



# DEVELOPMENT OF NEW CORRELATIONS FOR NUSSOLT NUMBER AND FRICTION FACTOR OF $\text{TiO}_2$ /WATER BASED NANOFLUID FLOW IN V-PATTERN PROTRUSION RIBBED SQUARE CHANNEL

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## ABSTRACT

Nanofluids play important roles in the heat transfer and flow characteristics in ribbed square channels. Systematic experiments are conducted to measure heat transfer enhancement and  $\text{TiO}_2 - \text{H}_2\text{O}$  based nanofluid flow characteristics on a protruded with combined V-type ribbed square channel. Reynolds number studied in the channel range from 4000-18000. Investigational parameters of square channel contain, volume fraction range of 1.0-4.0%, particle diameter range of 30nm-45nm, relative protruded rib height range of 0.10-0.25, ratio of protruded height to print diameter range of 0.8-2.0, relative protruded rib pitch ratio range of 2.0-3.5 and angle of attack of  $30^\circ$ - $75^\circ$  respectively. Results indicates that the thermal performance of the protruded combined V-type ribbed square channel falls with increasing Reynolds number. The highest value of thermal hydraulic performance parameter is found to be 3.21 for the range of parameters investigated. Correlations predicting Nusselt number and friction factor as a function of volume fraction, particle diameter range, relative protruded rib height, ratio of protruded height to print diameter, relative protruded rib pitch ratio, angle of attack and flow parameter. The correlations for Nusselt number and friction factor have also been developed on the basis of experimental data with considerable good accuracy.

**Keywords:** Nano powder, energy, flow duct, friction factor, single phase heat transfer.

## 1. INTRODUCTION

Nanofluids are attracting number of investigators for their better thermo-physical properties with low penalty flow friction. Nanofluids, the new generation heat transfer medium, are studied in many heat transfer processes due to its high heat transfer enhancement performance and stability. The successful employment of nanofluids makes the heat transfer equipment more portable and smaller, so the process of miniaturization of heat transfer devices is accelerated (Ganvir et al., 2017). In recent years, many studies have yet been conducted on the heat transfer performance of nanofluids. The subject of giving more powerful and reliable thermal system in reducing the size, weight, cost, and saving of energy has gotten significant consideration. Therefore, energy cost and ecological concerns are going ahead to urge endeavors to imagine better execution over the current outlines. Keeping in mind the end goal to accomplish these requests, numerous generation strategies have been investigated over the years (Fang et al., 2016; Sarkar, 2011). The repeated rib surfaces are known for their effective enhancement of heat transfer, which is widely demanded in many scientific and industrial applications (Kumar et al., 2017a; Kumar et al., 2017b). Utilizing ribs on the internal surface of heat exchangers has been one of the normal in dynamic strategies to breakdown the laminar sub-layer and produce local wall turbulence because of stream partition and reattachment between consecutive corrugations, which directs the thermal resistance and expressively expands heat exchange (Duangthongsuk and Wongwises, 2009). Pourdel et al. (2018) numerically analyzed the fluid flow and heat

transfer of a flat tube under the constant heat flux using finite volume method and SIMPLE algorithm. Their results showed that by increasing Reynolds number, the convection heat transfer (Nusselt number) and the friction factor rise. Also, by decreasing the dimensionless pitch and increasing the dimensionless depth of dimple, Nusselt number and friction factor increase.

Forced convection augmentation is obtained by means of several techniques in thermal devices development and it allows to realize size reduction and efficiency factor improvement. The grooved entry, because of its adequacy in heat exchange, is a decent hope for engineering applications for example cross-stream heat exchanger, gas turbine airfoil cooling plan, solar air heater blade cooling system and gas cooled atomic reactor (Tiwari et al., 2013). A lot of information are available is forced convection flow over a ribbed surface channels and tubes. Several scientists have investigated characteristics of nanofluids such as (Navaei et al., 2015) investigated the consequence of three different rib-groove shapes (rectangular, semi-circular and trapezoidal). Four different types of nanoparticles,  $\text{Al}_2\text{O}_3$ ,  $\text{CuO}$ ,  $\text{SiO}_2$  and  $\text{ZnO}$  with different volume fractions in the range of 1% to 4% and different nanoparticle diameters in the range of 20 nm to 60 nm. Simulation results reveal that the semi-circular rib-groove has the highest Nusselt number. The nanofluid containing  $\text{SiO}_2$  has the highest Nusselt number compared with other types. The Nusselt number rises as volume fraction increases, and it declines as the nanoparticle diameter increases. Ahn et al. (2010) examined the augmentation of nucleate boiling critical heat flux (CHF) by means of nanofluids stream in a pool boiling are well-known. The

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investigational outcomes showed that the nanofluid flow boiling CHF was improved under the forced convective stream conditions compared to that in pure water.

Behnampour et al. (2017) examined the outcome of rectangular, triangular, and trapezoidal ribs on the average heat transfer of  $H_2O - Ag$  nanofluid in a roughened triangular channel under a uniform heat flux. Among all of the investigated rib forms, heat transfer values are least for rectangular rib form and the triangular form has the best thermal performance. Hassan et al. (2016) numerical analysis is conducted to study the results of using nanofluids on impinging slot jet over a flat plate with a ribbed surface. Results indicated a marked improvement in average Nusselt number with the increase in the solid volume fraction. Vanaki and Mohammed (2015) examined the effect of dissimilar obstacle shapes and turbulent nanofluid flow on the efficiency factor through transversely ribbed rectangular ducts with flow parameter ranging from 5000 to 20000. It is found that the  $SiO_2 - H_2O$  shows the highest heat transfer enhancement compared with other ( $Al_2O_3, CuO, SiO_2$  and  $ZnO$ ) tested nanofluids. The  $Nu_{ave}$  through the ribbed channels was enhanced with the increase of  $\phi$  and  $Re_n$  and with the decrease of  $d_{np}$ .

Andreozzi et al. (2016) examined the effect of different rib shapes on heat transfer and  $Al_2O_3 - H_2O$  nanofluid flow characteristics of a channel. Simulations for different  $\phi$  from 0%, pure  $H_2O$ , to 4% and flow parameter between 20,000 and 60,000 are accomplished. They observed that, rectangular-trapezoidal-geometry rib highest efficiency factor at a pitch-rib-height equivalent to 10. Nassan et al. (2010) experimental studied  $Nu_{rs}$  and  $f_{rs}$  characteristics of  $Al_2O_3 - H_2O$  and  $CuO - H_2O$  nanofluids through a square channel in laminar flow under constant heat flux. Their results indicate that a considerable efficiency factor enhancement has been achieved by both nanofluids likened with base fluid.

Ahmed et al. (2015a) examined the  $Nu_{rs}$  and  $f_{rs}$  in an equilateral triangular channel using joint turbulence promoters and nanofluids. Both numerical and experimental results show a good enhancement in efficiency factor by using turbulence promoter with base fluid. Khoshvaght-Aliabadi et al. (2016) experimentally examined the performance of different metallic nanofluids on average heat transfer and flow friction mechanics in a pin rectangular channel. Their experimentations are performed using 0.1 % mass concentration of  $Cu - H_2O$ ,  $Fe - H_2O$ , and  $Ag - H_2O$  nanofluids. Results indicate that the use of pins inside the duct can noticeably improve the heat transfer performance.

Al-Shamani et al. (2015) carried mathematical optimization of heat transfer due to turbulent flow of nanofluids over rib-groove duct. Their outcomes showed that the average Nusselt number rises as the volume fraction rises and it reductions as the nanoparticle diameter rises. Abdolbaqi et al. (2014) investigated the efficiency factor improvement of nanofluids under turbulent flow over a conventional square duct under uniform heat flux conditions at the upper and lower walls. The results show that the heat transfer rates and wall shear stress increase by increasing the nanofluids volume concentration. Zeinali Heris et al. (2007) experimentally examined the laminar stream forced convection heat transfer of  $Al_2O_3 - H_2O$  nanofluid inside a ribbed tube with uniform wall temperature. Comparison of numerical data of ribbed tubes with plain tube shown that the heat transfer coefficient from 92% to 621% and friction factor from 25% to 241% compared to those obtained in smooth tube.

Wen and Ding (2004) determined that the efficiency factor of nanofluids, made of  $\gamma-Al_2O_3$  nanoparticles and de-ionized water, flowing through a copper round tube in the laminar flow regime. Their results showed considerable improvement of efficiency factor by means of the nanofluids. Xu and Pop (2014) examined the fully developed mixed bioconvection current in a horizontal duct filled with a nanofluid that contains together nanoparticles and gyrotactic micro-organisms. Ahmed et al. (2012) indicate the 2D-laminar flow of various nanofluids flow inside a triangular passage with the existence of turbulence

promoter is numerically examined. Their results show that for the case of  $SiO_2$  at  $\phi = 6\%$  and  $Re = 800$ , it is observed that the average heat transfer is about 50% better than the case of  $Re = 100$ .

Vatani and Mohammed (2013) investigated the effects of various rib-groove forms of arc, triangular and square, dissimilar geometrical parameters and also dissimilar types of nanofluids including  $Al_2O_3, CuO, ZnO$  and  $SiO_2$  on the efficiency factor of stream in a horizontal obstacle-grooved duct are numerically examined. Their outcomes revealed that rectangular obstacle- rectangular groove shows the finest performance having the uppermost efficiency factor. Heyhat et al. (2012) examined the effect of  $Nu_{rs}$  and  $f_{rs}$  of the nanofluids in a round tube with uniform wall temperature under turbulent stream conditions experimentally. Parsazadeh et al. (2013) reported that the nanofluid flow is numerically examined to analyses the outcomes of numerous types of nanoparticles with numerous nanoparticle parameters in a fully detached obstacle roughened duct. The consequences indicate that the highest heat transfer enhancement is achieved with  $SiO_2$  nanofluid and the flow friction not significantly change with by means of dissimilar types of nanoparticles in the base fluid. Vanaki et al. (2014) examined the outcomes of numerous nanofluids on the thermal and stream fields over transversely wavy wall passages with dissimilar phase shifts between the upper and lower wavy walls. Their results indicated that the wavy duct performance was importantly influenced by altering the phase shift and the wavy amplitude.

Ahmed et al. (2013) discussed the laminar turbulent heat transfer of  $CuO - H_2O$  nanofluid in trapezoidal-corrugated duct has been numerically examined. Their outcomes observed that the average  $Nu_{rs}$  improves with growth in nanoparticles volume fraction and with the amplitude of corrugated duct but this improvement accompanied by rises in flow friction. Ahmed et al. (2014) discussed the laminar forced convection stream of  $Al_2O_3 - H_2O$  nanofluid in sinusoidal-curved duct is numerically examined. Their outcomes that the numerical solution is obtained for flow parameter and nanoparticle fractions of volume ranges of 100-800 and 0-5%, respectively. Khdher et al. (2015) investigated experimentally and numerically the effect on  $Nu_{rs}$  and  $f_{rs}$  for  $Al_2O_3 - H_2O$  nanofluid flow inside circumferential roughened round tubes with various obstacle dimensions.

Manca et al. (2012) numerically studied the consequence hydrodynamic and thermal characteristics of turbulent nanofluids flow in a ribbed channel. Ahmed et al. (2015b) investigated experimentally the effect of nanofluid and turbulence promoter compound on average heat transfer and average friction factor in an equilateral triangular channel. Chokphomphun et al. (2015) discussed the influence of winglet vortex generators placed in the core flow area on thermal performance enhancement of a tube heat exchanger. Their results observed that thermal performance for the winglet vortex generators is found to be much superior to that for the wire coil and twisted tape and is in a variety of 1.35-1.59. Mohammed et al. (2012) examined the consequence hydrodynamic and thermal characteristics of turbulent nanofluids flow in a rib-groove duct are numerically studied. Their results that the  $SiO_2$  nanofluid has the supreme average heat transfer compared with other nanofluid types.

Haridas et al. (2015) presented a direct comparison of heat transfer characteristics of  $Al_2O_3$  and  $SiO_2$  nanoparticles on the basis of their relative influence on the thickness of thermal boundary layers, temperature gradients and the resultant heat transfer rates. The results of the study clearly reveal the effectiveness and higher heat transfer characteristics of  $Al_2O_3$  nanoparticles than that of  $SiO_2$  and experimentally demonstrated that the phenomena like increased thermal conductivity, boundary layer disruptions and advection effects primarily control the heat transfer rates in the lower range of Reynolds number.

Abed et al. (2015) numerically studied the fully developed turbulent flow and heat transfer behavior of  $Al_2O_3, CuO, SiO_2$  and  $ZnO$  in trapezoidal channels using nanofluids with different volume fractions (0-4%) and diameters (20-80nm) under constant heat flux. The results indicated that  $SiO_2$  has the highest Nusselt number among the nanofluids.

Enhancement of heat transfer increases with particle volume concentration, but a slight increase in pressure loss with decreasing nanoparticle diameter is also observed.

From this brief review [Navaei et al. (2015); Vanaki and Mohammed (2015); Al-Shamani et al. (2015); Parsazadeh et al. (2013); Vanaki et al. (2014); Ahmed et al. (2013); Manca et al. (2012); Mohammed et al. (2012); Haridas et al. (2015); Abed et al. (2015)] it follows that currently in the field of turbulent heat transfer of nanofluids there are still many unresolved issues. In particular, most of the works do not discuss in principle the issue of thermal-hydraulic efficiency of  $TiO_2 - H_2O$  based nanofluid flow in V-pattern protruded rib square channel on heat transfer and flow friction. The current analysis has been taken up to explore the effect of concentration, particle diameter, protrusion rib size, protrusion rib height, print diameter of rib and angle of attack on the thermal and hydrodynamic ribbed square channel. Extensive experimentations have been conducted to generate data pertaining to heat transfer and flow friction for fully developed turbulent nanofluid flow through V-pattern protrusion square channel.

The main objectives of the present experimental analysis are:

- To investigate the effect of  $TiO_2 - H_2O$  based nanofluid flow in V-pattern protruded rib square channel on heat transfer and flow friction.
- To find out superior thermal hydrodynamic performance of  $TiO_2 - H_2O$  based nanofluid flow in V-pattern protruded rib square channel.
- To develop correlations for Nusselt number and flow friction of  $TiO_2 - H_2O$  based nanofluid flow in V-pattern protruded rib square channel.

## 2. THERMO PHYSICAL PROPERTIES OF NANOFLUID

The following standard equations are used to compute density, specific heat, thermal conductivity and viscosity of the nanofluid.

The density of nanofluid is obtained by (Ho et al., 2008):

$$\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_{np} \quad (1)$$

The specific heat of the nanofluid is obtained by (Ho et al. 2008):

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_{bf} + \phi(\rho C_p)_{np} \quad (2)$$

The effective thermal conductivity of the nanofluid is obtained by following correlations (Vajjha and Das 2009):

$$k_{eff} = k_{static} + k_{Brownian}$$

$$k_{static} = k_{bf} \left[ \frac{(k_{np} + 2k_{bf}) - 2\phi(k_{bf} - k_{np})}{(k_{np} + 2k_{bf}) + \phi(k_{bf} - k_{np})} \right] \quad (3)$$

$$k_{Brownian} = 5 \times 10^4 \beta \phi \rho_{bf} C_{p,bf} \sqrt{\frac{KT}{2\rho_{np}R_{np}}} f(T, \phi) \quad (4)$$

where  $K$ (Boltzmann constant) =  $1.3807 \times 10^{-23} J/K$ ,  $\beta$  represents the fraction of liquid volume which travels with particle and  $T$  is the temperature.

$$\text{Modeling, } f(T, \phi) = (2.8217 \times 10^{-2} \phi + 3.917 \times 10^{-3}) \left( \frac{T}{T_0} \right) + (-3.0669 \times 10^{-2} \phi - 3.91123 \times 10^{-3}) \quad (5)$$

For  $1\% \leq \phi \leq 4\%$  and  $300K < T < 325K, T_0 = 293K$ .

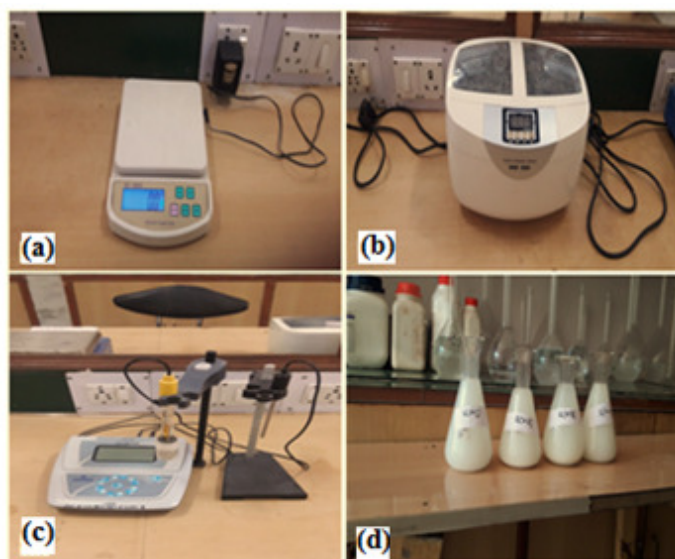
Thermophysical properties of nanoparticle and water are listed in Table.1.

**Table.1** Thermophysical properties of nanoparticle and water.

Type of base fluid and nanoparticles	$\rho(kgm^{-3})$	$k(Wm^{-1}K^{-1})$	$C_p(Jkg^{-1}K^{-1})$	$\mu(Nsm^{-2})$
Water	998.20	0.613	4182	0.001003
$TiO_2$	3940	36.00	710	0.00102

## 3. PREPARATION OF NANOFLUID

Titanium oxide nanofluid preparation must ensure proper dispersion of nanoparticles in the liquid and proper mechanism is needed to attain the stability of the suspension against sedimentation. The size of nanoparticles is 20nm-80nm. The ultrasonic homogenizer, digital weighing machine, pH adjusted apparatus, Nanofluid ready flasks are depicted in Fig. 1 respectively. The digital weighing machine used in the present study is shown in Fig. 1(a). The nanoparticles and base fluid is mixed using mechanical stirrer and immersed in ultrasonic vibration as shown in Fig. 1(b) for 1.5 hours. After making the proper solid/liquid mixture, the flask was kept under ultrasonic vibration for 60 min and there was no sedimentation observed for long time (more than 140 hrs) and there after minor sedimentation was observed for 1 vol. % (1wt %) of  $TiO_2$ -water nanofluid only. The pH value of the base fluid water is adjusted with the addition of small amount of hydrochloric acid. The pH meter used in the present study is shown in Fig. 1(c). The prepared nanofluid flasks are shown in Fig.1(d). The unknown weight of the nano powder is estimated based on the known percentage of volume fraction, density of the particle and density of the water by law of mixtures.



**Fig. 1** Photographic view (a) Digital weighing machine (b) Ultrasonic homogenizer (c) pH adjusted machine (d) Nanofluid prepared flasks.

## 4. ROUGHNESS PARAMETERS

In current experimental analysis the  $TiO_2 - H_2O$  based nanofluid flow in V-type protruded rib square channel is selected; channel width ( $W_c$ )=10mm, channel height ( $H_c$ )=10mm and total length of channel is 340 mm. The length of test section is  $L_t = 108 mm$ . The results are formed on square channel with V-type protruded obstacle having four values of volume fraction( $\phi$ ), four values of particle diameter ( $n_p$ ), four values of relative V-pattern protruded- rib height ( $H_a/D_h$ ), four values of ratio of V-pattern protruded height-to-print diameter( $H_a/d_p$ ), four values of relative V-pattern protruded rib pitch ( $P_a/H_a$ ) and four values of angle-of-attack ( $\alpha_a$ ) respectively. The profile and photographic view of V-type protruded ribs are depicted in Fig. 2 respectively. The values and ranges of variant experimental ribbed parameters are presented in Table 2.

## 5. EXPERIMENTAL SETUP

To analysis the results of V-pattern protruded obstacle square duct on the  $Nu_{rs}$  and  $f_{rs}$  of air stream an experimental setup was intended and made-up accordance with guidelines recommended in ASHRAE standard (Lehtimaki et al. 2002). A schematic diagram of experimental setup is presented in Fig. 3. The experimental arrangement consists of a test

section with installed heater, a control panel, a collecting tank, a circulating pump, a condenser (cooling unit).

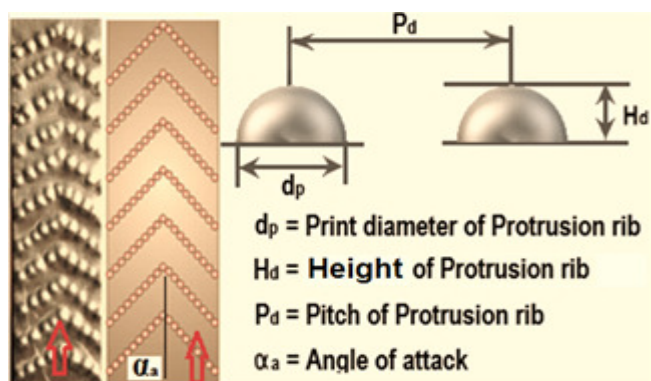


Fig. 2 Discuss V-type protrusion rib shape parameters.

Table.2 Range of experimental parameters.

Sr. No.	Parameters	Range
1.	$\phi$	1 –4%
2.	$n_p$	30 – 45nm
3.	$H_d/D_h$	0.10 – 0.25
4.	$H_d/d_p$	0.8 – 2.0
5.	$P_d/H_d$	2 – 3.5
6.	$\alpha_a$	30° – 75°
7.	$Re$	4000-8000

The length of test section is  $L_t = 108 \text{ mm}$ , with an upstream length of  $200 \text{ mm}$  to confirm a fully developed stream in the test unit. The fluid was forced to the developing unit and then to the test unit in order to get a fully developed stream before incoming the test unit. The entrance unit ensures a hydro dynamically developed stream before incoming the test unit. A fixed heat flux of  $10 \text{ kW/m}^2$  is provided with the support of Nicrome wire as heating elements and variac transformer. T-category thermocouple was used to calculate the temperature reading with the support of data loggers. Seven thermocouples were used, out of which four were located on heated wall, one at inlet and two at the exit. The nanofluid fluid was forced to the developing unit and then to the test unit in order to get a fully developed stream before inflowing the test unit. A switch panel was arranged as the main connector of the scheme to the power supply. The pressure losses over the square heat exchanger duct developed was determined by a U-tube manometer. Submersible pump was used to drive the employed fluid from tank of 10- litre capacity to stream through the test unit. Electrical heater plate was used to heat the top surface of the test unit. The electrical heater was connected to AC power regulator unit which was used to control the input voltage and current to the heaters. A flow meter was linked between the pump and the inlet of the developing unit to calculate the mass flow rate of the fluid. A cooler with  $1 \text{ kW}$  cooling capacity was immersed inside the tank to adjust the temperature of the working fluid enters the test section. Before consecutively each experimentation, enormous cautions were taken to insure suitable performance of entire test setup and there was no leakage at joints in experimental setup. The similar devices propose the consequences under steady states which were expected to be attained while there was no substantial variation in nanofluid flow duct and roughened plate temperature was noted above duration of more than 15 min.

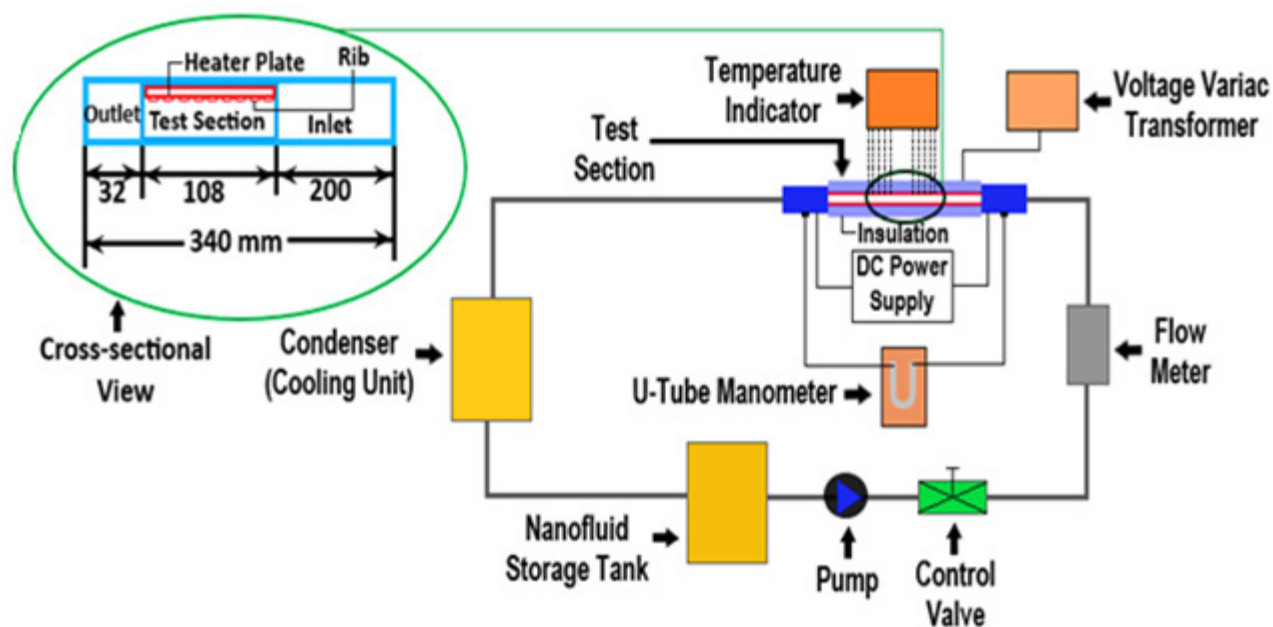


Fig. 3 Schematic diagram of experimental setup.

## 6. DATA REDUCTION

Raw data collected by experimentation are used to estimate the values of dimensionless numbers like Reynolds number ( $Re$ ), Nusselt number ( $Nu_{rs}$ ), friction factor ( $f_{rs}$ ), hydraulic diameter ( $D_h$ ), heat transfer coefficient ( $h_c$ ) and thermal hydrodynamic performance ( $\eta_p$ ) have been determined by employing the following equations as depicted below: The  $Re$  is calculating as:

$$Re = \frac{\rho_{nf} \cdot v_x \cdot d_H}{\mu_{nf}} \quad (6)$$

Here,  $D_h$  is determined as follow:

$$D_h = \frac{4 \cdot W_c \cdot H_c}{2 \cdot (W_c + H_c)} \quad (7)$$

The  $Nu_{rs}$  is determined as:



$$Nu_{rs} = \frac{h_c \cdot L_t}{k_{nf}} \quad (8)$$

where  $k_{nf}$  is the thermal conductivity of nanofluid.

Here,  $h_c$  is determined as:

$$h_c = \frac{Q}{A_p \cdot (T_{pt} - T_{ft})} \quad (9)$$

Here,  $T_{pt}$  is the area of weighted average plate temperature and  $T_{ft}$  is the average fluid temperature respectively.

The  $f_{rs}$  is calculated as:

$$f_{rs} = \frac{2\Delta P D_h}{\rho_{nf} L_t v_x^2} \quad (10)$$

The  $\eta_p$  is determined as:

$$\eta_p = \frac{\left(\frac{Nu_{rs}}{Nu_{ss}}\right)}{\left(\frac{f_{rs}}{f_{ss}}\right)^{1/3}} \quad (11)$$

### 7. VALIDATION OF EXPERIMENTAL SETUP

The results of  $Nu_{ss}$  and  $f_{ss}$  collected from experimental designs for plain square channel has been compared among the results obtained from the Gnielinski correlation equation for  $Nu_{ss}$  depicted in equation (12) and Petukhov correlation equation for  $f_{ss}$  depicted in equation (13) respectively.

The  $Nu_{ss}$  for smooth wall square duct is obtainable by using Gnielinski correlation equation as follow:

$$Nu_{ss} = \frac{\left(\frac{f_{rs}}{8}\right) (Re - 1000) Pr^{0.4}}{1 + 12.7 \left(\frac{f_{ss}}{8}\right)^{1/2} (Pr^{2/3} - 1)} \quad \text{for } 3,000 \leq Re \leq 21,000 \quad (12)$$

The  $f_{ss}$  for smooth wall square channel is obtainable by using Petukhov correlation equation as follow:

$$f_{ss} = (0.079 \ln Re - 1.64)^{-2} \quad (13)$$

The intended results of experimental and predicted outcomes of  $Nu_{ss}$  and  $f_{ss}$  as a function of  $Re$  is presented in Figs. 4 and 5 respectively. In order to show the variation of  $Nu_{ss}$  and  $f_{ss}$  as a function of Reynolds number the error bars are drawn and shown in Figs. 6 and 7. The average absolute deviation among experimental and predicted values of  $Nu_{ss}$  and  $f_{ss}$  has been found to be 4.38% and 3.19% respectively. This reasonably good accuracy among the two sets of values, confirms the correctness of the data being arranged using this experimental setup.

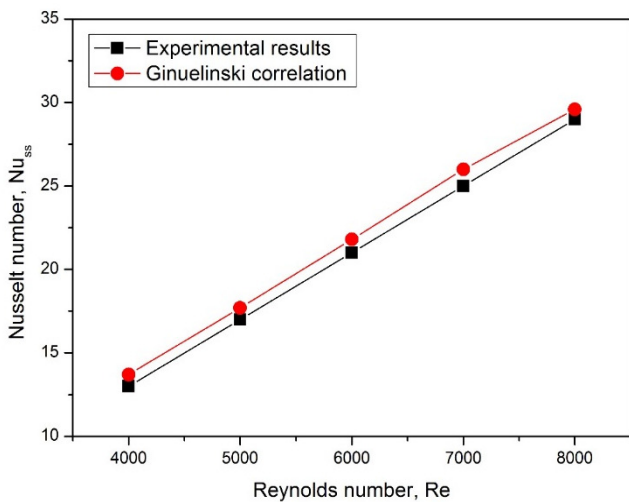


Fig. 4 Comparison of experimental and predicted values of  $Nu_{ss}$  for smooth surface square channel.

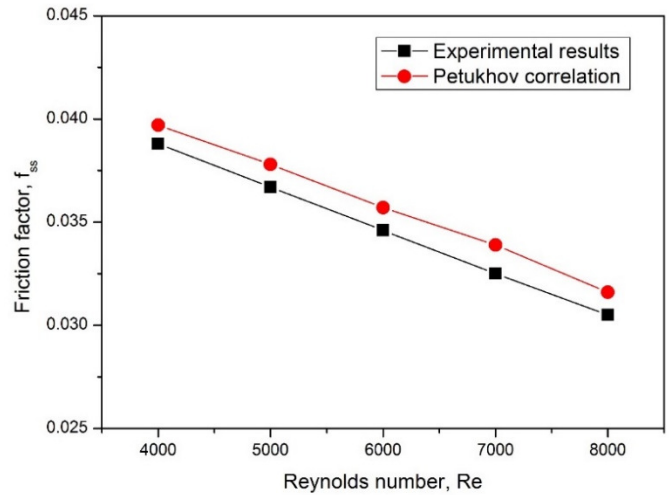


Fig. 5 Comparison of experimental and predicted values of  $f_{ss}$  for smooth surface square channel.

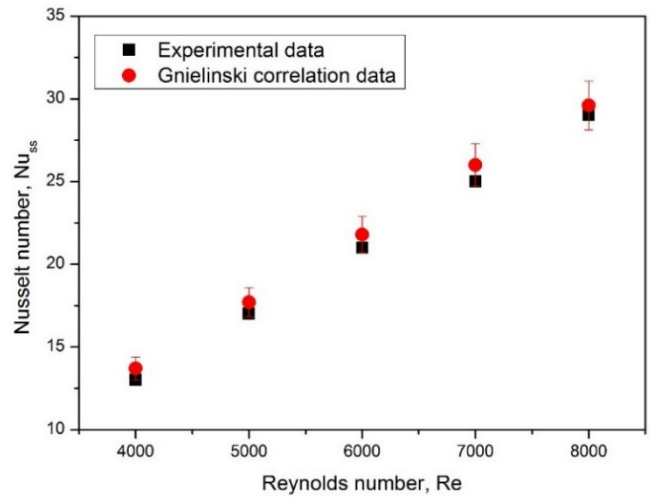


Fig. 6 Error bar analysis of smooth surface square channel for Nusselt number.

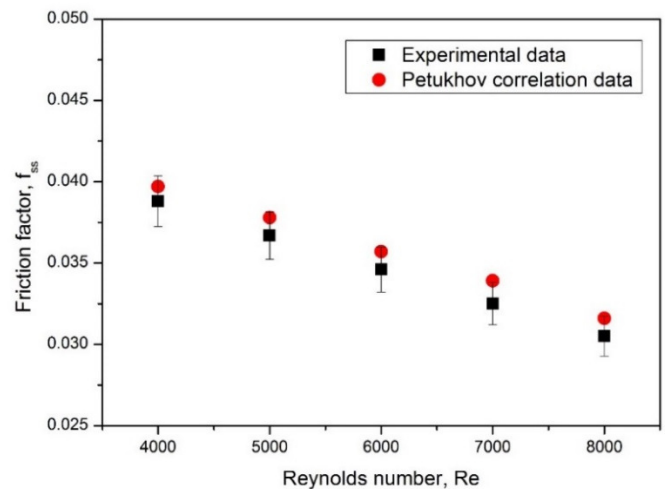
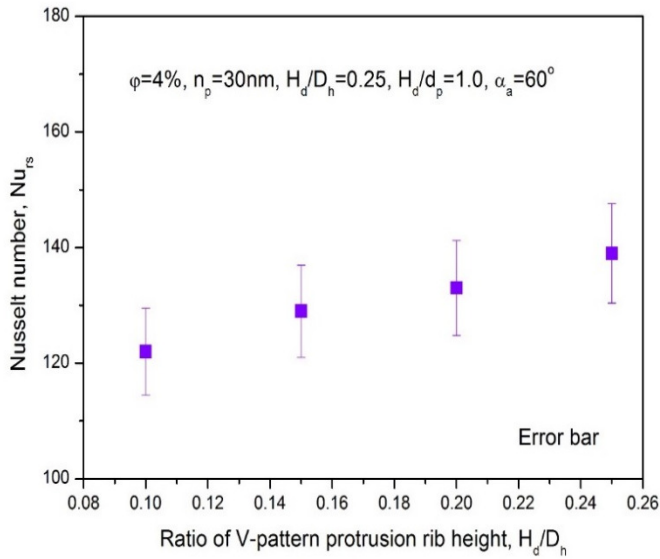
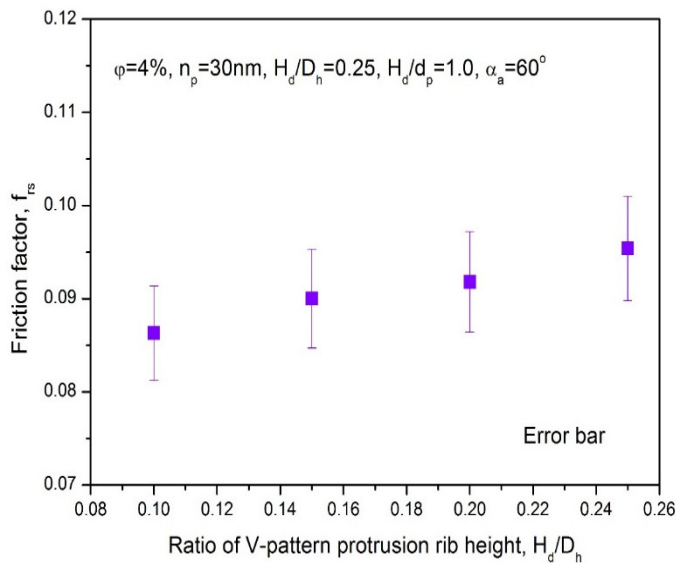


Fig. 7 Error bar analysis of smooth surface square channel for friction factor.



**Fig. 8** Error bar analysis of V-pattern protrusion rib for Nusselt number.



**Fig. 9** Error bar analysis of V-pattern protrusion rib for friction factor.

## 8. UNCERTAINTY ANALYSIS

The measurement uncertainty of various important quantities has been evaluated (Coleman and Steele 2009) and is in control limits. The procedure for computation of uncertainty is discussed below:

The uncertainty in measurement of "y" when the parameter is calculated using specific measured quantities is given by:

$$\frac{\delta y}{y} = \left[ \left( \frac{\delta y}{\delta x_1} \delta x_1 \right)^2 + \left( \frac{\delta y}{\delta x_2} \delta x_2 \right)^2 + \left( \frac{\delta y}{\delta x_3} \delta x_3 \right)^2 + \dots + \left( \frac{\delta y}{\delta x_n} \delta x_n \right)^2 \right]^{0.5} \quad (14)$$

where,  $\delta x_1, \delta x_2, \delta x_3, \dots, \delta x_n$  are the possible error in measurement of  $x_1, x_2, x_3, \dots, x_n$ .

$\delta y$  is absolute uncertainty and  $\frac{\delta y}{y}$  the relative uncertainty.

**Uncertainty in area of heated plate ( $A_p$ )**

$$A_p = W_c \times L_t \quad (15)$$

$$A_p = \left[ \left( \frac{\delta A_p}{\delta L_t} L_t \right)^2 + \left( \frac{\delta A_p}{\delta W_c} W_c \right)^2 \right]^{0.5} \quad (16)$$

$$\frac{\delta A_p}{A_p} = \left[ \left( \frac{\delta L_t}{L_t} \right)^2 + \left( \frac{\delta W_c}{W_c} \right)^2 \right]^{0.5} \quad (17)$$

**Uncertainty in area of flow ( $A_f$ )**

$$A_f = W_c \times H_c \quad (18)$$

$$A_f = \left[ \left( \frac{\delta A_f}{\delta W_c} W_c \right)^2 + \left( \frac{\delta A_f}{\delta H_c} H_c \right)^2 \right]^{0.5} \quad (19)$$

$$\frac{\delta A_f}{A_f} = \left[ \left( \frac{W_c \times \delta H_c}{W_c \times H_c} \right)^2 + \left( \frac{H_c \times \delta W_c}{W_c \times H_c} \right)^2 \right]^{0.5} \quad (20)$$

**Uncertainty in measurement of Hydraulic diameter ( $D_h$ )**

$$D_h = \frac{4 \times (W_c \times H_c)}{2 \times (W_c + H_c)} = 2(W_c H_c)(W_c + H_c)^{-2} \quad (21)$$

$$\frac{\delta D_h}{D_h} = [2(W_c H_c)(-1)(W_c + H_c)^{-2}] + [(W_c + H_c)^{-1}(2W_c)] \quad (22)$$

**Uncertainty in useful heat gain ( $Q_u$ )**

$$Q_u = m_a c_p \Delta T \quad (23)$$

$$\frac{\delta Q_u}{Q_u} = \left[ \left( \frac{\delta m_a}{m_a} \right)^2 + \left( \frac{\delta c_p}{c_p} \right)^2 + \left( \frac{\delta \Delta T}{\Delta T} \right)^2 \right]^{0.5} \quad (24)$$

**Uncertainty in heat transfer coefficient ( $h_t$ )**

$$h_c = \frac{Q_u}{A_p \cdot (T_{pt} - T_{ft})} = \frac{Q_u}{A_p \times \Delta T_f} \quad (25)$$

$$\frac{\delta h_c}{h_c} = \left[ \left( \frac{\delta Q_u}{Q_u} \right)^2 + \left( \frac{\delta A_p}{A_p} \right)^2 + \left( \frac{\delta \Delta T_f}{\Delta T_f} \right)^2 \right]^{0.5} \quad (26)$$

**Uncertainty in Nusselt number ( $Nu_{rs}$ )**

$$Nu_{rs} = \frac{h_c \cdot L_t}{k_{nf}} \quad (27)$$

$$\frac{\delta Nu_{rs}}{Nu_{rs}} = \left[ \left( \frac{\delta D_h}{D_h} \right)^2 + \left( \frac{\delta h_c}{h_c} \right)^2 + \left( \frac{\delta k_{nf}}{k_{nf}} \right)^2 \right]^{0.5} \quad (28)$$

**Uncertainty in Reynolds Number (Re)**

$$Re = \frac{\rho_{nf} \cdot v_x \cdot D_h}{\mu_{nf}} \quad (29)$$

$$\frac{\delta Re}{Re} = \left[ \left( \frac{\delta D_h}{D_h} \right)^2 + \left( \frac{\delta v_x}{v_x} \right)^2 + \left( \frac{\delta \rho_{nf}}{\rho_{nf}} \right)^2 + \left( \frac{\delta \mu_{nf}}{\mu_{nf}} \right)^2 \right]^{0.5} \quad (30)$$

**Uncertainty in friction factor ( $f_{rs}$ )**

$$f_{rs} = \frac{2\Delta P_0 D_h}{\rho_{nf} L_t v_x^2} \quad (31)$$

$$\frac{\delta f_{rs}}{f_{rs}} = \left[ \left( \frac{\delta D_h}{D_h} \right)^2 + \left( \frac{\delta v_x}{v_x} \right)^2 + \left( \frac{\delta L_t}{L_t} \right)^2 + \left( \frac{\delta \rho_{nf}}{\rho_{nf}} \right)^2 + \left( \frac{\delta \Delta P_0}{\Delta P_0} \right)^2 \right]^{0.5} \quad (32)$$

**Uncertainty in thermal hydraulic performance parameter ( $\eta_p$ )**

$$\eta_p = \frac{Nu_{rs}}{Nu_{ss}} \left( \frac{f_{rs}}{f_{ss}} \right)^{0.33} \quad (33)$$

$$\frac{\delta \eta_p}{\eta_p} = \left[ \left( \frac{\delta Nu_{rs}}{Nu_{rs}} \right)^2 + \left( \frac{\delta f_{rs}}{f_{rs}} \right)^2 \right]^{0.5} \quad (34)$$

Based on the examined of the errors in the experimental measurements using different instruments, the uncertainties in the calculated value of Nusselt number and friction factor have been estimated as  $\pm 6.18\%$  and  $\pm 5.89\%$ , respectively. The variation of Nusselt number and friction factor as a function of ratio of V-pattern

protrusion rib height the error bars are drawn and shown in Figs. 8 and 9.

## 9. RESULTS AND DISCUSSIONS

The consequence of V-pattern protruded rib and flow parameters on heat transfer rates to the  $\text{TiO}_2 - \text{H}_2\text{O}$  based nanofluid with square channel is discussed in this section. The consequence of V-pattern protruded rib profile on heat transfer and pressure drop has been presented for dissimilar factors like  $\varphi$ ,  $n_p$ ,  $H_d/D_h$ ,  $H_d/d_p$ ,  $P_d/H_d$  and  $\alpha_a$ .

### 9.1 HEAT TRANSFER AND FLUID FLOW

The schemes for Nusselt number as significance  $\varphi$  for dissimilar values of  $Re$  are depicted in Fig. 10(a) and the remaining parameters are constant such as;  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$ . The above graph depicts that  $Nu_{rs}$  increased due to increase in  $\varphi$  from 1.0% to 4.0% and attains the maximum resultant values at  $\varphi = 4\%$ , and after that enhancement the data plotted in graph starts to decrease. It has been found that in each data of  $\varphi$  the V-type protruded obstacle wall generates superior heat transfer than that of without V-type protruded obstacle surface square duct.

The secondary stream has two variant types of revolving vortices which transmits the cold nano fluid from inner central part region to the protrusion among V-pattern protruded obstacle wall and these streams combined among the major stream. Such internal flow  $\text{TiO}_2 - \text{H}_2\text{O}$  based nano fluid mixed among the inner flow, effect stream reconnection and retransmission within the V-pattern protruded obstacle and the suspended boundary conditions a developed to the downstream of reattachment regions. When  $Re$  raised the width of boundary layer reduced due to reduction in convective resistance and hence  $Nu_{rs}$  is enhanced correspondingly. The V-type protrusion ribs produce a strong secondary stream at the back of limbs and sustains the turbulence accumulation while  $\text{TiO}_2 - \text{H}_2\text{O}$  nano fluid detach from single V-shape protrusion ribs and mixing with the main flow. The effect of  $Nu_{rs}$  with  $Re$  for variant data of  $n_p$  is presented in Fig. 10 (b). The values of other roughness parameters are fixed such as;  $\varphi = 4.0\%$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$ . It is obtained that  $Nu_{rs}$  raises with raise in the data of  $n_p$  for every value of  $Re$  due to the expanded flow velocity which is directed to maximum turbulence, hence resulting an enhancement in  $Nu_{rs}$ . Fig. 10(c) demonstrations the variation of Nusselt number as a function of  $Re$  for different data  $H_d/D_h$  and the values of other roughness parameters are kept such as  $\varphi = 4.0\%$ ,  $n_p = 30$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$ .

It is clearly seen that the  $Nu_{rs}$  increase with increase in  $H_d/D_h$ . It is believed that an artificial roughness on the heat transfer surface in the form of V-pattern protrusion rib creates local wall turbulence or breaks the laminar sub layer due to flow separation and reattachment between the consecutive ribs, which reduces thermal resistance and greatly enhance rate of heat transfer. For each  $Re$ , the highest results of  $Nu_{rs}$  are achieved at  $H_d/D_h = 0.25$ , whereas the smaller resultant values of  $Nu_{rs}$  are realised to occur at  $H_d/D_h = 0.10$  for the range of parameters investigated. Maximum enhancement in the heat transfer of four times as compared to a smooth plate is observed for the  $H_d/D_h$  value of 0.25 at  $Re$  of 8,000.

The deviations of  $Nu_{rs}$  as a function of  $Re$  for variant values of  $H_d/d_p$  are showed in Fig. 10(d). The remaining experimental factors like  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  are fixed. The above graph depicts that  $Nu_{rs}$  increases with increase in the values of  $H_d/d_p$  for all data of  $Re$  due to elevated flow which is prone to highest turbulence, therefore providing an optimal increase in  $Nu_{rs}$ . The maximum  $Nu_{rs}$  is achieved at  $H_d/d_p = 1.0$ , after this enhancement in  $Nu_{rs}$  it begins to reduce with increase in the data of  $H_d/d_p$  and least value of  $Nu_{rs}$  is obtained at  $H_d/d_p = 2.0$  respectively.

The  $Nu_{rs}$  is observed for increase in  $H_d/d_p$  from 0.8 to 1.0, however, Nusselt number increased considerably for  $H_d/d_p$  value of 1.0

and decreased for further increase in the value to 1.3. Further increase in the  $H_d/d_p$  value to 1.3, causes decrease in Nusselt number values. This may be due to the fact that for ratio of V-pattern protrusion height to print diameter values much more than the thickness of the laminar sublayer, increase in the turbulence is more prominent in the core flow rather than within the boundary layer, which is not much beneficial for enhancing the convective heat transfer from the heated plate. Maximum enhancement in the heat transfer of four times as compared to a smooth plate is observed for the  $H_d/d_p$  value of 1.0 at  $Re$  of 8,000.

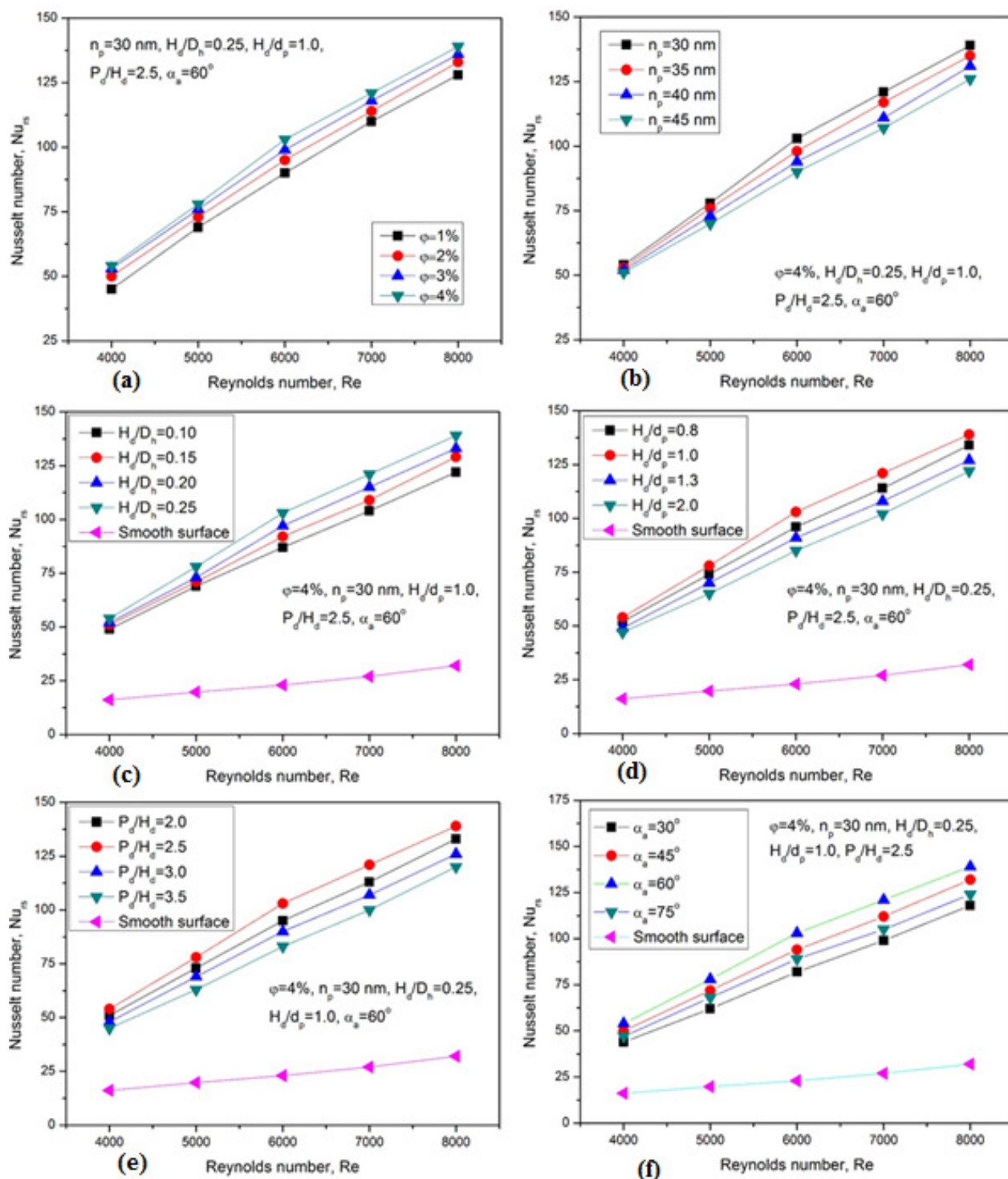
The dissimilarity of  $Nu_{rs}$  as a meaning of  $Re$  for different data of  $P_d/H_d$  are presented in Fig. 10(e). The values of other roughness parameters are fixed such as  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$  and  $\alpha_a = 60^\circ$ . The above plot depicts that  $Nu_{rs}$  rise due to rise the value of  $P_d/H_d$  and attain a maximum at  $P_d/H_d = 2.5$ , and after that rise the value plotted in graph starts to decrease. It can be seen that from graph the  $Nu_{rs}$  attain maxima corresponding to  $P_d/H_d = 2.5$  and on either side of this value, decrease in  $Nu_{rs}$  has been observed. Maximum heat transfer has been found to occur in the vicinity on a reattachment point. It is reasonable to expect that a similar effect can be produced by decreasing  $P_d/H_d$  for a fixed value of relative roughness height. For  $P_d/H_d$  value considerably less than about 2.5, the reattachment will not occur at all resultant in the decrease of heat transfer enhancement.

For higher values of  $P_d/H_d = 3.0$  and  $3.5$ ,  $Nu_{rs}$  value starts to decline sharply, because number of roughness elements becomes very less at these pitch values and hence overall number of reattachment points decreases. Higher pitch values also allow nanofluid to move longer after reattachment till it encounters the next rib and thereby leading to re-development of the laminar sublayer on some portion of the heated plate surface. Re-development of boundary layer adversely affects the heat transfer from the heated plate and hence decrease the heat transfer rate.

Fig. 10(f) demonstrations the variation of  $Nu_{rs}$  with  $Re$  for different data of  $\alpha_a$  and the other V-pattern combined with protruded rib parameter fixed such as  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$  and  $P_d/H_d = 2.5$  respectively. The  $Nu_{rs}$  raises because of the rise in  $\alpha_a$  and attains the uppermost value correspondent to flow attack angle of  $60^\circ$ , and after that increment it decreases due to increase in the data of  $\alpha_a$ . From graph at all  $Re$ , the highest heat transfer is observed at  $\alpha_a = 60^\circ$ , and it reduces on both sides of this value. This variation in  $Nu_{rs}$  is caused by interaction of secondary flow along the V-pattern protrusion rib and boundary layer on upstream side of the V-pattern rib. The boundary layer is done to main flow with the roughened surface and originates from flow reattachment point between the ribs up to the succeeding downstream rib. The strength of secondary flow along the rib changes with change in values of flow attack angle. For the present study, the value of  $Nu_{rs}$  has been observed to be highest at  $\alpha_a = 60^\circ$ .

Application of V-pattern combined with protrusion rib roughness is supposed to result in more complicated flow patterns near the wall as shown in Fig. 11. It is believed that the flow patterns responsible for the enhancement of heat transfer are 1) reattachment of the primary flow behind the V-pattern protrusion rib 2) acceleration of the primary flow through the protrusions. 3) generation of the counter rotating secondary flow vortices across the cross section of the square duct. Relative contribution of these factors towards the heat transfer improvement depends upon the geometrical parameters of the V-pattern protrusion rib roughness.

On the other hand, presence of the recirculation region behind the V-pattern protrusion rib lowers the heat transfer from the heated plate. Shaping of an extended-inclined rib into V-type obstacle helps in the creation of two leading ends (where heat transfer rate is high) and a single trailing end (where heat transfer rate is low) resultant in much larger increase of heat transfer. V-shaped ribs form two secondary stream cells as compared to one in case of an inclined rib resultant in higher efficiency factor in case of V-shaped rib as shown in Fig. 11.



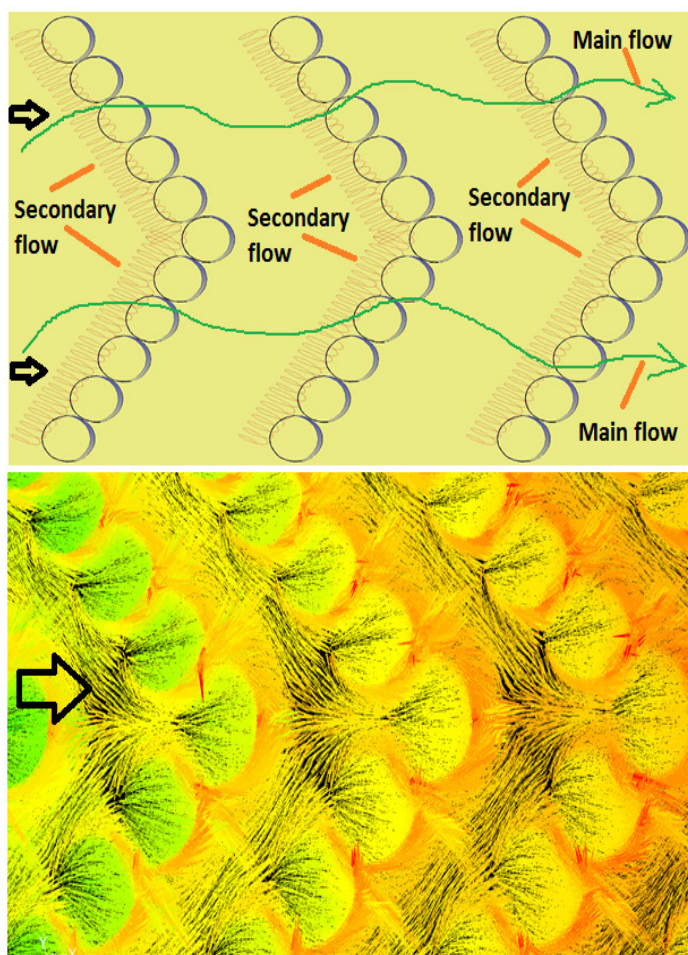
**Fig. 10** (a) Effect of  $\phi$  on  $Nu_{rs}$  (b) Effect of  $n_p$  on  $Nu_{rs}$  (c) Effect of  $H_d/D_h$  on  $Nu_{rs}$  (d) Effect of  $H_d/d_p$  on  $Nu_{rs}$  (e) Effect of  $P_d/H_d$  on  $Nu_{rs}$  (f) Effect of  $\alpha_a$  on  $Nu_{rs}$ .

The roughness applied in the form of V-type protrusion ribs on a heated surface significantly improves  $Nu_{rs}$  from the excited surface of V-type  $TiO_2 - H_2O$  based nano fluid square channel; however a significant increment takes place in  $f_{rs}$  losses. In the present experimental examination, it has been found that how  $f_{rs}$  characteristics of  $TiO_2 - H_2O$  nano fluid square channel were effected while V-type protrusion ribs roughness parameter  $\phi$  was varied and operating parameters are constant  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$  and

$P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  were reserved constant as shown in Fig. 12(a). The plot depicts that  $f_{rs}$  alters among  $Re$  for different data of  $\phi$  and hence  $f_{rs}$  improves with decrease in  $Re$  for each case. With raise in  $\phi$  the data of  $f_{rs}$  increases and obtains a highest value like  $\phi$  value of 4% in the range of operating parameters. Several of the vortices increases the mixing of  $TiO_2 - H_2O$  nano fluid and because of this  $Nu_{rs}$  enhanced. So, the  $Nu_{rs}$  increases from the target plate to nano fluid and these vortices also improve the highest  $Nu_{rs}$  during stream across the nano



fluid mini square passage. The quantity of vortices is higher in case of  $\varphi = 4\%$  as compared to  $\varphi = 3\%$ . Therefore, in case of  $\varphi = 4\%$  TiO<sub>2</sub> – H<sub>2</sub>O nano fluid and vortices mixing is higher, that gives a highest  $f_{rs}$ . The variation of  $f_{rs}$  with  $Re$  for different data of  $n_p$  has been presented in Fig. 12(b). The remaining rib roughness parameters like  $\varphi = 4\%$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 3.5$  and  $\alpha_a = 60^\circ$  are fixed. It can be observed that  $f_{rs}$  rises with rise in  $n_p$  and the highest rate of  $f_{rs}$  correspondent to  $n_p$  will be equal to 30. The reason behind this is that due to rise in range of  $n_p$ , the TiO<sub>2</sub> – H<sub>2</sub>O nano fluid spread out up to a huge area into the core of stream, results an enhancement in the data of  $f_{rs}$ . Fig. 12(c) presents the effect of  $Re$  on  $f_{rs}$  for various values of  $H_d/D_h$ . The remaining rib roughness parameters like  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  are fixed. It has been observed that the highest value of  $f_{rs}$  at  $H_d/D_h = 0.25$  and least value of  $f_{rs}$  at  $H_d/D_h = 0.10$ .



**Fig. 11** Secondary flow pattern in V-type protruded ribbed square channel.

The distinction of  $f_{rs}$  with  $Re$  for different value of  $H_d/d_p$  is presented in Fig. 12(d) and the remaining of the roughness parameters like  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  were constant. It clearly shows that for individual value of  $H_d/d_p$ ,  $f_{rs}$  increases and the highest value for  $f_{rs}$  correspondent to  $H_d/d_p$  was achieved at a value of 1.0. It may be noted that friction factor trends exactly complement the trend of Nusselt number i.e., Nusselt number is high when turbulence level in the flow is high and increase in turbulence

increases the friction factor. Friction is low at  $H_d/d_p$  value of 2.0 since proper flow reattachment does not take place at such a low rib height (with increase protrusion rib print diameter) of nanofluid flow over the ribs with low separation and reattachment between the upstream and downstream V-pattern protrusion rib. Flow friction increases sharply at  $H_d/d_p$  value of 1.0 due to increase in turbulence owing to the onset of separation-reattachment flow pattern.

Fig. 12(e) depicted the characteristics of  $f_{rs}$  with  $Re$  for different data  $P_d/H_d$  and the rib roughness parameters such as  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$  and  $\alpha_a = 60^\circ$  were fixed. It depicts that for individual value of  $P_d/H_d$ ,  $f_{rs}$  increases and the highest value for  $f_{rs}$  correspondent to  $P_d/H_d$  was achieved at a value of 2.5. It may be noted that friction factor trends exactly complement the trend of Nusselt number i.e., Nusselt number is high when turbulence level in the flow is high and increase in turbulence increases the friction factor. Friction is low at  $P_d/H_d$  value of 2.0, since proper flow reattachment does not take place at such a low value of  $P_d/H_d$  and nanofluid flows over the ribs without touching the heated plate between the upstream and downstream ribs. Flow friction increases sharply at  $P_d/H_d$  value of 2.5 due to increase turbulence owing to the onset of separation-reattachment flow pattern. For  $P_d/H_d$  values of 3.0 and 3.5 flow friction decreases due to decrease in the number of roughness elements per unit length of the heated plate. Further increase in the  $P_d/H_d$  value of 3.0 and 3.5 results in the drop-in friction factor, because of decrease in the number of roughness elements per unit length of heated plate start dominating the influence of the V-pattern protrusion ribs hence friction factor start decreasing.

The difference of  $f_{rs}$  with  $Re$  for different data of  $\alpha_a$  and remaining of the roughness parameters like  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$  and  $P_d/H_d = 2.5$  were kept as shown in Fig. 12(f). It has been obtained that for each value of  $\alpha_a$ ,  $f_{rs}$  reduces due to rise in  $Re$  and an enhancement is observed in  $f_{rs}$  at a value of  $\alpha_a = 60^\circ$ , however after this increment it begins to decrease with rise in  $\alpha_a$  and attains highest possible data correspondent to  $\alpha_a$  of  $60^\circ$ .

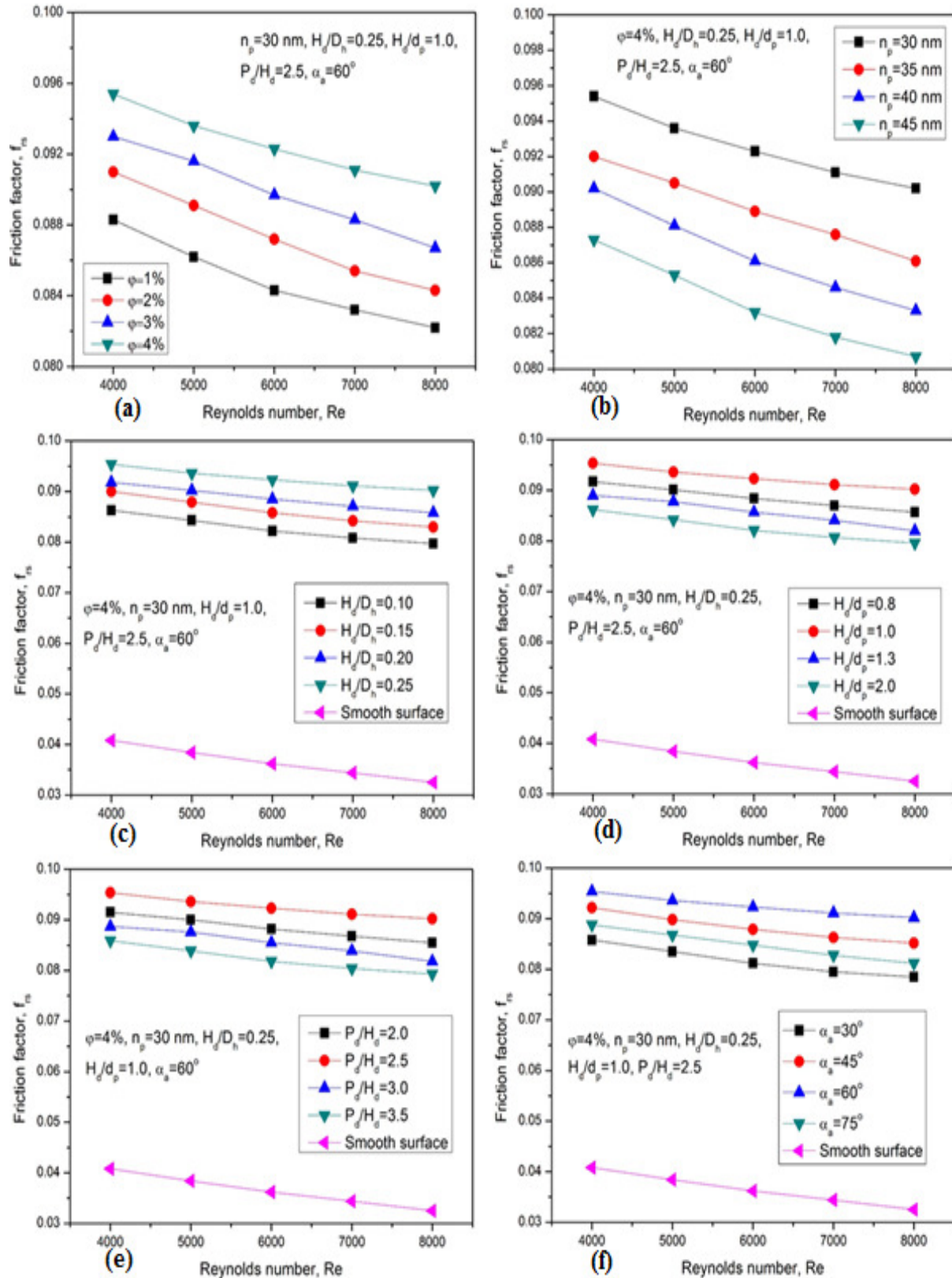
It is believed that the combined effect of all the other fixed geometry parameters contributes toward the favorable effect of the secondary flows along the V-pattern with combined protrusion rib for better mixing and fast heat transfer for  $\alpha_a$  of  $60^\circ$ . Further increase in the V-pattern rib inclination to  $75^\circ$  results in considerable decrease in the strength of secondary flow vortices moving along the V-pattern protrusion ribs, because rib inclination approaches  $90^\circ$  for which vortices behind the rib becomes almost stationary leading to decrease in the friction factor.

## 9.2 THERMAL HYDRODYNAMIC PERFORMANCE

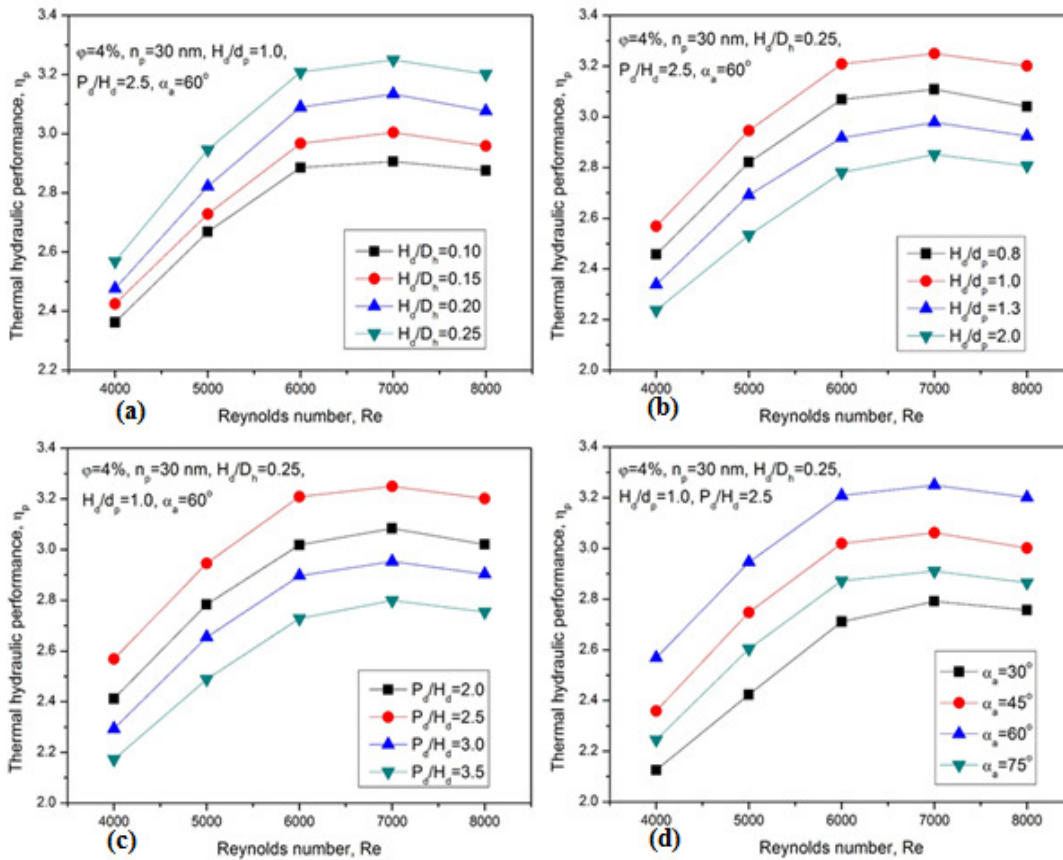
Experimental outcomes pertinent to protruded with combined V-pattern roughness shapes for the present analysis manifest significant enhancement in heat transfer rates with corresponding rise in pressure drop. To accomplish such function of concurrent consideration of  $\eta_p$ , (Webb and Eckert 1972) presents a thermal hydrodynamic performance parameters  $\eta_p$  deciding the  $Nu_{rs}$  via  $\varphi$ ,  $n_p$ ,  $H_d/D_h$ ,  $H_d/d_p$ ,  $P_d/H_d$  and  $\alpha_a$  of V-pattern protruded ribs per unit propelling power estimated among  $Nu_{rs}$  for the perfectly formed turbulent stream in the passage among smooth features are presented as follow:

$$\eta_p = (Nu_{rs}/Nu_{ss})/(f_{rs}/f_{ss})^{0.33} \quad (35)$$

Fig.13(a-d) indicates the effect of protruded with combined V-pattern rib parameters on  $(Nu_{rs}/Nu_{ss})/(f_{rs}/f_{ss})^{0.33}$  as a function of  $Re$ . The value of protruded rib with combined V-pattern surface factors in which  $(Nu_{rs}/Nu_{ss})/(f_{rs}/f_{ss})^{0.33}$  values were achieved to be supreme is depicted in Fig.13. The uppermost outcomes of  $(Nu_{rs}/Nu_{ss})/(f_{rs}/f_{ss})^{0.33}$  are attained to be 3.21 corresponding to  $\varphi = 4\%$ ,  $n_p = 30$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  for whole value of  $Re$  selected for current examination. The greatest results of thermal hydrodynamic performance parameter for different factors are represents in Table.3.



**Fig. 12** (a) Effect of  $\varphi$  on  $f_{rs}$  (b) Effect of  $n_p$  on  $f_{rs}$  (c) Effect of  $H_d/D_h$  on  $f_{rs}$  (d) Effect of  $H_d/d_p$  on  $f_{rs}$  (e) Effect of  $P_d/H_d$  on  $f_{rs}$  (f) Effect of  $\alpha_a$  on  $f_{rs}$ .



**Fig. 13** (a) Effect of  $H_d/D_h$  on  $\eta_p$  (b) Effect of  $H_d/d_p$  on  $\eta_p$  (c) Effect of  $P_d/H_d$  on  $\eta_p$  (d) Effect of  $\alpha_a$  on  $\eta_p$ .

**Table. 3** V-type protruded rib parameter corresponding to maximum of  $\eta_p = (Nu_{rs}/Nu_{ss})/(f_{rs}/f_{ss})^{0.33}$ .

Sr. No.	Fixed parameters	Varied parameter	Thermal hydrodynamic performance
1.	$\varphi = 4\%, n_p = 30, H_d/d_p = 1.0, P_d/H_d = 2.5$ and $\alpha_a = 60^\circ$	$H_d/D_h$	3.21
2.	$\varphi = 4\%, n_p = 30, H_d/D_h = 0.25, P_d/H_d = 2.5$ and $\alpha_a = 60^\circ$	$H_d/d_p$	3.21
3.	$\varphi = 4\%, n_p = 30, H_d/D_h = 0.25, H_d/d_p = 1.0$ and $\alpha_a = 60^\circ$	$P_d/H_d$	3.21
4.	$\varphi = 4\%, n_p = 30, H_d/D_h = 0.25, H_d/d_p = 1.0$ and $P_d/H_d = 2.5$	$\alpha_a = 60^\circ$	3.21

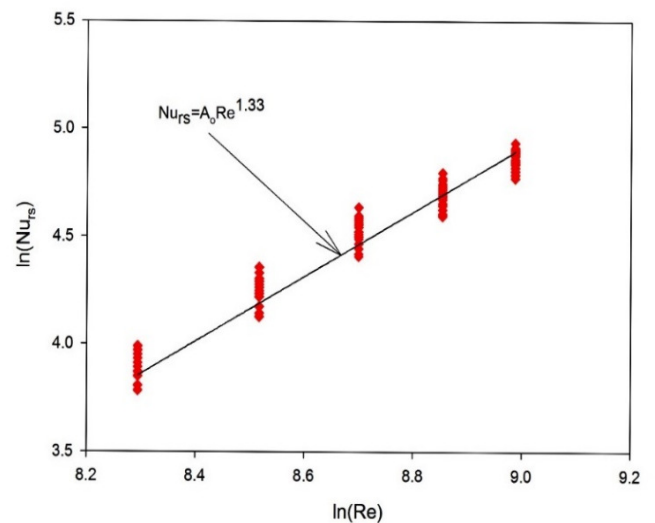
### 10. DEVELOPMENT OF CORRELATIONS FOR $Nu_{rs}$ AND $f_{rs}$

The experimental data of the investigation reveals that the  $Nu_{rs}$  and  $f_{rs}$  are strong functions of the system and the operating parameters. The relationship of  $Nu_{rs}$  and  $f_{rs}$  with the governing parameters can be functionally written as

$$Nu_{rs} = f_1(Re, \varphi, n_p, H_d/D_h, H_d/d_p, P_d/H_d, \alpha_a/60) \quad (36)$$

$$f_{rs} = f_2(Re, \varphi, n_p, H_d/D_h, H_d/d_p, P_d/H_d, \alpha_a/60) \quad (37)$$

The data collected have been utilized to develop statistical correlations for the Nusselt number and friction factor using regression analysis. The development of correlation starts with plotting  $\ln(Nu_{rs})$  and  $\ln(Re)$  for all the data points. Functional relationship between  $Nu_{rs}$  and  $Re$  is obtained by regression analysis as shown in Fig. 14.



**Fig. 14** Plot of  $\ln(Nu_{rs})$  as function of  $\ln(Re)$  for all the experimental data.



By curve fitting a straight line is generated through these points and is given by

$$Nu_{rs} = A_0 Re^{1.33} \quad (38)$$

where  $A_0$  is functionally depending on other system parameter i.e.  $\phi$ . The value of  $(Nu_{rs}/Re^{1.33} = A_0)$  is plotted on the log-log scale, as a function of  $\phi$  as shown in Fig. 15.

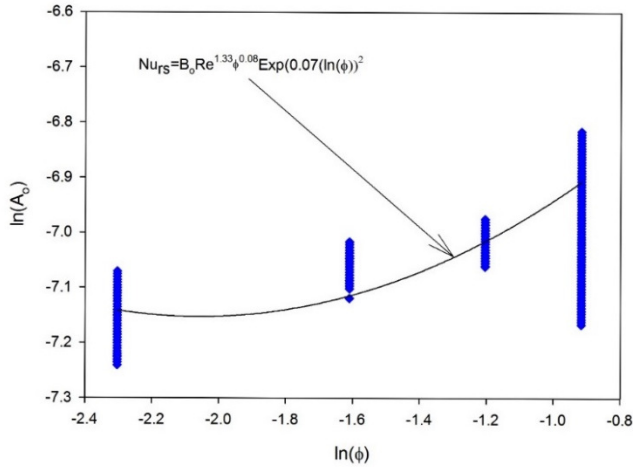


Fig. 15 Plot of  $\ln(A_0)$  as function of  $\ln(\phi)$ .

A second-order polynomial equation using regression analysis is generated and is given by

$$\ln[Nu_{rs}/Re^{1.33}] = \ln B_0 + A_{11} \ln(\phi) + A_{12} (\ln(\phi))^2 \quad (39)$$

The above equation can be rearranged as

$$[Nu_{rs}/Re^{1.33}] = B_0(\phi)^{0.08} \exp[0.07(\ln(\phi))^2] \quad (40)$$

where  $B_0$  is the function of system parameter  $n_p$ . A log-log scale plot between  $B_0$ , and  $n_p$  is shown in Fig. 16. A curve fitting by second-order polynomial is given by:

$$\ln \left[ \frac{Nu_{rs}}{Re^{1.33} \phi^{0.08} \exp[0.07(\ln(\phi))^2]} \right] = C_0 n_p^{1.25} \exp(-0.21(\ln(n_p))^2) \quad (41)$$

where  $C_0$  is the function of  $H_d/D_h$  and is plotted on a log-log scale.

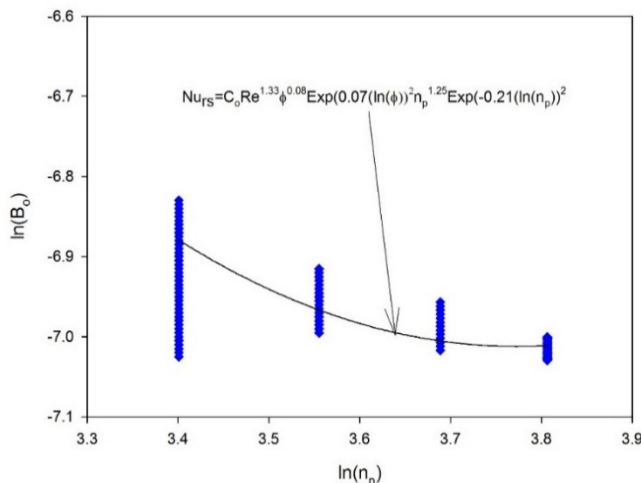


Fig. 16 Plot of  $\ln(B_0)$  as function of  $\ln(n_p)$ .

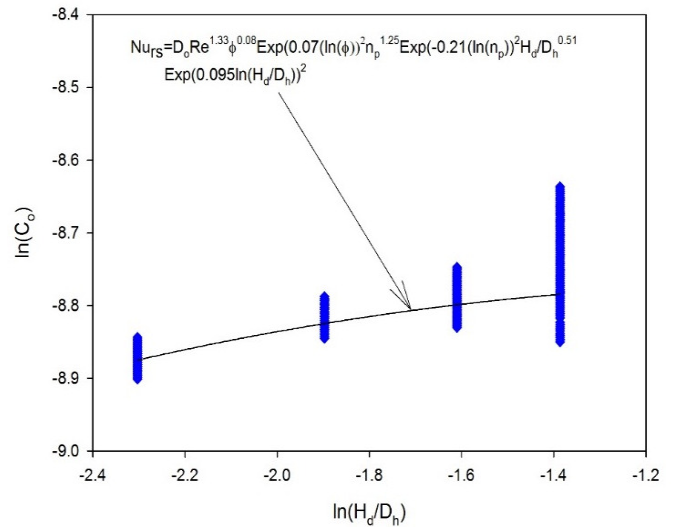


Fig. 17 Plot of  $\ln(C_0)$  as function of  $\ln(H_d/D_h)$ .

Fig. 17 shows a regression analysis second-order polynomial curve fitting and the equation is given by

$$\ln \left[ \frac{Nu_{rs}}{Re^{1.33} \phi^{0.08} \exp[0.07(\ln(\phi))^2] n_p^{1.25} \exp(-0.21(\ln(n_p))^2)} \right] = D_0 \left( \frac{H_d}{D_h} \right)^{0.51} \exp(0.095(\ln(H_d/D_h)))^2 \quad (42)$$

where  $D_0$  is the function of  $H_d/d_p$  and is plotted on a log-log scale.

Fig. 18 shows a regression analysis second-order polynomial curve fitting and the equation is given by

$$\ln \left[ \frac{Nu_{rs}}{Re^{1.33} \phi^{0.08} \exp[0.07(\ln(\phi))^2] n_p^{1.25} \exp(-0.21(\ln(n_p))^2)} \right] = D_0 \left( \frac{H_d}{d_p} \right)^{-0.07} \exp(-0.19(\ln(H_d/d_p)))^2 \quad (43)$$

where  $E_0$  is the function of  $P_d/H_d$  and is plotted on a log-log scale.

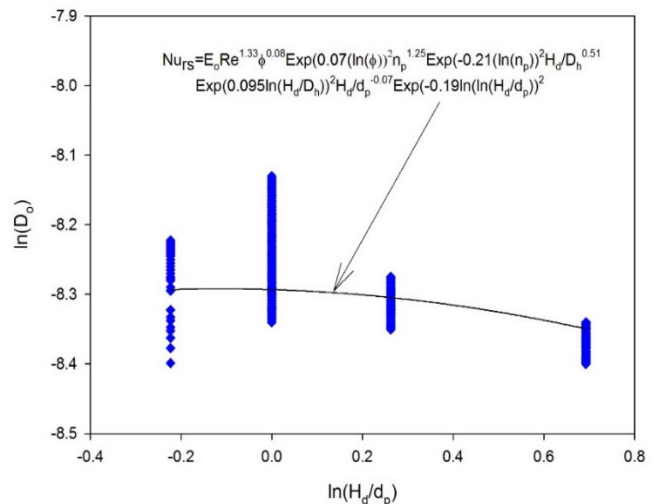


Fig. 18 Plot of  $\ln(D_0)$  as function of  $\ln(H_d/d_p)$ .



Fig. 19 shows a regression analysis second-order polynomial curve fitting and the equation is given by

$$\ln \left[ \frac{Nu_{rs}}{Re^{1.33} \phi^{0.08} \text{Exp}[0.07(\ln(\phi))^2] n_p^{1.25} \text{Exp}(-0.21(\ln(n_p))^2)} \right. \\ \left. \frac{(H_d/D_h)^{0.51} \text{Exp}(0.095(\ln(H_d/D_h)))^2}{(H_d/d_p)^{-0.07} \text{Exp}(=0.19(\ln(H_d/d_p)))^2} \right] \\ = E_0 \left( \frac{P_d}{H_d} \right)^{2.35} \text{Exp}(-1.35(\ln(P_d/H_d))^2) \quad (44)$$

where  $F_0$  is the function of  $\alpha_a$  and is plotted on a log-log scale.

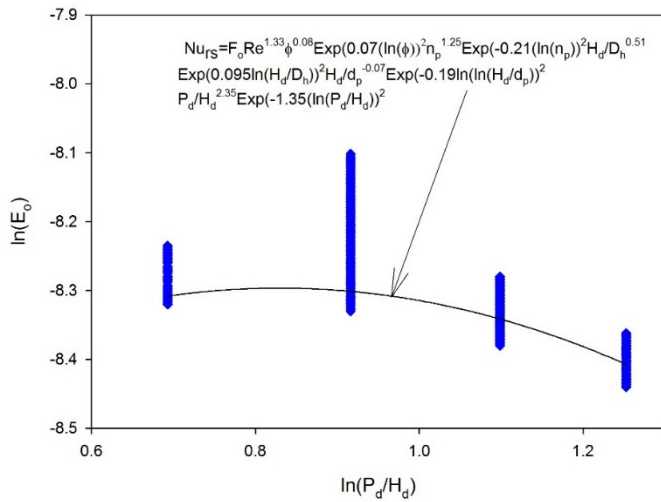


Fig. 19 Plot of  $\ln(E_0)$  as function of  $\ln(P_d/H_d)$ .

Fig. 20 shows a regression analysis second-order polynomial curve fitting and the equation is given by

$$\ln \left[ \frac{Nu_{rs}}{Re^{1.33} \phi^{0.08} \text{Exp}[0.07(\ln(\phi))^2] n_p^{1.25} \text{Exp}(-0.21(\ln(n_p))^2)} \right. \\ \left. \frac{(H_d/D_h)^{0.51} \text{Exp}(0.095(\ln(H_d/D_h)))^2}{(H_d/d_p)^{-0.07} \text{Exp}(=0.19(\ln(H_d/d_p)))^2} \right] \\ = 0.19(\ln(H_d/d_p))^2 (P_d/H_d)^{2.35} \text{Exp}(-1.35(\ln(P_d/H_d))^2) \\ = F_0 \left( \frac{\alpha_a}{60} \right)^{-0.17} \text{Exp}(-0.54(\ln(\alpha_a/60))^2) \quad (45)$$

The regression analysis yields the values of coefficients as:  
 $A_0 = -7.1$ ,  $B_0 = -6.9$ ,  $C_0 = -8.65$ ,  $D_0 = -4.23$ ,  $E_0 = -8.23$ ,  $F_0 = -9.22$  and  $G_0 = -0.54$

The values of coefficients result in the correlation for the Nusselt number as

$$Nu_{rs} = 9.23 \times 10^{-7} Re^{1.33} \phi^{0.08} \text{Exp}[0.07(\ln(\phi))^2] n_p^{1.25} \text{Exp}(-0.21(\ln(n_p))^2)$$

$$(H_d/D_h)^{0.51} \text{Exp}(0.095(\ln(H_d/D_h)))^2 (H_d/d_p)^{-0.07} \text{Exp}(-0.19(\ln(H_d/d_p)))^2 \\ \frac{(P_d/H_d)^{2.35}}{\text{Exp}(-1.35(\ln(P_d/H_d))^2)} (\alpha_a/60)^{-0.17} \text{Exp}(-0.54(\ln(\alpha_a/60))^2) \quad (46)$$

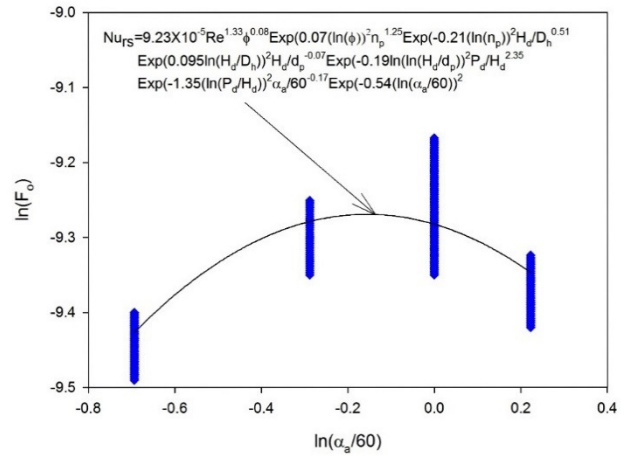


Fig. 20 Plot of  $\ln(F_0)$  as function of  $\ln(\alpha_a/60)$ .

A similarly regression analysis of experimental data of friction factor is carried out that yields to the correlation for the friction factor as given below and Fig.21 shows the regression analysis.

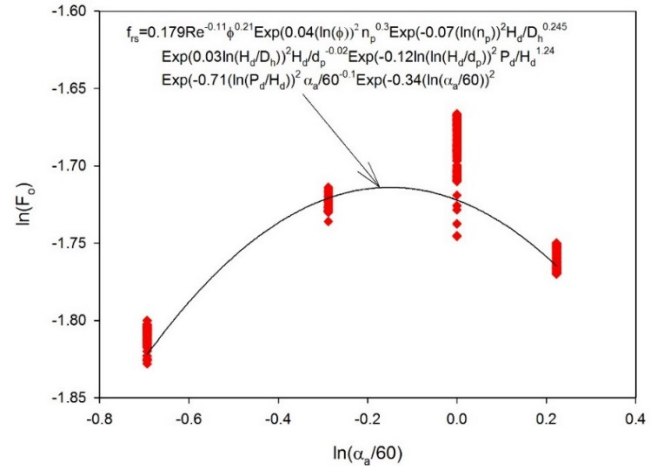


Fig. 21 Plot of  $\ln(F_0)$  as function of  $\ln(\alpha_a/60)$ .

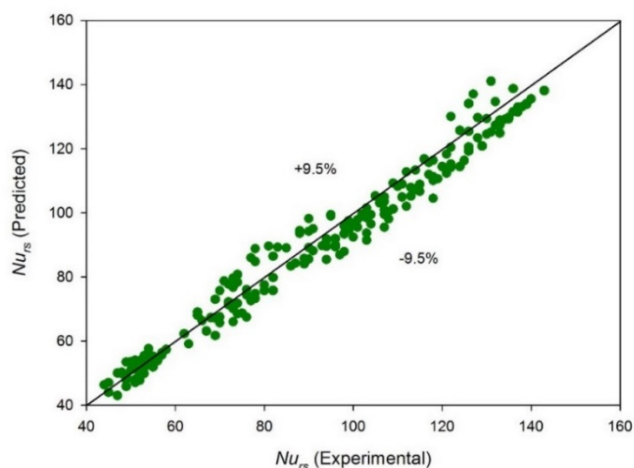
$$f_{rs} = 0.179 Re^{-0.11} \phi^{0.21} \text{Exp}[0.04(\ln(\phi))^2] n_p^{0.307} \text{Exp}(-0.07(\ln(n_p))^2) \\ \frac{(H_d/D_h)^{0.24}}{\text{Exp}(0.03(\ln(H_d/D_h)))^2} (H_d/d_p)^{-0.02} \text{Exp}(-0.12(\ln(H_d/d_p)))^2 (P_d/H_d)^{1.24} \\ \text{Exp}(-0.71(\ln(P_d/H_d))^2) (\alpha_a/60)^{-0.109} \text{Exp}(-0.34(\ln(\alpha_a/60))^2) \quad (47)$$

The  $Nu_{rs}$  and  $f_{rs}$  values predicted from the correlations were compared with that of experimental values. Figs. 22 and 23 show the comparative graph of  $Nu_{rs}$  and  $f_{rs}$ , respectively. A deviation of experimental and predicted values of the  $Nu_{rs}$  and  $f_{rs}$  comes out to be  $\pm 9.5\%$  and  $\pm 8.8\%$ , respectively. The results show an acceptable agreement between the experimental and the predicted values of  $Nu_{rs}$  and  $f_{rs}$ .

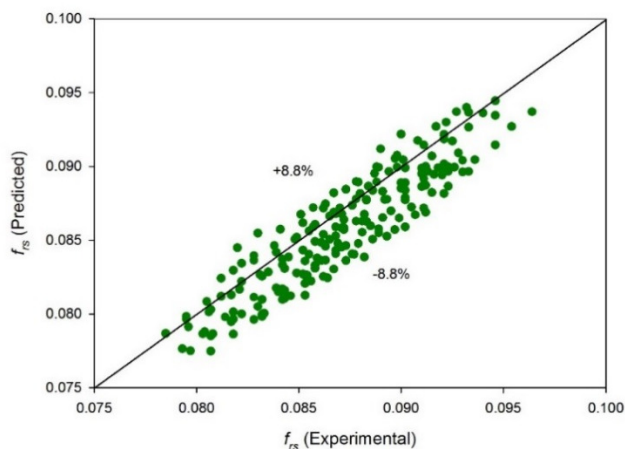
### 11. CONCLUSIONS

This an experimental analysis of the  $Nu_{rs}$  and  $f_{rs}$  of  $TiO_2 - H_2O$  based nanofluid flowing through a V-pattern protrusion ribbed square channel. The following conclusions can be drawn this analysis:

1. The effect of concentration, particle diameter, protrusion rib height, protrusion rib pitch, protrusion print diameter, flow attack angle and Reynolds number on thermal and hydrodynamics performance has been studied.
2. As compared to the smooth surface the V-pattern protrusion rib surface yields an increase of about 4.34 and 2.33 times in the  $Nu_{rs}$  and  $f_{rs}$ , respectively for the range of parameters investigated.
3. The highest improvement of thermal hydrodynamic performance is obtained at a range of  $\varphi = 4\%$ ,  $n_p = 30nm$ ,  $H_d/D_h = 0.25$ ,  $H_d/d_p = 1.0$ ,  $P_d/H_d = 2.5$  and  $\alpha_a = 60^\circ$  respectively. The maximum value of the thermal hydrodynamic performance was found to be 3.21 for the range of parameters examined.
4. Correlations have been developed for  $Nu_{rs}$  and  $f_{rs}$  as function of roughness and operating parameters. These correlations have been found predict the  $Nu_{rs}$  and  $f_{rs}$  values with deviation for  $\pm 9.5\%$  and  $\pm 8.8\%$  respectively.



**Fig. 22** Comparison of experimental and predicted values of Nusselt number.



**Fig. 23** Comparison of experimental and predicted values of friction factor.

### NOMENCLATURE

$A_p$	Area of heated plate, $m^2$
$d_H$	Hydraulic diameter, $m$
$f_{rs}$	Friction factor, dimensionless ( $f_{rs} = \frac{2\Delta P_0 D_h}{\rho_{nf} L_t v_x^2}$ )
$f_{av}$	Average friction factor, dimensionless
$h_c$	Heat transfer coefficient, $W/m^2K$ ( $\frac{Q_u}{A_p \times \Delta T_f}$ )
$H_d/D_h$	Ratio of V-pattern protrusion rib height
$H_c$	Height of channel, $m$
$H_d/d_p$	Ratio of V-pattern protrusion height to print diameter
$k_{nf}$	Thermal conductivity of nanofluid flow, $W/mK$
$L_t$	Length of test section, $m$
$n_p$	Particle diameter, $nm$
$Nu_{rs}$	Nusselt number of V-pattern protrusion rib
$Nu_{av}$	Average Nusselt number
$P_d/H_d$	Relative V-pattern protrusion rib pitch ratio
$\Delta P$	Pressure drop across the test section, $Pa$
$Q_u$	Heat gain, $W$ ( $Q_u = m_a c_p \Delta T$ )
$Re$	Reynolds number ( $Re = \frac{\rho_{nf} \cdot v_x \cdot D_h}{\mu_{nf}}$ )
$T_{pt}$	Weighted average plate temperature, $K$
$T_{ft}$	Average fluid temperature, $K$
$W_c$	Width of channel, $m$

### Greek Letters

$\alpha_a$	V-pattern protrusion rib flow attack angle, degree
$\rho_{nf}$	Density of nano fluid, $kg/m^3$
$\mu_{nf}$	Dynamic viscosity of nano fluid, $Ns/m^2$
$\eta_p$	Thermal hydrodynamic performance, dimensionless
$\varphi$	Volume fraction, %

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