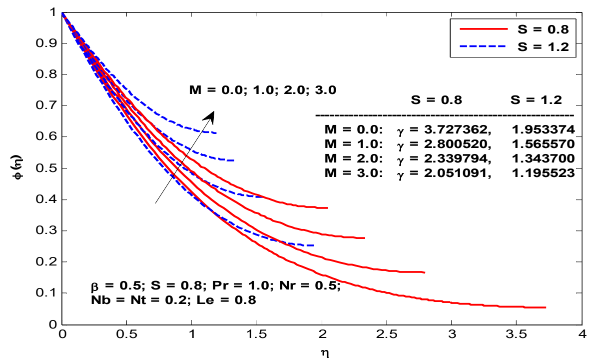

Fig. 10 Dominance of M on $\phi(\eta)$ for S=0.8 &1.2

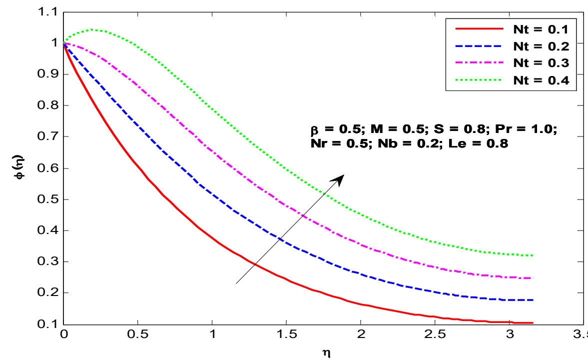

Fig. 11 Dominance of Nt on $\phi(\eta)$

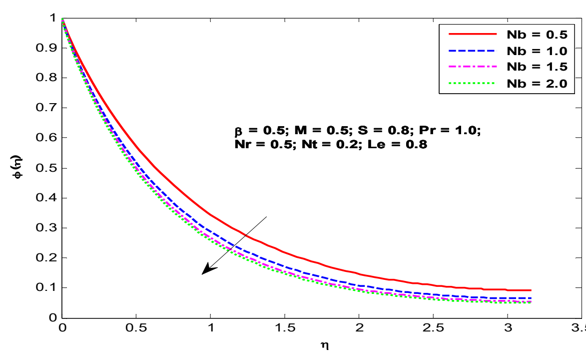

Fig. 12 Dominance of Nb on $\phi(\eta)$

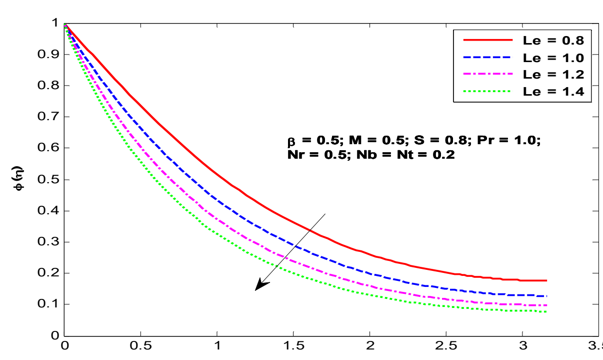

Fig. 13: Dominance of Le on $\phi(\eta)$

iv) ***Influence of governing parameters on skin friction coefficient, Nusselt number and Sherwood number.***

The combined effect of β and M is to decrease skin friction coefficient predominantly because of the retardation in the flow due to higher Lorentz force and smaller velocities which is shown in Fig.14. The collective result of Pr, and Nb can be observed in Fig.15. Brownian force plays a significant role for particle decomposition and increases Nusselt number, however near the wall there is steady change and also it is seen that higher values of Pr enhances Nusselt number. Sherwood number increase for increasing Le and higher values of Nt gradually decreases it which can be observed from Fig. 16.

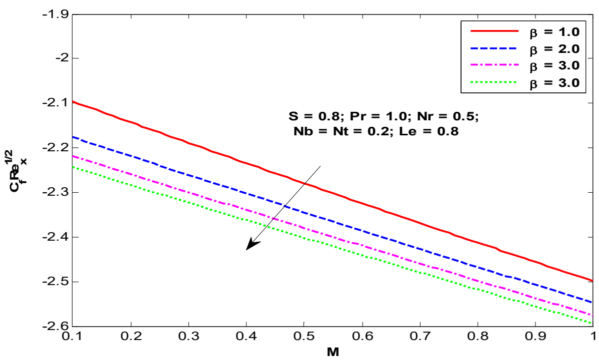

Fig. 14: Dominance of M on skin friction Coefficient for distinct values of β

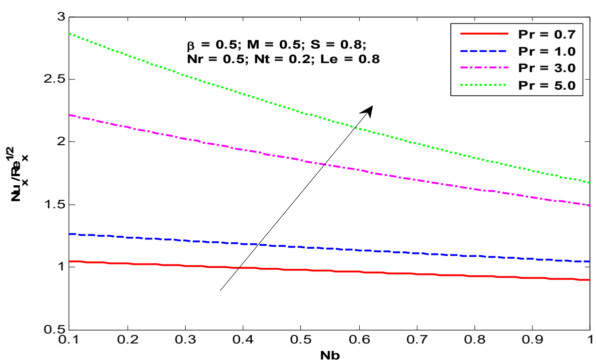

Fig. 15 Dominance of Nb on Nusselt number for different values of Pr

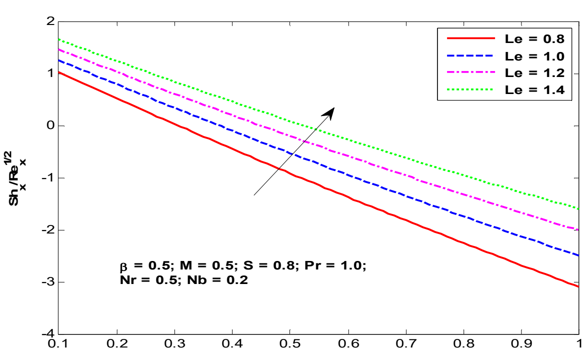

Figure. 16 Dominance of Nt on Sherwood number for different values of L

5. b) Numerical Analysis

Accuracy of the present scheme is ensured by comparing the present results, viz., non dimensional thickness of the film γ, surface skin friction coefficient $f''(0)$ with the corresponding values evaluated by Wang (1990), Abel et al. (2009) and Megahed (2015) in the absence of magnetic field parameter and $\beta \to \infty$ for different values of unsteady parameter. It is pertinent to mention that as Wang (1990) used different similarity variable, the values of $\frac{f''(0)}{\gamma}$ obtained by Wang (1990), shall be same as $f''(0)$of the present study. These values are presented in **Table.1** and it seen that they are in excellent agreement.

6. CONCLUSIONS

Liquid thin film flow of Casson Nano fluid over elongated expanse under the impact of thermophorosis and Brownian motion is studied extensively. Influence of critical parameters on velocity, temperature, and concentrations are expressed realistically via graphs. The result of this analysis can be concisely given as follows

- Non – Newtonian nature of the Casson nano fluid is clearly observed. There is a gradual decrease in velocity and increase in temperature for higher values of Casson parameter β.
- A comparative analysis for the stretching sheet and unsteadiness parameters S against Magnetic parameter M is done. It is detected that at higher values of M there is increase in temperature and concentration but velocity is reduced. The important fact to note down is that film thickness decreases if M is increases.
- Brownian motion through Nb, and thermophorosis influence through Nt on Nano particles is observed in the gesture of the Casson nanofluid. Temparature accelerates at higher values of Nb, and Nt.
- The behavior of nano particles is clearly observed at larger values of Nt and Nb.At higher values of Nt concentration accelerates and at higher values of Nb concentration profiles decelerates.
- The physical parameters skin friction coefficient decreases at higher values of Casson Parameter$\beta$and magnetic parameter M.
- Brownian Parameter increases Nusselt number and Sherwood number increase for increasing Le.

7. NOMENCLATURE

$\bar{u}$	Velocity component along x-axis
$\bar{v}$	Velocity component along y-axis
ν	Kinematic viscosity of the fluid
ρ	Density of the nanofluid
σ	Electrical conductivity
T	Fluid temperature
C	Fluid concentration
k	Thermal conductivity
c_p	Specific heat at constant pressure
σ^*	Stefen-Boltzman constant,
k^*	absorption coefficient
ρ^*	Density of nano-particles
${c_p}^*$	Specific heat of the nano-particles,
D_B	Mass diffusivity
D_T	Thermophoresis diffusion coefficient

Table .1: Assessment of γ and $f''(0)$ with M = 0 and $\beta \to \infty$ for distinct S values

S	Wang (1990)		Abel et al (2009)		Megahed (2015)		Present study	
	γ	$f''(0)/\gamma$	γ	$f''(0)$	γ	$f''(0)$	γ	$f''(0)$
0.4	5.122490	-1.307785	4.981455	-1.134098	4.98145	-1.134096	4.981455	-1.134098
0.6	3.131250	-1.195155	3.131710	-1.195128	3.131710	-1.195126	3.131710	-1.195128
0.8	2.151990	-1.245795	2.151990	-1.245805	2.151994	-1.245806	2.151990	-1.245805
1.0	2.543620	-1.277762	1.543617	-1.277769	1.543616	-1.277769	1.543617	-1.277769
1.2	1.127780	-1.279177	1.127780	-1.279171	1.127781	-1.279172	1.127780	-1.279171
1.4	0821032	-1.233549	0.821033	-1.233545	0.821032	-1.233545	0.821033	-1.233545
1.6	0.576173	-1.491137	0.576176	-1.114937	0.576173	-1.114938	0.576176	-1.114937
1.8	0.356389	-0.867414	0.356390	-0.867416	0.356389	-0.867414	0.356390	-0.867416

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