



SORET AND DUFOUR EFFECTS ON UNSTEADY HYDROMAGNETIC DUSTY FLUID FLOW PAST AN EXPONENTIALLY ACCELERATED PLATE WITH VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY

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ABSTRACT

Soret and Dufour effects on the unsteady flow of a viscous incompressible dusty fluid past an exponentially accelerated vertical plate with viscous dissipation have been considered in the presence of heat source and magnetic field. The viscosity and thermal conductivity of the fluid are assumed to be varying with respect to temperature. Saffman model of dusty fluid is considered for the investigation. The non-linear partial differential equations with prescribed boundary conditions governing the flow are discretized using Crank-Nicolson formula and the resulting finite difference equations are solved by an iterative scheme based on the Gauss-Seidel method by developing computer codes for MATLAB software. Numerical results are obtained for different values of the viscosity variation parameter, thermal conductivity variation parameter, Soret number, Dufour number and magnetic parameter. The overall investigation of variation of velocity, temperature, and species concentration profiles is presented graphically. Finally, numerical values of skin friction coefficient, Nusselt number, and Sherwood number are obtained and presented in tabular form for different values of physical parameters. It is found that skin friction coefficient increases with an increase in thermal conductivity variation parameter and magnetic parameter and decreases as the values of viscosity variation parameter, Soret number and Dufour number increase. Moreover, an increase in the Dufour number tends to decrease the Nusselt number and to increase the Sherwood number. But, a contrary tendency is observed with Soret number.

Keywords: *Dusty fluid, Soret and Dufour effects, variable viscosity and thermal conductivity, viscous dissipation, finite difference method.*

1. INTRODUCTION

The study of momentum, heat and mass transfer of dusty fluids has great practical importance due to tremendous applications in sciences and engineering. In the past few decades, researchers have been focusing their research work on analyzing the heat and mass transfer characteristics of dusty fluids through different channels. In the present study, we are taking initiation to discuss the momentum, and heat and mass transfer characteristics of a dusty fluid flows over an exponentially stretching surface. The convective flow of dusty viscous fluids has a variety of applications like wastewater treatment, combustion and petroleum transport, power plant piping etc. Heat transport in dusty fluids plays a major role in heat transfer enhancement in the renewable energy systems, material processing and industrial thermal management like aerodynamic extrusion of plastic sheets, manufacturing and rolling of plastic films, cooling of metal sheets etc.

Sakiadis (1961) analyzed the pioneering work on the flow past a continuous moving surface with a constant velocity in boundary layer. He formulated the equations governing the two-dimensional flow problems. Magyari and Keller (1999) were the first authors who investigate the boundary layer flow due to an exponentially stretching continuous surface. They analyzed the problem both analytically and numerically. The effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface was studied by Partha *et al.* (2005). Khan (2006) also presented the visco-elastic boundary layer flow and heat transfer characteristics over an exponentially stretching sheet. Al-odat *et al.* (2006) investigated the

effect of magnetic field on the flow and heat transfer over an exponentially stretching continuous surface. Ishak (2005) investigated the radiation effect on MHD boundary layer flow due to an exponentially stretching. Soret and Dufour effects on mixed convection flow and heat transfer from an exponentially stretching surface were studied by Srinivasacharya and RamReddy (2011). They obtained numerical solution for the problem using Keller-box method.

Important applications of dust particles in a boundary layer include a wide range of real world applications. Initially, Saffman (1962) worked on the laminar flow of a gas containing dust particles and stability, which describes the fluid-particle system. He derived the equations of motion for a flow of gas carrying the dust particles. The flow of dusty fluid in the boundary layer over a semi-infinite flat plate was studied by Datta and Mishra (1982). Vajravelu and Nayfeh (1992) discussed the hydromagnetic flow of a dusty fluid over a stretching sheet with the effects of particle loading, fluid-particle interaction and suction on the flow characteristics. Also, they compared their analytical solution with numerical ones. Flow of an unsteady viscous incompressible fluid with dust particles through a rectangular channel was studied by Gireesha *et al.* (2007). An analytical study of unsteady viscous dusty fluid flow with uniform distribution of dust particles between two infinite parallel plates was also carried out by Gireesha *et al.* (2007). Unsteady MHD boundary layer flow and heat transfer characteristics of dusty fluid over a stretching sheet with

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variable wall temperature (VWT) and variable heat flux (VHF) were observed by Gireesha *et al.* (2011; 2013). In these papers, they analyzed the effect of magnetic field on the flow and heat transfer within the boundary layer of dusty fluid. Pavithra and Gireesha (2013) investigated boundary layer flow problem of dusty fluid for an exponentially stretching sheet by considering the internal heat generation/absorption and viscous dissipation. Ramana Reddy *et al.* (2014) studied the laminar convective flow of a dusty viscous fluid of non conducting walls in presence of aligned magnetic field with volume fraction, radiation, heat absorption along with chemical reaction. Ramana Reddy *et al.* (2014) also studied the solet, radiation and chemical reaction effects on laminar convective flow of a dusty viscous fluid of non conducting walls in presence of transverse magnetic field. Flow of a conducting dusty fluid due to linearly stretching cylinder immersed in a porous media with the effect of radiation was analyzed by Manjunatha *et al.* (2015).

Recently, a study on a convective heat transfer characteristics of an incompressible viscous dusty fluid over an exponentially stretching surface has been carried out by Izani and Ali (2016) with an exponential temperature distribution. Ramana Reddy *et al.* (2016) analyzed the momentum, heat and mass transfer behavior of a chemically reacting MHD nanofluid flow embedded with conducting dust particles past a cone in the presence of non-uniform heat source/sink. Marangoni thermal convective boundary layer dusty nanoliquid flow across a flat surface in the presence of solar radiation was discussed by Mahanthesh *et al.* (2017).

The effect of a magnetic field on a boundary layer flow of an electrically conducting dusty fluid over a stretching surface has been investigated by Jalil *et al.* (2017). Konch and Hazarika (2017) have investigated effects of variable viscosity and thermal conductivity on momentum, heat and mass transfer characteristics of a hydromagnetic Newtonian dusty fluid flow due to a rotating disk with radiation and viscous dissipation.

In most of the studies mentioned above are carried out under a steady-state condition. But, in many cases, flow becomes time dependent due to a sudden stretching of the flat sheet or heat flux of the sheet or by a step change of the temperature, and consequently, it becomes an unsteady flow problem. Furthermore, in all the flow problems mentioned above, the viscosity and thermal conductivity of fluid were considered as constants. However, a more accurate prediction for the flow, heat and mass transfer can be obtained by taking into account the variation of such properties with temperature.

A quick review of the literature shows that the effects of Soret and Dufour on flow, heat and mass transfer over an exponentially stretching sheet was not taken into consideration for an unsteady dusty fluid with variable viscosity and thermal conductivity. Therefore, the goal of this chapter is to study the effects of temperature dependent viscosity and thermal conductivity, and Soret and Dufour on an unsteady hydromagnetic boundary layer flow over an exponentially stretching sheet.

2. MATHEMATICAL FORMULATION

Consider an unsteady flow, heat and mass transfer flow of an incompressible viscous electrically conducting and radiating dusty fluid past an exponentially accelerated infinite isothermal vertical plate. It is assumed that a temperature dependent heat source present in the flow and dust particles are assumed to be electrically nonconductive, spherical in shape having the same radius and mass, and un-deformable. Also, the fluid is supposed to be gray, absorbing-emitting but non-scattering. At the beginning, the fluid is considered to be at rest. The x' -axis is taken along the plate in the vertically upward direction and

the y' -axis is taken normal to the plate as shown in Fig. 1. A magnetic field of strength $\vec{B}(0, B_0)$ is applied perpendicular to the plate. Reynolds number is supposed to be so small that the induced magnetic field can be neglected (Sutton (1965)).

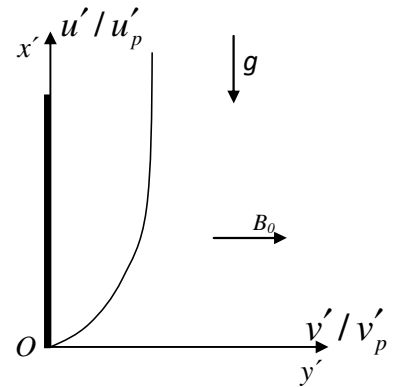


Fig. 1 Geometry of the problem

At the time $t' = 0$, the temperature of the plate and species concentration of the fluid are T'_∞ and C'_∞ , respectively. At time $t' > 0$, the plate is exponentially accelerated in its own plane with a velocity $u = u_0 e^{at'}$ and the plate temperature and species concentration of the fluid upstanding to T'_w and C'_w , and are maintained constantly thereafter.

Under the above assumptions, using the usual Boussinesq's approximation and the Saffman (1962) model, the governing equations for the two-phase flow are:

For the fluid phase:

$$\frac{\partial u'}{\partial t'} = \frac{\partial}{\partial y'} \left(\nu \frac{\partial u'}{\partial y'} \right) + g\beta^* (T' - T'_\infty) + g\beta^{**} (C' - C'_\infty) - \frac{\sigma B_0^2}{\rho} u' + \frac{KN}{\rho} (u'_p - u') \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \frac{\partial}{\partial y'} \left(\lambda \frac{\partial T'}{\partial y'} \right) - \frac{\partial q_r}{\partial y'} + Q_0 (T' - T'_\infty) + \frac{N c_p}{\tau_T} (T'_p - T') + \frac{\rho D_{md} K_T}{C_s} \frac{\partial^2 C'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 \quad (2)$$

$$\frac{\partial C'}{\partial t'} = \frac{\partial}{\partial y'} \left(D_m \frac{\partial C'}{\partial y'} \right) + \frac{mN}{\rho \tau_c} (C'_p - C') + \frac{D_{md} K_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

For the dust phase:

$$\frac{\partial u'_p}{\partial t'} = -\frac{K}{m} (u'_p - u') \quad (4)$$

$$\frac{\partial T'_p}{\partial t'} = -\frac{c_p}{c_m \tau_T} (T'_p - T') \quad (5)$$

$$\frac{\partial C'_p}{\partial t'} = -\frac{mN}{\rho \tau_c} (C'_p - C') \quad (6)$$

The initial and boundary conditions are [Gireesha *et al.* (2012) and Manjunatha and Gireesha (2016)]:

$$\left. \begin{aligned} t' = 0 : u' = 0, T' = T'_{\infty}, C' = C'_{\infty} \text{ for } y' \geq 0 \\ t' > 0 : u' = u_0 e^{at'}, T' = T'_w, C' = C'_w \text{ at } y' = 0 \\ u' \rightarrow 0, u'_p \rightarrow u', T' \rightarrow T'_{\infty}, T'_p \rightarrow T'_{\infty}, C' \rightarrow C'_{\infty}, \\ C'_p \rightarrow C'_{\infty} \text{ as } y' \rightarrow \infty. \end{aligned} \right\} \quad (7)$$

Where T' and C' are the temperature and species concentration of fluid, respectively. T'_p and C'_p are the temperature and species concentration of dust phase, respectively. u' and u'_p are the velocities of fluid and dust phases, respectively.

Using Rosseland approximation for radiation we can write radiative heat flux as (Necati (1973)):

$$q_r = -\frac{4\sigma^* \partial T'^4}{3k^* \partial y'} \quad (8)$$

where σ^* and k^* are the Stefan-Boltzmann constant and mean absorption coefficient, respectively.

It is assumed that temperature difference within the flow such that the term T'^4_{∞} can be expressed as a linear function of temperature. This is accomplished by expanding T'^4_{∞} in a Taylor series about T'_{∞} and neglecting the second and higher order terms, we get:

$$T'^4 \approx 4T'^3_{\infty} T' - 3T'^4_{\infty} \quad (9)$$

Now we introduce the following non-dimensional quantities to make the governing equations dimensionless:

$$\left. \begin{aligned} u = \frac{u'}{u_0}, u_p = \frac{u'_p}{u_0}, t = \frac{t' u_0}{v_{\infty}}, y = \frac{y' u_0}{v_{\infty}}, a = \frac{a' v_{\infty}}{u_0^2}, \\ \theta = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}, \theta_p = \frac{T'_p - T'_{\infty}}{T'_w - T'_{\infty}}, \phi = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}}, \phi_p = \frac{C'_p - C'_{\infty}}{C'_w - C'_{\infty}}. \end{aligned} \right\} \quad (10)$$

Viscosity of the fluid is assumed to be an inverse linear function of temperature, and it can be expressed as (following Lai and Kulacki (1990)):

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} [1 + \delta(T' - T'_{\infty})] \quad \text{or,} \quad \frac{1}{\mu} = \alpha(T' - T'_r) \quad (11)$$

where $\alpha = \frac{\delta}{\mu_{\infty}}$ and $T'_r = T'_{\infty} - \frac{1}{\delta}$.

Moreover, thermal conductivity of the fluid varies with temperature. Following Choudhury and Hazarika (2013), we assumed thermal conductivity of the fluid as:

$$\frac{1}{\lambda} = \frac{1}{\lambda_{\infty}} [1 + \xi(T' - T'_{\infty})] \quad \text{or,} \quad \frac{1}{\lambda} = \zeta(T' - T'_c) \quad (12)$$

where $\zeta = \frac{\xi}{\lambda_{\infty}}$ and $T'_c = T'_{\infty} - \frac{1}{\xi}$.

Here, $\alpha, \delta, \xi, \zeta, T'_r$ and T'_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., ν (kinematic viscosity) and λ (thermal conductivity).

Substituting equations (8) to (12) into equations (1) to (6), we get the following dimensionless equations:

For the fluid phase:

$$\frac{\partial u}{\partial t} = \frac{\theta_r}{(\theta - \theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} - \frac{\theta_r}{\theta - \theta_r} \frac{\partial^2 u}{\partial y^2} + Gr \theta + Gm \phi - M u + R(u_p - u) \quad (13)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\theta_c}{(\theta - \theta_c)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 - \frac{1}{Pr} \frac{\theta_c}{\theta - \theta_c} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{Pr} \theta + Q \theta + \frac{2R}{3Pr} (\theta_p - \theta) + \frac{1}{Pr} Du \frac{\partial^2 \phi}{\partial y^2} - Ec \frac{\theta_r}{\theta - \theta_r} \left(\frac{\partial u}{\partial y} \right)^2 \quad (14)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\theta_r}{(\theta - \theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial y} - \frac{1}{Sc} \frac{\theta_r}{\theta - \theta_r} \frac{\partial^2 \phi}{\partial y^2} - Ra_1 (\phi_p - \phi) + Sr \frac{\partial^2 \theta}{\partial y^2} \quad (15)$$

For the dust phase:

$$\frac{\partial u_p}{\partial t} = -G(u_p - u) \quad (16)$$

$$\frac{\partial \theta_p}{\partial t} = -L_T (\theta_p - \theta) \quad (17)$$

$$\frac{\partial \phi_p}{\partial t} = -L_C (\phi_p - \phi) \quad (18)$$

Corresponding initial and boundary conditions (7) are reduced to:

$$\left. \begin{aligned} t = 0 : u = 0, \theta = 0, \phi = 0 \text{ for all } y \\ t > 0 : u = e^{at}, \theta = 1, \phi = 1 \text{ at } y = 0 \\ u \rightarrow 0, u_p \rightarrow 0, \theta \rightarrow 0, \theta_p \rightarrow 0, \phi \rightarrow 0, \phi_p \rightarrow 0 \text{ as } y \rightarrow \infty. \end{aligned} \right\} \quad (19)$$

where the dimensionless parameters are defined as follows:

$\tau_v = m/K$ is the velocity equilibration time, $\rho_r = \rho_p / \rho$ is the relative density, $R = \frac{KN v_{\infty}}{\rho u_0^2}$ is the particle concentration parameter, $a_1 = \frac{\tau_v}{\tau_c}$

is a constant, $Gr = \frac{g \beta^* (T'_w - T'_{\infty}) v_{\infty}}{u_0^3}$ is the Grashof number;

$Gm = \frac{g \beta^{**} (C'_w - C'_{\infty}) v_{\infty}}{u_0^3}$ is the local mass Grashof number,

$Ec = \frac{u_0^2}{c_p (T'_w - T'_{\infty})}$ is the Eckert number, $M = \frac{\sigma B_0^2 v_{\infty}}{u_0^2}$ is the

magnetic parameter, $Pr = \frac{\mu_{\infty} c_p}{\lambda_{\infty}}$ is the Prandtl number, $Q = \frac{Q_0 v_{\infty}}{\rho c_p u_0^2}$ is

the heat generation/absorption coefficient, $G = \frac{K v_{\infty}}{\mu u_0^2}$ is the particle

mass parameter, $L_T = \frac{c_p v_{\infty}}{c_m \tau_T u_0^2}$ is the temperature relaxation time

parameter, $L_C = \frac{m N v_{\infty}}{\rho \tau_c u_0^2}$ is the species concentration relaxation time

parameter, $Sr = \frac{D_{md} K_T T'_w - T'_{\infty}}{T_m v_{\infty} C'_w - C'_{\infty}}$ is the Soret number,

$Du = \frac{D_{md} K_T C'_w - C'_{\infty}}{\alpha^* C_s c_p T'_w - T'_{\infty}}$ is the Dufour number, $\alpha^* = \frac{\lambda_{\infty}}{\rho c_p}$ is the

thermal diffusivity. $\theta_r = \frac{T'_r - T'_{\infty}}{T'_w - T'_{\infty}}$ and $\theta_c = \frac{T'_c - T'_{\infty}}{T'_w - T'_{\infty}}$ are the viscosity

variation parameter and thermal conductivity variation parameter, respectively. It is also important to note that θ_r and θ_c are negative for liquids and positive for gases (Kuppalapalle *et al.* (2013)).

2.1 Skin friction coefficient, Nusselt Number and Sherwood Number

Skin friction coefficient (C_f), Nusselt number (Nu) and Sherwood number (Sh) are the parameters of physical and engineering interest for the present problem, which physically indicate the wall shear stress, rate of heat and mass transfer, respectively.

In this problem, dimensionless skin friction coefficient, Nusselt number and Sherwood number are given respectively by:

$$C_f = -\left. \frac{\theta_r}{1-\theta_r} \frac{\partial u}{\partial y} \right|_{y=0}, \quad Nu = -\left. \frac{\theta_c}{1-\theta_c} \frac{\partial \theta}{\partial y} \right|_{y=0} \quad \text{and} \quad Sh = -\left. \frac{\theta_r}{1-\theta_r} \frac{\partial \phi}{\partial y} \right|_{y=0}.$$

3. METHOD OF SOLUTION

The system of equations (13) to (18) governing the flow represents a system of coupled nonlinear partial differential equations, which are solved numerically under the boundary conditions (19) by adopting Crank-Nicolson implicit finite difference scheme, which is always unconditionally stable. This method is discussed by many authors, namely, Soundalgekar and Ganesan (1981), Ganesan and Rani (1999), Muthucumaraswamy and Ganesan (1999, 2000), Ganesan and Palani (2004). The scheme of this method is unconditionally stable and is described by Bapuji *et al.* (2008). To obtain the difference equations, the region of the flow is divided into a grid or mesh lines parallel to y and t axes.

The region of integration is considered as a rectangle with sides $y_{\max}(=5)$ and $t_{\max}(=1)$, where y_{\max} corresponds to $y \rightarrow \infty$, which lies very well outside the momentum, energy and species concentration boundary layers. The maximum of y has been chosen as 5 after some preliminary investigations so that the last two of the boundary conditions (19) are satisfied within the tolerance limit 10^{-5} . After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta y = 0.17$ with time step $\Delta t = 0.0031$. In this case, the computations are carried out first by reducing the spatial mesh sizes by 50 % in one direction, and later in both directions by 50 %. The results are compared. It is observed that, in all cases, the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate.

Solutions of difference equations are obtained at the intersection of these mesh lines called nodes. In this problem, the values of the dependent variables u, θ and ϕ at the nodal points along the $y=0$ are given by $u(0,t), \theta(0,t)$ and $\phi(0,t)$, hence are known from the boundary conditions. But, $u_p(0,t), \theta_p(0,t)$ and $\phi_p(0,t)$ are unknown, which are estimated by applying trial and error, in such a way that, for those values, all the boundary conditions are satisfied at the other boundary with a good accuracy (error less than 10^{-5}). Δy and Δt are taken as the constant mesh sizes along y and t directions respectively. We need the scheme to find single values at next time level in terms of known values at an earlier time level.

In order to obtain the finite difference equations for the equations (13) to (18), we have adopted forward difference approximation for the first order partial derivatives of $u, \theta, \phi, u_p, \theta_p$ and ϕ_p with respect to t and y and a central difference approximation for the second order partial derivatives of u, θ and ϕ with respect to y .

Knowing the values of $u, \theta, \phi, u_p, \theta_p$ and ϕ_p at time t we can calculate the values at time $t + \Delta t$. Using initial and boundary conditions, the system can be solved based on an

iterative scheme and developing suitable programming codes for the method in MATLAB software.

The truncation error in the finite difference approximation is $O(\Delta t^2 + \Delta y^2)$ and it tends to zero as Δt and Δy tend to zero. Hence the scheme is compatible. Thus, stability and compatibility ensure convergence. Time required to complete the computation and to draw a graph for a particular parameter is 8 seconds (approximately) in MATLAB 2017a.

4. RESULTS AND DISCUSSION

In order to analyze the problem physically, numerical computations are carried out to explain the effects of different parameters governing the flow upon the nature of the flow, heat and mass transport phenomenon. The numerical values of the velocity, temperature, species concentration, skin-friction coefficient, Nusselt number and Sherwood number are obtained for different physical parameters like the viscosity variation parameter θ_r , thermal conductivity variation parameter θ_c , Soret number Sr , Dufour number Du , magnetic parameter M and time t . A representative set of numerical results is presented graphically in Figs. 2 to 17 and in Tables 1 to 3. Numerical values of the parameters used for simulation are: $M=1, Pr=0.71, Ec=0.05, a_1=1, Q=0.5, Gr=5, Gm=5, R=1, Sc=0.22, Sr=0.3, Du=0.5, \theta_r=3$ and $\theta_c=5$, unless otherwise stated.

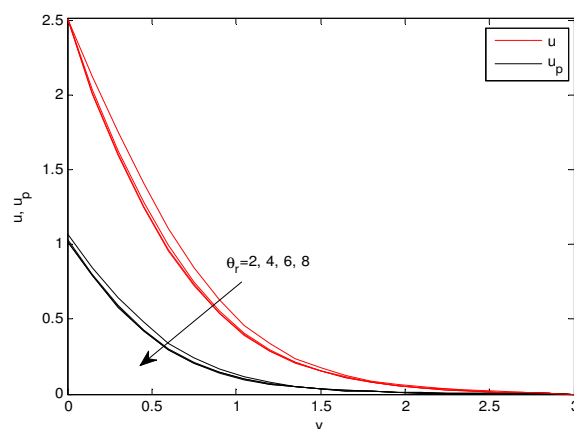


Fig. 2 Velocity profile for different θ_r .

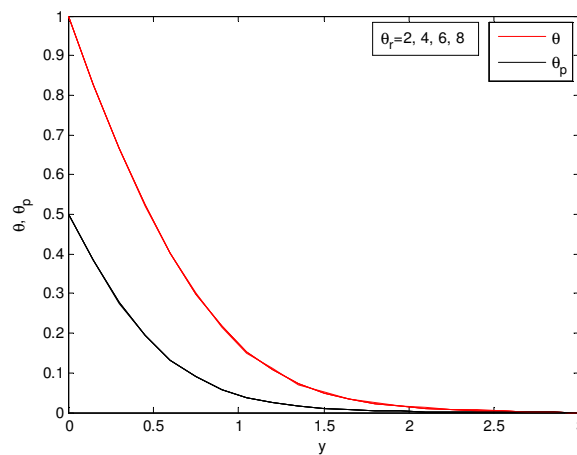


Fig. 3 Temperature profile for different θ_r .

The effect of the viscosity variation parameter θ_r on the velocity, temperature, and species concentration profiles is shown in Figs. 2 to 4. It is observed from these figures that the

velocity and species concentration decreases with the increase of the viscosity variation parameter. Physically, as the values of viscosity variation parameter θ_v increases, the resistance to the relative motion of the different layers of fluid increases due to the increase of viscous force, as a result, velocity of fluid phase decreases. Thus, the increase of θ_v decelerates the fluid motion and reduces the species concentration profile of the fluid and dust phases along the wall. Also, one can see that the temperature of both phases is almost not affected by the increase of the viscosity variation parameter θ_v (Fig. 3).

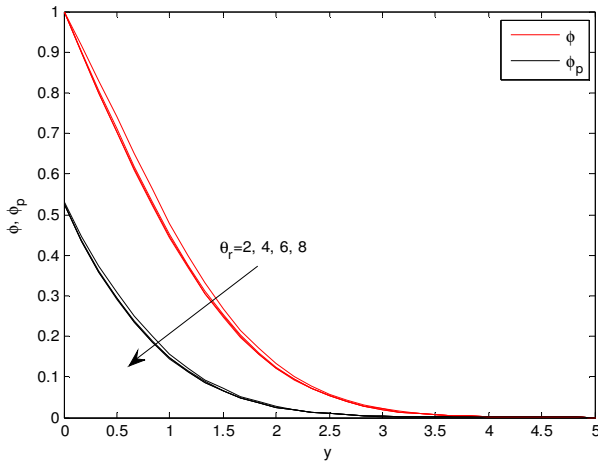


Fig. 4 Species concentration profile for different θ_v .

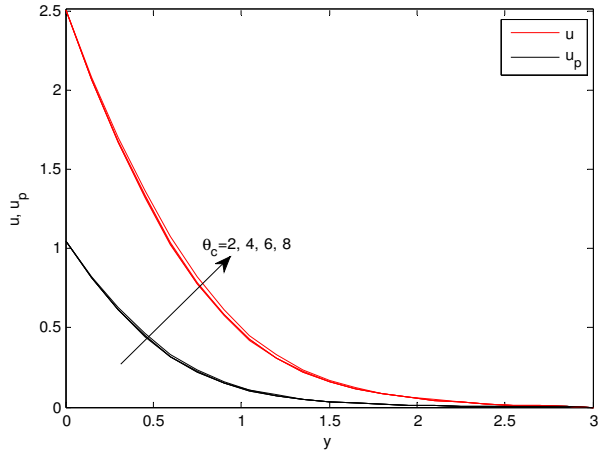


Fig. 5 Velocity profile for different θ_c .

The effect of thermal conductivity variation parameter θ_c on velocity, temperature, and species concentration profiles is shown in Figs. 5 to 7. Figure 5 has shown that viscosity increases for both the fluid and dust phases with increasing values of θ_c . On the other hand, a reverse trend is noticed with the temperature profile (Fig. 6). It is due to the reason that thermal conductivity is an inverse linear function of temperature, hence the result. Thermal conductivity variation parameter has no significant effect on species concentration profile (Fig. 7).

Figures 8 to 10 depict the effect of Soret number Sr on velocity, temperature, and species concentration regimes of the fluid and dust phases. It is explicitly observed from Fig. 10 that species concentration profiles of both the fluid and dust phases grow up with the increment in Sr , while temperature behaves in opposite manner for

increasing variation of Sr (Fig. 9). Physically, Soret number signifies the ratio of temperature gradients and mass diffusion effects. Hence for mounting values of Sr , thermal flux increases and subsequently causes a considerable downfall in the temperature distribution. In Fig. 8, it is seen that velocity increases with increasing values of Sr . The reason behind is that temperature falls for rising values of Sr and in case of gases, viscosity decreases with decreasing temperature, as a result velocity increases.

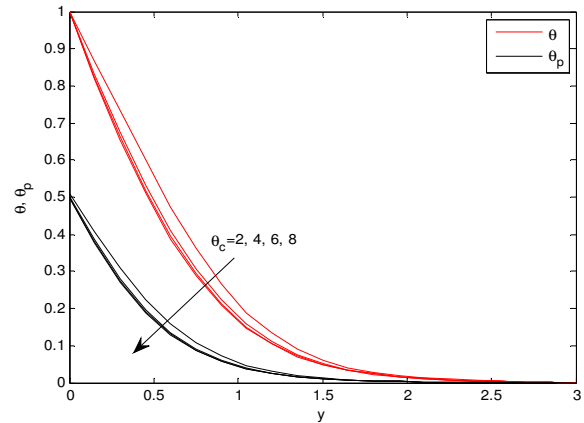


Fig. 6 Temperature profile for different θ_c .

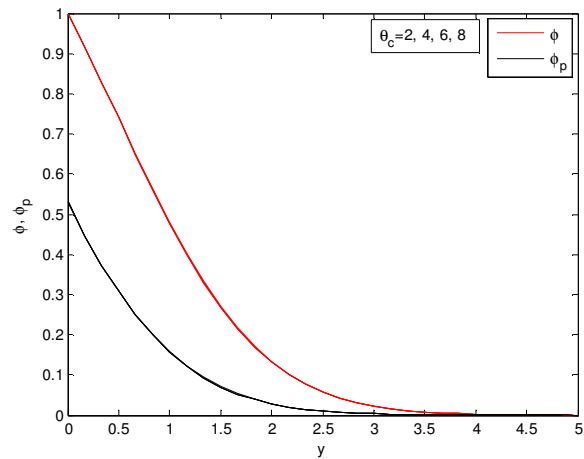


Fig. 7 Species concentration profile for different θ_c .

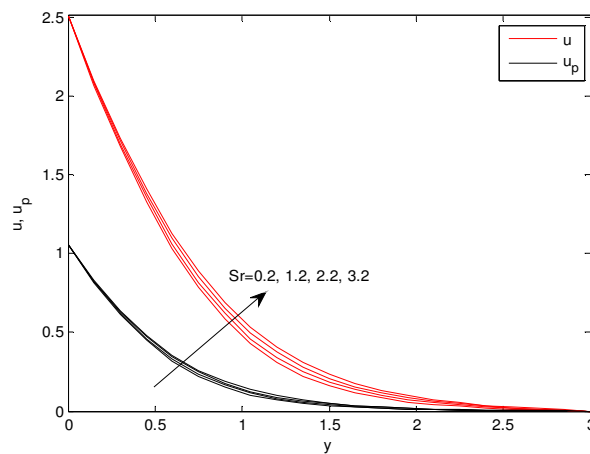


Fig. 8 Velocity profile for different Sr .

Figures 11 to 13 have depicted the effects of Dufour number (Du) on velocity, temperature, and species concentration fields. It is clearly seen that Dufour number leads to increase the velocity and temperature distributions (Figs. 11 and 12). But species concentration distribution decreases with the enhancement of the Dufour number (Fig. 13). Physically, Dufour number presents the relation among the contribution of concentration gradient to the temperature gradient.

Figure 14 illustrates the effect of magnetic field on velocity profiles through the magnetic parameter (M). As the increasing values of magnetic parameter M , velocity decreases for both the fluid and dust phases. The presence of magnetic field sets a resistive force, called Lorentz force, which opposes the velocity field. Therefore, as the values of M increase so does the retarding force and hence the velocity decreases.

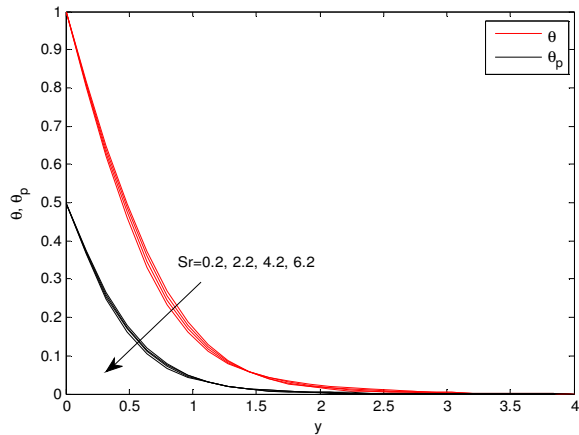


Fig. 9 Temperature profile for different Sr

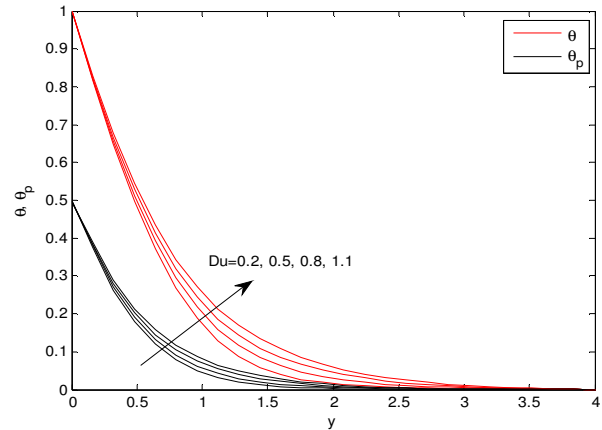


Fig. 12 Temperature profile for different Du

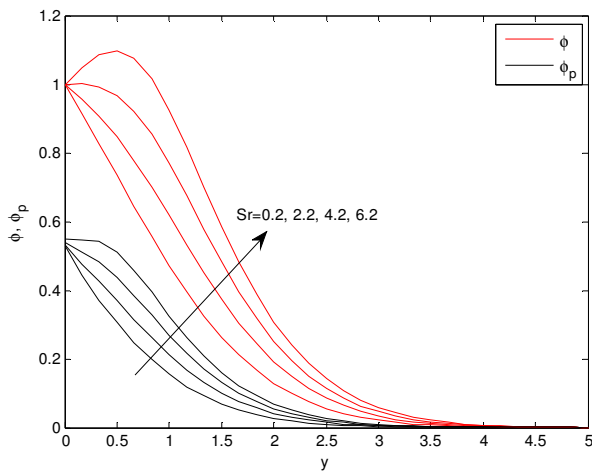


Fig. 10 Species concentration profile for different Sr

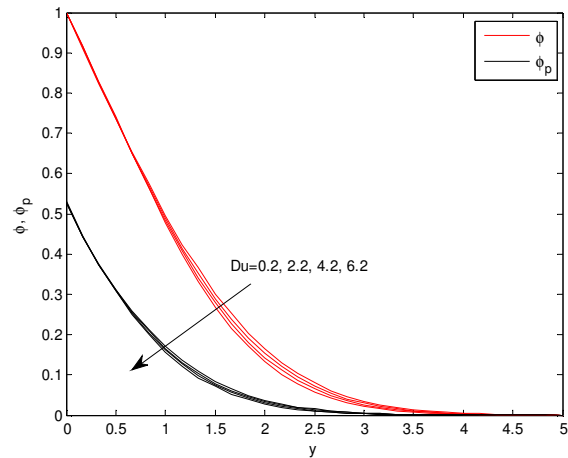


Fig. 13 Species Concentration profile for different Du

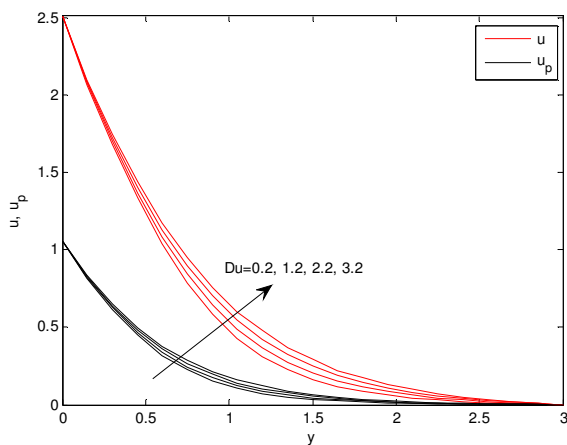


Fig. 11 Velocity profile for different Du

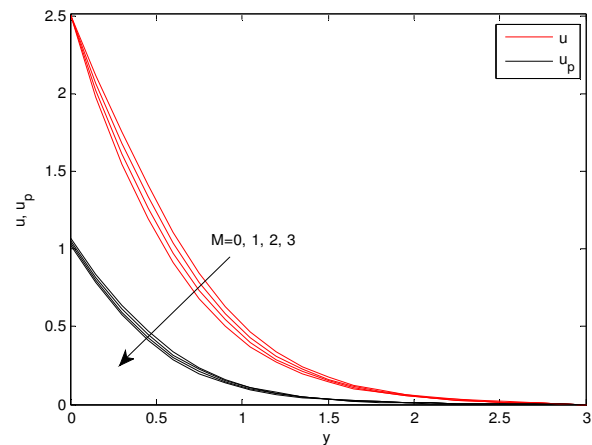


Fig. 14 Velocity profile for different Du

The behaviors of velocity, temperature and species concentration profiles for different values of the time t are presented in Figs. 15 to 17, respectively. It is observed that velocity, temperature, and species concentration increases with increasing values of time t for both the fluid and dust phases.

Table 1 shows the increment of velocity, temperature and species concentration with constant time intervals. From the table, it has been found that the percentage of increase in the velocity, temperature and species concentration of both phases has decreased with increasing time. This means that there is no change in the remarkable properties over time after the specified time interval. Thus, one can say that after a while an unsteady flow becomes steady.

Table 2 presents values of C_f , Nu and Sh for both the constant and variable cases of viscosity and thermal conductivity. The values of C_f , Nu and Sh , when variable viscosity and thermal conductivity are taken into account, are smaller than the values when viscosity and thermal conductivity are taken as constant. Thus, it shows that it is better to have viscosity and thermal conductivity as function of temperature to achieve accurate results of flow, heat and mass transfer properties.

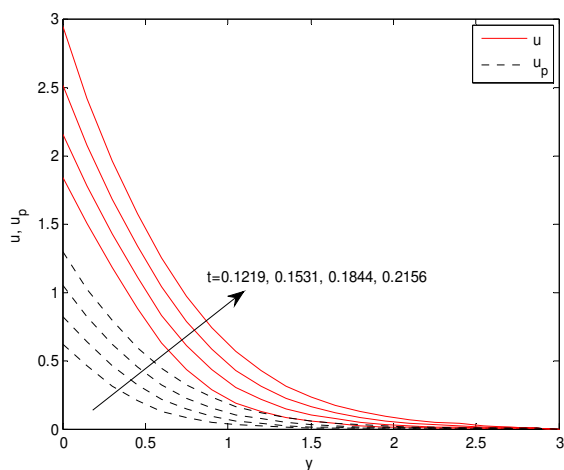


Fig. 15 Velocity profile for different t

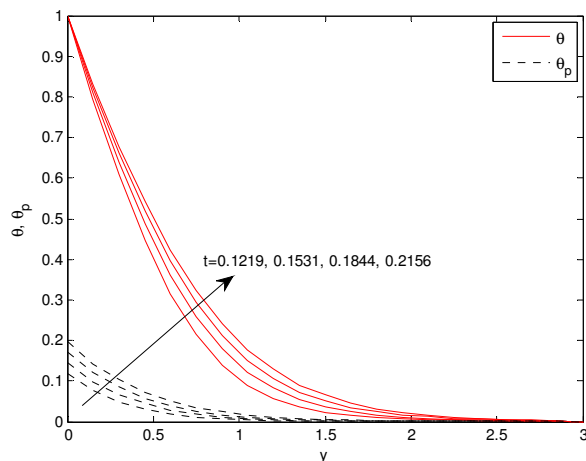


Fig. 16 Temperature profile for different t

The variation of the skin friction coefficient, Nusselt number and Sherwood number for various flow governing parameters, viz. θ_f , θ_c , Sr , Du and M is shown in Table 3. From the table, it is noticed that magnitude of skin friction coefficient decreases when viscosity variation parameter (θ_f) is increased. This is due to the reason that fluids having higher viscosity have relatively lower surface velocity gradient, for which skin friction coefficient decreases. On the contrary, the Nusselt number increases with the increasing values of the same parameter. This is due to the fact that, the friction of the fluid increases due to viscosity so the heat is generated from the friction on the surface, which leads to a rise in the magnitude of heat transfer rate, and hence Nusselt number increases. Also, Sherwood number decreases when viscosity variation parameter increases.

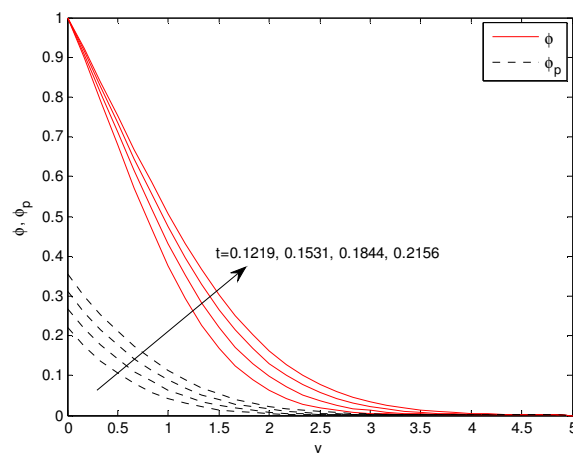


Fig. 17 Species concentration profile for different t

In the same table, we have seen that the magnitude of skin friction coefficient increases and the Nusselt number decreases for increasing thermal conductivity variation parameter θ_c . The reason behind is that viscosity decreases with increasing thermal conductivity variation parameter, which enhances the magnitude of velocity gradient at the surface of the sheet and retards the magnitude of the heat transfer rate. Hence, Nusselt number decreases and skin friction coefficient increases with increasing θ_c .

Table 1 Variation of velocity, temperature and species concentration with time t when $y=1.5$

Time Interval (Δt)	Increment (%)					
	Velocity		Temperature		Species concentration	
	u	u_p	θ	θ_p	ϕ	ϕ_p
0.1219–0.1531	78.85	132.14	51.42	100.00	31.05	78.57
0.1531–0.1844	53.52	88.45	34.90	73.08	20.04	53.20
0.1844–0.2156	39.40	64.77	24.90	51.11	14.10	39.16

Table 3 also demonstrates that rise in Dufour number (Du) results a reduction in the Nusselt number (i.e., the rate of heat transfer), while a reverse pattern is established for Soret number (Sr). The reducing behavior of Nusselt number for intensifying values of Dufour number (Du) is explained from the

mathematical relation $Du = \frac{D_{md}K_T C'_w - C'_\infty}{\alpha^* C_s c_p T'_w - T'_\infty}$. It is evident from

the expression that Dufour number inversely relates with the temperature gradient. Thus, with the increase of Dufour number, the temperature difference between ambient and local temperature depreciates and hence temperature flux minimizes. The increasing behavior of heat transfer with variation in Soret number (Sr) can be understood from the defined relation for Soret number i.e. $Sr = \frac{D_{md}K_T T'_w - T'_\infty}{T_m v_\infty C'_w - C'_\infty}$. Here, it is crystal clear from the relation that Soret number (Sr) gives support to the temperature gradient and mount it.

Table 2 Comparison of skin friction coefficient (C_f), Nusselt number (Nu) and Sherwood number (Sh)

Sr	Du	when viscosity (μ) and thermal conductivity (λ) are constants			when viscosity (μ) and thermal conductivity (λ) are variables		
		C_f	Nu	Sh	C_f	Nu	Sh
0	0.2	-3.4634	1.2210	0.5504	-4.3479	1.4066	0.7737
1		-3.3725	1.2469	0.3614	-4.2606	1.4216	0.6007
2		-3.2786	1.2745	0.1612	-4.1715	1.4372	0.4222
0.3	0.0	-3.4548	1.2516	0.4935	-4.3395	1.4292	0.7208
	0.3	-3.4272	1.2170	0.4955	-4.3131	1.4017	0.7231
	0.6	-3.3992	1.1816	0.4975	-4.2864	1.3733	0.7255

An enhancement of Sherwood number (i.e., the rate of mass transfer) is observed for increasing Dufour number (Du), but reducing behavior is seen for a rising Soret number (Sr).

Magnetic parameter leads to enhance the magnitude of skin friction coefficient and Sherwood number, because of the Lorentz force.

Table 3 Numerical values of skin friction coefficient(C_f), Nusselt number(Nu) and Sherwood number (Sh) for different values of θ_r , θ_c , Sr , Du and M

θ_r	θ_c	Sr	Du	M	C_f	Nu	Sh	
3	5	0.3	0.2	1	-4.32189	1.411027	0.722343	
6					-3.82188	1.413097	0.671041	
9					-3.68569	1.41366	0.656459	
12					-3.62203	1.41392	0.649553	
3	3	0.3	0.2	1	-4.27776	1.527918	0.728956	
	6				-4.33205	1.385589	0.720805	
	9				-4.34832	1.345937	0.71833	
	12				-4.35617	1.327278	0.717131	
3	5	0	0.2	1	-4.34789	1.406566	0.773702	
					1	-4.26061	1.421621	0.600661
					2	-4.1715	1.43722	0.422182
					3	-4.08044	1.4534	0.23794
3	5	0.3	0.0	1	-4.33951	1.429267	0.720804	
					0.3	-4.31305	1.401749	0.723121
					0.6	-4.28641	1.373261	0.72549
					0.9	-4.25959	1.343766	0.727914
3	5	0.3	0.2	0.0	-3.8433	1.413115	0.722264	
				0.5	-4.08658	1.412071	0.722303	
				1.0	-4.32189	1.411027	0.722343	
				1.5	-4.54963	1.409985	0.722382	

5. CONCLUSIONS

The Soret and Dufour effects on the flow, heat and mass transfer characteristics of an unsteady hydromagnetic flow due to an exponentially stretching sheet with viscous dissipation, Joule heating, variable viscosity and variable thermal conductivity have been made theoretically. The solution of this problem is obtained numerically using an implicit finite difference scheme based on Crank-Nicolson scheme, with Gauss-Seidel iteration method, by developing computer codes for MATLAB software. Graphical and tabular mode of presentation of the computed results illustrates the details of flow, heat and mass transfer characteristics and their dependence on the physical parameters involved in the problem. The important findings of this analysis are listed below:

- From the study, we have noticed that the effect of temperature on viscosity and thermal conductivity of the fluid is very significant.
- It is found that both fluid and dust phase velocity decreases with the enhancement of viscosity variation parameter and magnetic parameter.
- Thermal conductivity variation parameter and Soret number have increasing effects on the velocity profile, while an opposite trend is observed for temperature profile.
- An increase in the value of viscosity variation parameter leads to decrease the concentration profile of both the fluid and dust phases.
- The concentration profile is decreased with increases in Dufour number (or decrease in Soret number) while the temperature profile is enhanced consistently.
- Velocity, temperature and species concentration raise with higher values of time t for both the phases.
- Soret and Dufour numbers have opposite effects on the concentration as well as temperature profiles.
- The magnitude of skin friction coefficient increases with increases in the value of thermal conductivity variation parameter and magnetic parameter; whereas it decreases for increasing viscosity variation parameter, Soret number and Dufour number.
- It is noticed that rate of mass transfer decreases with increasing viscosity variation parameter and thermal conductivity variation parameter.
- An enhancement of the viscosity variation parameter leads to increase rate of heat transfer slightly.
- An increase in the Dufour number tends to decrease the Nusselt number and to increase the Sherwood number slightly. An opposite trend is observed in Soret number.

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NOMENCLATURE

- c_m specific heat of dust particles at constant pressure, $M^2T^2K^{-1}$
- c_p specific heat of fluid at constant pressure, $M^2T^2K^{-1}$
- C_s concentration susceptibility
- Dm mass diffusivity, M^2T^{-1}
- D_{md} coefficient of mean diffusivity
- K Stokes' resistance (drag co-efficient), MT^{-1}
- K_T thermal diffusion ratio, ML^{-3}
- l mass concentration
- m mass of the dust particle, M
- N number density of the particle phase
- T_m mean fluid temperature, K

Greek symbols

λ	thermal conductivity, $MLT^{-3}I^2$
λ_{∞}	thermal conductivity of the ambient fluid, $MLT^{-3}I^2$
μ	dynamic viscosity, $ML^{-1}T^{-1}$
μ_{∞}	dynamic viscosity of the ambient fluid, $ML^{-1}T^{-1}$
ρ	density of the fluid, ML^{-3}
ρ_p	density of the dust phase, ML^{-3}
σ	electrical conductivity, $M^{-1}L^{-3}T^3I$
τ_v	relaxation time of the dust particles i.e., the time required by a dust particle to adjust its velocity to the fluid
τ_T	thermal equilibrium time and is the time required by the dust cloud to adjust its temperature relative to the fluid
τ_c	solutal equilibrium time i.e., the time required by the dust particle to adjust its concentration relative to the fluid
ν_{∞}	kinematic viscosity of the fluid in the free stream, L^2T^{-1}
ω	constant density ratio

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