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THERMAL AND MOMENTUM SLIP EFFECTS ON HYDROMAGNETIC CONVECTION FLOW OF A WILLIAMSON FLUID PAST A VERTICAL TRUNCATED CONE

CH. Amanulla [a, b*], N. Nagendra [a], M. Suryanarayana Reddy [b]

[a] Department of Mathematics, Madanapalle Institute of Technology and Science, Madanapalle-517325, India.
[b] Department of Mathematics, JNTUA College of Engineering, Pulivendula-516390, Andhra Pradesh, India.

ABSTRACT

In this article, the combined theoretical and computational study of the magneto hydrodynamic heat transfer in an electro-conductive polymer on the external surface of a vertical truncated cone under radial magnetic field is presented. Thermal and velocity (hydrodynamic) slip are considered at the vertical truncated cone surface via modified boundary conditions. The Williamson viscoelastic model is employed which is representative of certain industrial polymers. The governing partial differential equations (PDEs) are transformed into highly nonlinear, coupled, multi-degree non-similar partial differential equations consisting of the momentum and energy equations via appropriate non-similarity transformations. These transformed conservation equations are solved subject to appropriate boundary conditions with a second order accurate finite difference method of the implicit type. Validation of the numerical solutions is achieved via benchmarking with earlier published results. The influence of Williamson viscoelastic fluid parameter, magnetic body force parameter, Thermal and velocity (hydrodynamic) slip parameters, stream wise variable and Prandtl number on thermos-fluid characteristics are studied graphically. The model is relevant to the simulation of magnetic polymer materials processing.

Keywords: *Magneto-hydrodynamics; Thermal convection; Williamson fluid parameter; Williamson model; Thermal and momentum slip.*

1. INTRODUCTION

Many modern engineering applications involve the study of non-Newtonian fluids. These include environmental engineering (2001), biological gels (2010), polymer processing (2009) and energy generation (2008). Most commonly, the viscosity of non-Newtonian fluids is dependent on shear rate. Some non- Newtonian fluids with shear-independent viscosity, however, still exhibit normal stress differences or other non-Newtonian behavior. Several salt solutions and molten polymers are non-Newtonian fluids, as are many other liquids encountered in science and technology such as dental creams, physiological fluids, detergents and paints. In a non-Newtonian fluid, the relation between the shear stress and the shear rate is generally non-linear and can even be time dependent. The Williamson viscoelastic model although simple is useful in simulating a number of polymers, for example alcoflood polymers employed in enhanced oil recovery (2002). Williamson (1929) first studied the flow of pseudo plastic materials. The Williamson fluid model tends to a Newtonian fluid at a very high wall shear stress i.e. when the wall stress is far greater than yield stress. To improve processing of many types of polymers, numerous investigators have conducted simulations of Williamson flow dynamics using many computational and analytical methods. These studies have included heat transfer (important for thermal treatment), mass transfer (critical to doping modification of polymers), viscous heating, Magnetohydrodynamic (for electro-conductive polymers) and many other phenomena. Hering and Grosh (1962) presented an early classical study on natural convection boundary layers' non-isothermal cone, showing that similarity solutions exist when the wall temperature distribution is a power function of distance along a cone ray. They further documented solutions for an isothermal surface as well as for the surface maintained at the temperature varying linearly with the

distance measured from the apex of the cone for Prandtl number of 0.7. Hering (1965) re-visited the problem in (1962) to consider low Prandtl number fluids (i.e. high thermal conductivity fluids such as liquid metals) for constant wall heat flux conditions. Further studies of heat transfer from conical bodies have been communicated by Alamgir (1979) who used an integral method, Hossain and Paul (2001) who considered surface blowing/suction effects and Chamkha (2001) who studied the case of heat transfer from a truncated cone with Magnetohydrodynamic and radiation flux effects. Bég et al. (2016) used the network electro-thermal PSPICE code to elaborate the influence of buoyancy, wall mass flux, pressure work and dissipation on hydromagnetic convection from a non-isothermal cone. Saleem and Nadeem (2015) used an optimized Homotopy method to derive series solutions for thermal convection slip flow from a rotating one, observing that increasing hydrodynamic slip decelerates the primary flow nearer the cone surface, accelerates the primary flow further from the cone surface and consistently reduces secondary velocity. Very recently Bég et al. (2016) presented novel solutions based on shooting quadrature for the heat transfer from a rotating cone in anisotropic porous media.

These studies however did not consider the Williamson model. This is a shear-thinning non-Newtonian model which quite accurately simulates polymer viscoelastic flows over a wide spectrum of shear rates. In Williamson fluids the viscosity is reduced with rising shear stress rates. This model has found some popularity in engineering simulations. In any case, by and large polymers are known to display non-Newtonian qualities. Engineers have thusly built up an assortment of constitutive models to investigate the shear stress strain attributes of these liquids, including viscoplastic, viscoelastic, small scale auxiliary and power-law models. Both purely fluid flow and heat transfer from a semi-infinite vertical plate to non-Newtonian fluids have been reported

* Corresponding author. Email: amanullamsc@gmail.com

in a number of theoretical investigations. Verma (1977) reported analytical solutions for viscoelastic boundary layer flow from a sphere. Dasman (2010) considered heat transfer in external boundary layer flows from a sphere for a variety of viscoelastic fluids. Subba Rao et al. (2016) investigated slip effects on Rheological Casson fluid flow from an Inclined vertical plate. They notice that the behavior of fluid on velocity and temperature distributions when thermal and velocity slips are considered. These studies however did not consider the Williamson model. This is a shear-thinning non-Newtonian model which quite accurately simulates polymer viscoelastic flows over a wide spectrum of shear rates. In Williamson fluids the viscosity is reduced with rising shear stress rates. This model has found some popularity in engineering simulations. Prasannakumara et al. (2016) analyzed reactive-radiative flow of Williamson viscoelastic fluid from a stretching sheet in a permeable Nano-materials with the Runge-Kutta-Fehlberg shooting method. Khan and Khan (2014) investigated Blasius, Sakiadis, stretching and stagnation point flows of Williamson fluid using the Homotopy analysis method, over a range of Weissenberg numbers. Bég et al. (2013) presented extensive numerical solutions for hydromagnetic pumping of a Williamson fluid using a modified differential transform method, observing that a change in Weissenberg number strongly modifies the pressure difference and axial velocity. Further studies of transport phenomena in Williamson fluids include Rao and Rao (2014) and Dapra and Scarpi (2007), Abegunrin et al. (2016).

The previous studies invariably assumed the classical "no-slip" condition at the boundary. Slip effects have however shown to be important in numerous polymeric transport processes including the production stage of polymers from the raw (monomeric) materials and in converting high-molecular-weight products into specific products (2000). Many researchers, primarily in chemical engineering have therefore studied, both experimentally and numerically, the influence of wall slip on polymer dynamics. Important works in this regard include Wang et al. (1996) who considered low density polyethylene liquids, Piau et al. (1995) who addressed polymer extrudates, Piau and Kissi (1994) who quantified macroscopic wall slip in polymer melts, Lim and Schowalter (1989) who studied boundary slip in polybutadiene flows and Hatzikiriakos and Kalogerakis (1994) who also studied molten polymer wall slip. Wall slip in thermal polymer processing was considered by Liu and Gehde (2016) in which slip was shown to significantly modify temperature distribution in polymers. Hatzikiriakos and Mitsoulis (2009) presented closed form solutions and finite element computations for wall slip effects on pressure drop of power law fluids in tapered dies. Many studies of both momentum (hydrodynamic or velocity) slip and thermal slip on transport phenomena have also been reported. Sparrow et al. (1962) presented the first significant analysis of laminar slip-flow heat transfer for tubes with uniform heat flux, observing that momentum slip acts to improve heat transfer whereas thermal slip (or "temperature jump") reduces heat transfer. Subba Rao et al. (2017) considered velocity on thermal slip effects on thermal convection boundary layer flow of Casson nanofluids with buoyancy effects. Uddin et al. (2016) used Maple software to compute the influence of anisotropic momentum, thermal, and multiple species slip on three-dimensional stagnation point boundary layers in nanofluid bio-convection flows.

Heat transfer with and without slip in external boundary layers from curved bodies are also of interest in polymeric enrobing systems. Subba Rao et al. (2016) studied numerically the combined effects of thermal and velocity slip and radiative flux on steady Casson enrobing boundary layer from a horizontal cylinder in porous media. They noted that greater velocity slip increases thermal boundary layer thickness and also momentum boundary layer thickness, whereas greater thermal slip decreases momentum boundary layer thickness (i.e. decelerates the flow) and cools the regime. Basir et al. (2016) considered transient nanofluid bioconvection boundary layer flow from a stretching horizontal cylinder with four slip mechanisms- thermal, velocity, nano-particle mass and micro-organism slip. Prasad et al. (2013) obtained finite difference solutions for nonlinear heat and momentum transfer

from a sphere in Darcy-Forchheimer porous media with velocity and thermal slip effects. Ahmed and Mahdy (2012) investigated magnetic free convection boundary layer nanofluid flow past a truncated cone. Elbashbeshy et al. (2016) analyzed the effects of heat generation/absorption and radiation on free convection flow over a truncated cone. Ram Reddy and Pradeepa (2016) studied viscous disspation and sorret effecte on natural convection flow over a truncated cone in the presence of Biot number. Subba Rao et al. (2015) reported on multiple slip effects in Casson boundary layer convection from a sphere observing that both thermal and hydrodynamic slip significantly alter surface skin friction and Nusselt numbers. Other studies on Williamson viscoelastic fluid include Hayat et al. (2016), Subba Rao et al. (2017).

In the present work, a mathematical model is developed for steady, free convection boundary layer flow in a Williamson viscoelastic polymeric fluid external to a semi-infinite vertical truncated cone. A finite difference numerical solution is obtained for the transformed nonlinear two-point boundary value problem subject to physically appropriate boundary conditions at the cone surface and in the free stream. The impact of the emerging thermo-physical parameters i.e. Weissenberg parameter, magnetic body force parameter, Prandtl number thermal and velocity (hydrodynamic) slips on velocity and temperature are presented graphically. The present problem has to the authors' knowledge not appeared thus far in the scientific literature and is relevant to electro-conductive thermal treatment of polymeric enrobing systems.

2. MATHEMATICAL THERMO-VISCOELASTIC FLOW MODEL

The regime under investigation is illustrated in Fig. 1. Steady, incompressible hydromagnetic Williamson non-Newtonian boundary layer flow and heat transfer from vertical cone body under radial magnetic field is considered. For an incompressible Williamson fluid, the continuity (mass conservation) and momentum equations are given as:

$$\text{div } V = 0, \tag{1}$$

$$\rho \frac{dV}{dt} = \text{div} S + \rho b, \tag{2}$$

Where ρ is the density of the fluid, V is the velocity vector, S is the Cauchy stress tensor, b represents the specific body force vector, and d/dt represents the material time derivate. The constitutive equations of the Williamson fluid model (2016, 2014, 2013, 2007) are given as:

$$S = -pI + \tau \tag{3}$$

$$\tau = \left[\mu_\infty + \frac{(\mu_0 - \mu_\infty)}{1 - \Gamma\dot{\gamma}} \right] A_1, \tag{4}$$

Here p is the pressure, I is the identity vector, τ is the extra stress tensor, μ_0 are the limiting viscosities at zero and at infinite shear rate, Γ is the time constant (>0), A_1 is the first Rivlin-Erickson tensor and $\dot{\gamma}$ is defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\pi} \tag{5}$$

$$\pi = trace(A_1^2) \tag{6}$$

Here we considered the case for which $\mu_\infty = 0$ and $\Gamma\dot{\gamma} < 1$. Thus eq. (4) can be written as:

$$\tau = \left(\frac{\mu_0}{1 - \Gamma\dot{\gamma}} \right) A_1 \tag{7}$$

Or by using binomial expansion we get:

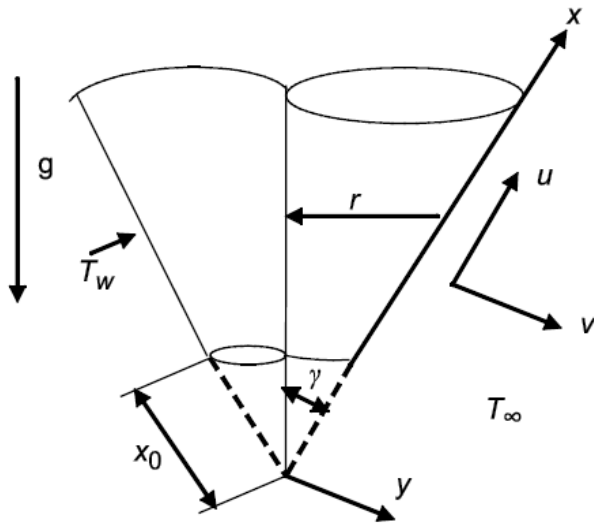$$\tau = \mu_0 \left[1 + \Gamma \dot{\gamma} \right] A_1 \tag{8}$$

Fig. 1 Magnetohydrodynamic non-Newtonian heat transfer from a vertical truncated cone

The two-dimensional mass, momentum and energy boundary layer equations governing the flow in an (x,y) coordinate system may be shown to take the form:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}$$
$$+ g\beta(T - T_\infty)\cos\gamma - \frac{\sigma B_0^2}{\rho}u \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{11}$$

The boundary conditions for the considered flow with velocity and thermal slip are:

At $y = 0, u = N_0\frac{\partial u}{\partial y}, v = 0, T = T_w + K_0\frac{\partial T}{\partial y}$

As $y \to \infty, u \to 0, v \to 0, T \to T_\infty$ \tag{12}

Here N_0 is the velocity slip factor, K_0 is the thermal slip factor and T_∞ is the free stream temperature. For $N_0 = 0 = K_0$, one can recover the no-slip case. The stream function ψ is defined by $ru = \frac{\partial\psi}{\partial y}$ and $rv = -\frac{\partial\psi}{\partial x}$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced:

$$\xi = \frac{\bar{x}}{x_0} = \frac{x - x_0}{x_0}, \ \eta = \frac{y}{x}(Gr_x)^{1/4}, \ \psi = r\nu(Gr_x)^{1/4}f(\xi,\eta)$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ Gr_x = \frac{g\beta\cos\gamma(T_w - T_\infty)x^{-3}}{\nu^2}, r = x\sin\gamma \tag{13}$$

The emerging momentum and heat (energy) conservation equations in dimensionless from assume the following form:

$$f''' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right)ff'' - f'^2 + Wef''f''' + \theta - Mf' =$$
$$\xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \tag{14}$$

$$\frac{1}{Pr}\theta'' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right)f\theta' = \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{15}$$

The transformed dimensionless boundary conditions are reduced to:

At $\eta = 0, f = 0, f' = S_f f''(0), \theta = 1 + S_T\theta'(0)$

As $\eta \to \infty, f' \to 0, \theta \to 0$ \tag{16}

The skin-friction coefficient (cone surface shear stress) and the local Nusselt number (cone surface heat transfer rate) can be defined, respectively, using the transformations described above with the following expressions:

$$\frac{1}{2}Gr_x^{1/4}C_f = f''(\xi,0)\left(1 + \frac{We}{2}f''(\xi,0)\right) \tag{17}$$

$$Gr_x^{-1/4}Nu = -\theta'(\xi,0) \tag{18}$$

All parameters are defined in the nomenclature.

3. COMPUTATIONAL SOLUTION WITH KELLER BOX IMPLICT METHOD

The coupled boundary layer equations in a (ξ,η) coordinate system remain strongly nonlinear. A numerical method, the Keller-Box implicit difference method, is therefore deployed to solve the boundary value problem defined by Ens. (14) - (15) with boundary conditions (16). This technique has been described succinctly in Cebeci and Bradshaw (1984) and Keller (1970). It has been used recently in polymeric flow dynamics by Subba Rao et al. (2017) and Amanulla et al. (2017) for viscoelastic models. The key stages involved are as follows:
a. Reduction of the Nth order partial differential equation system to N first order equations
b. Finite difference discretization
c. Quasilinearization of non-linear Keller algebraic equations
d. Block-tridiagonal elimination of linear Keller algebraic equations

4. VALIDATION OF KELLER BOX SOLUTIONS

The present Keller box solutions have been validated for the special case of non-magnetic ($M = 0$) Newtonian flow ($We = 0$) in the absence of slip effects ($S_f = S_T = 0$). This case was considered earlier by Na and Chiou (1979), Yih (1999), and Ram Reddy and Venkata Rao (2017). Furthermore, it is also possible to make a comparison as the momentum equation and boundary conditions assume the following reduced form:

$$f''' + \left(\frac{3}{4} + \frac{\xi}{1+\xi}\right)ff'' - f'^2 + \theta = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \tag{19}$$

At $\eta = 0; f = 0; f' = 0, \theta = 1$

As $\eta \to \infty : f' \to 0; \theta \to 0$.

The energy equation (16) is identical to that considered in Na and Chiou (1979), Yih (1999). The comparison of solutions is documented in **Table 1 & 2**. Excellent correlation is achieved and confidence in the present solutions is therefore justifiably high.

Table 1 Comparison of values of Cf for different values of Pr when $We = 0$, $S_f = S_T = 0.0$, $M = 0.0$.

Pr	Cf		
	Yih (1999)	Reddy and Rao (2017)	Present study
0.1	1.2144	1.21446	1.2144612
1.0	0.9084	0.90819	0.908081
10.0	0.5927	0.59283	0.592903
100.0	0.3559	0.35587	0.355771
1000.0	0.2049	0.19648	0.196307

Table 2 Comparison of values of Nu for different values of Pr when $We = 0$, $S_f = S_T = 0.0$, $M = 0.0$

| Pr	Nu			
	Na and Chiou (1979)	Yih (1999)	Reddy and Rao (2017)	Present study
0.1	-	0.1629	0.16275	0.1626481
1.0	0.4010	0.4012	0.40103	0.400763
10.0	0.8269	0.8258	0.82684	0.826930
100.0	1.5493	1.5493	1.54948	1.549484
1000.0	-	2.8035	2.79895	2.797412

5. RESULTS AND DISCUSSION

Extensive computations have been conducted using the Keller box code to study the influence of the key thermo-physical parameters on velocity, temperature, skin friction and Nusselt number. These are visualized in **figs. 2a-b** to **7a-b.**
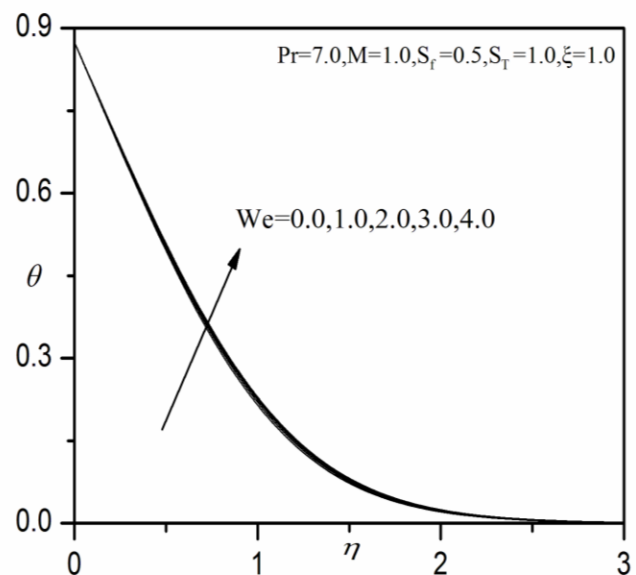
Figs 2- 8 present velocity and temperature distributions for variations in specific key thermo physical parameters, namely magnetic body force parameter (M), hydrodynamic slip (S_f), thermal slip (S_T), Williamson viscoelastic fluid parameter (We) and Prandtl number (Pr). In all graphs ξ is constrained as unity. **Fig. 5a** exhibits the effect of velocity slip parameter, S_f on the velocity. It is seen apparent that the flow is markedly decelerated in the vicinity of the cone surface with greater wall momentum slip. Velocity peaks some distance from the wall and thereafter momentum slip induces a notable acceleration in the flow i.e. decreases momentum boundary layer thickness The slip effect is therefore opposing near the cone surface and assistive further into the boundary layer transverse to the cone surface. Greater momentum slip retards the fluid motion near and at the upper wall. Moreover, dragging of the fluid adjacent to the cone surface is partially transmitted into the fluid partially which induces a deceleration near the wall; however, this is eliminated and reversed further from the cone surface. The peak velocity is observed to migrate with greater momentum slip further from the cone surface.

Figs. 2a-b, depict the effect of Williamson viscoelastic fluid parameter, We on velocity and temperature profiles. It is shown that the effect of We reduces velocity near the cone surface but depletes it further away. Increasing Williamson viscoelastic fluid parameter however consistently weakly increases temperatures throughout the boundary layer. The influence on velocity field is significantly greater however since the viscoelastic effect is simulated solely in the momentum equation (14) via the shear term (mixed derivative)

$We f'' f'''$. Williamson viscoelastic fluid parameter (We) measures the relative effects of viscosity to elasticity. Williamson viscoelastic fluid parameter of zero corresponds to a purely Newtonian fluid, and infinite Weissenberg number corresponds to a purely elastic solid. The effect of viscoplastic parameter is indirectly transmitted to the temperature field. Since the Williamson viscoelastic fluid parameter is also present in the wall boundary condition, the acceleration effect is only confined to the region close to the cone surface. Overall however the dominant influence of We, is near the wall and is found to be assistive to momentum development. Only a very small decrease in temperature is observed with a large enhancement in Weissenberg parameter, as shown in Fig.3b. Thermal boundary layer thickness is therefore enhanced with increasing We values i.e. decreasing viscosity and increasing elastic effects. Effectively therefore Newtonian fluids ($We = 0$) achieve lower velocities and temperatures than Williamson fluids. Similar trends have been reported by Hayat et al. (2016) and Khan and Khan (2014).
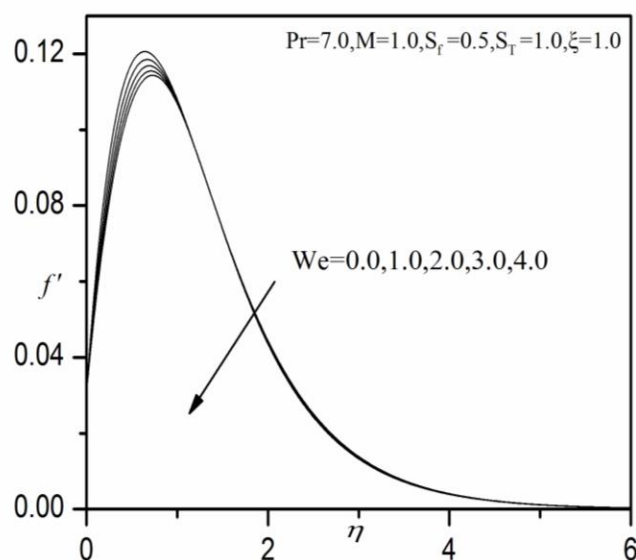
(a)

(b)

Fig. 2 Effect of We on (a) velocity profiles and (b) temperature profiles
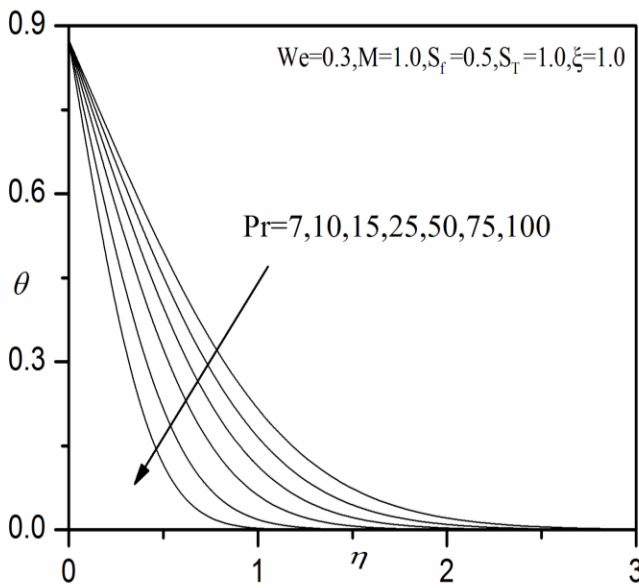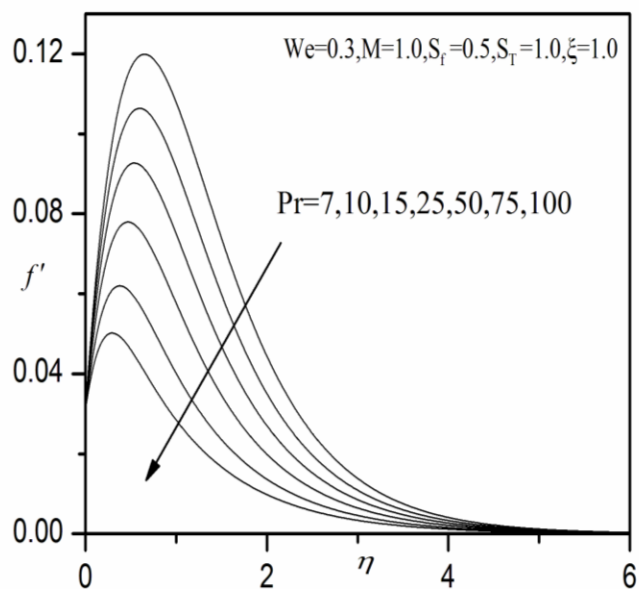
(a)

(b)

Fig.3 Effect of *Pr* on (a) velocity profiles and (b) temperature profiles

Figs. 3a − 3b, present the impact of Prandtl number (*Pr*) on the velocity and temperature profiles along the transverse coordinate i.e. normal to the cone surface. Prandtl number epitomizes the ratio of momentum diffusion to thermal diffusion in the boundary layer regime. It also represents the ratio of the product of specific heat capacity and dynamic viscosity, to the fluid thermal conductivity. For *Pr equal to unity* both the momentum and thermal diffusion rates are the same, as are the momentum and thermal boundary layer thicknesses. An increment in *Pr* from 7.0 through 10, 15, 25, 50, 75 to 100, which corresponds to increasing momentum diffusivity and decreasing thermal diffusivity, results in a tangible reduction in velocity magnitudes throughout the boundary layer. For *Pr* <*1*, thermal diffusivity surpasses momentum diffusivity i.e. heat will diffuse at a faster rate than momentum. In this manner for lower *Pr* fluids (e.g. *Pr = 0.01* which physically relate to liquid metals), the flow will be accelerates whereas for greater *Pr* fluids (e.g. *Pr = 1* for low weight molecular polymers (2004,2015) it will be strongly decelerated, as observed in Fig.3b. For *Pr < 1*, the momentum boundary layer thickness is lesser than thermal boundary layer thickness. The

asymptotically smooth profiles in the free stream (high η values) confirm that an adequately large infinity boundary condition has been imposed in the Keller box numerical code

Fig.4a-b illustrates that the velocity is reduced with an increase in *M*. This indicates that the transverse magnetic field opposes the transport phenomena since an increase in *M* leads to an increase in the Lorentz force, which opposes the transport process. This stronger Lorentz force produces more resistance to the transport. The higher the value of *M*, the more prominent is the reduction in hydrodynamic boundary layer thickness. But from Figs.4b, the opposite phenomenon is observed with an increase in Magnetic field parameter *M* on temperature filed. The excess work expended in dragging the polymer against the action of the magnetic field is dissipated as thermal energy (heat). This energizes the boundary layer and increases thermal boundary layer thickness. Again the influence of magnetic field is sustained throughout the entire boundary layer domain. These results concur with other investigations of magnetic non-Newtonian heat transfer including Kasim *et al.* (2013) and Megahed (2012).
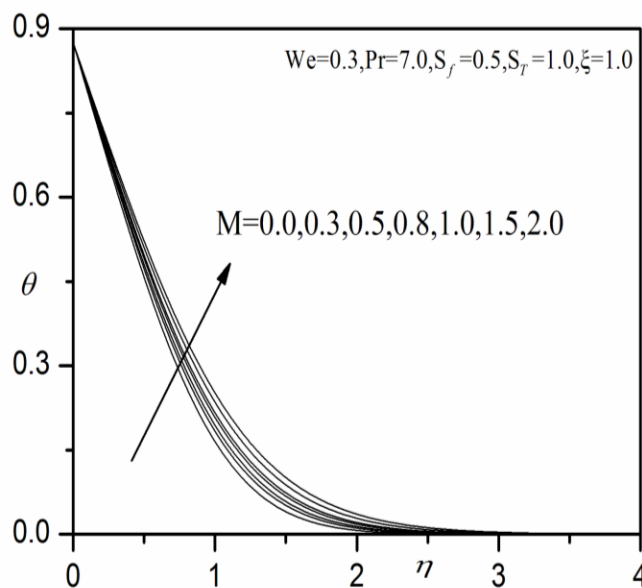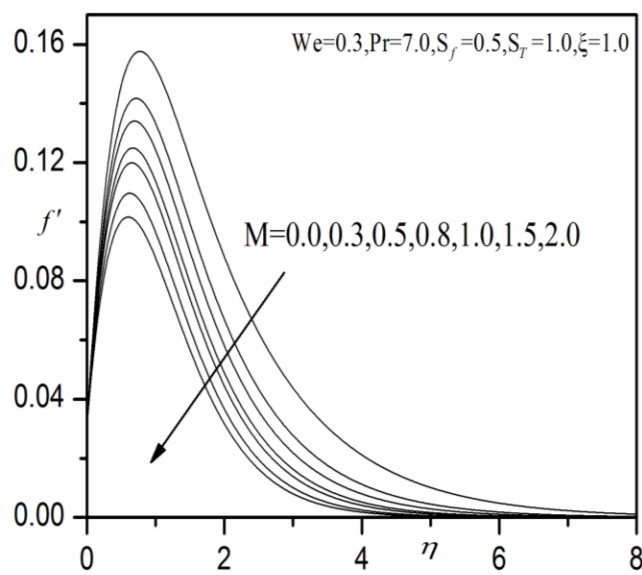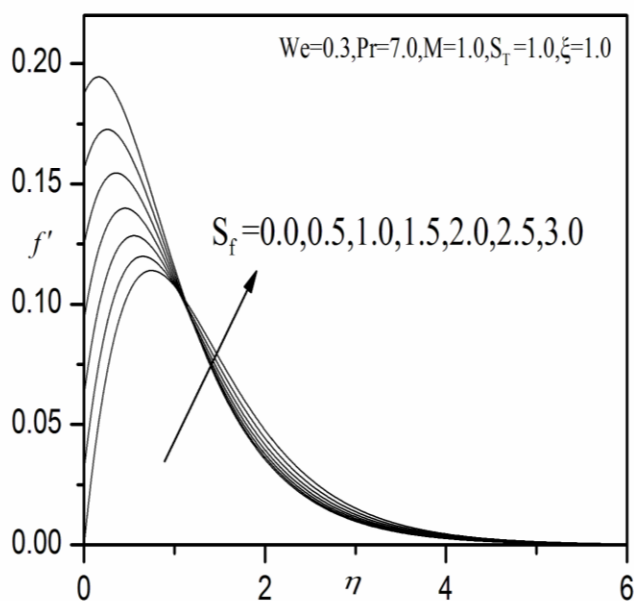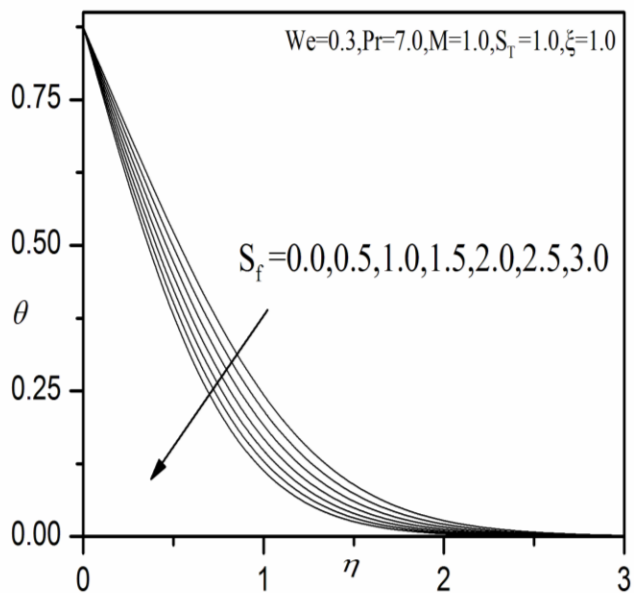

(a)

(b)

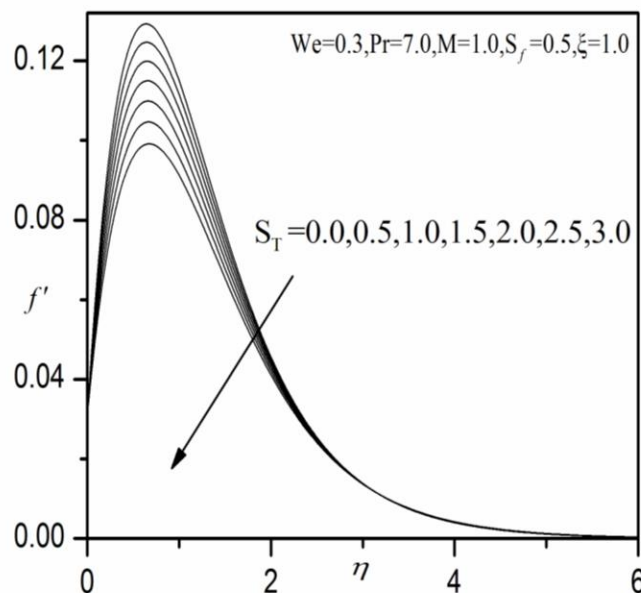**Fig. 4** Effect of *M* on (a) velocity profiles and (b) temperature profiles

Momentum boundary layer thickness is enhanced whereas thermal boundary layer thickness is decreased with increasing thermal slip. A similar response has been observed by Basir *et al.* (2016). Physically, as the thermal slip parameter rises, the fluid flow within the boundary layer becomes progressively less sensitive to the heating effects at the cone surface and a decreased quantity of thermal energy (heat) is transferred from the hot cone surface to the fluid, resulting in a fall in temperatures, manifesting in a cooling and thinning of the thermal boundary layer. This has important implications in thermal polymer enrobing, since thermal slip modifies the heat transferred to the polymer material which in turn alters characteristics of the final product (1990).

(a)

(b)

Fig. 5 Effect of S_f on (a) velocity profiles and (b) temperature profiles
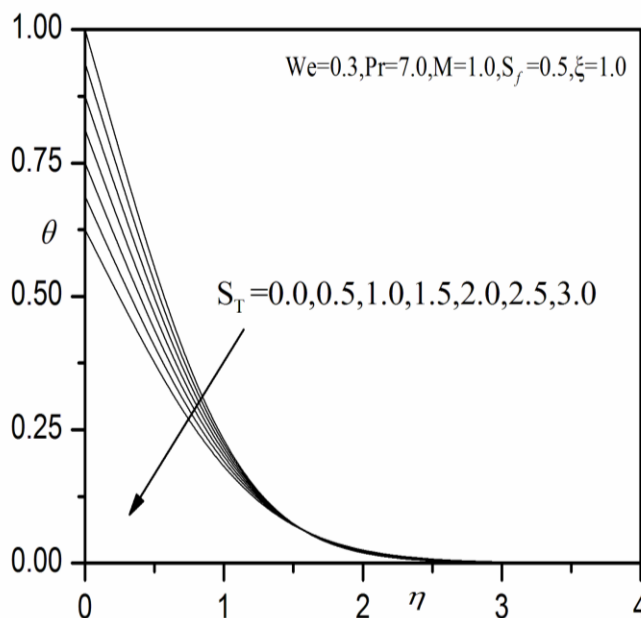
Fig.5a-b presents the evolution in temperature function, $\theta(\eta)$, with transverse coordinate η with variation in hydrodynamic slip parameter, S_f. Temperature profiles consistently decay monotonically from a maximum at the cone surface to the free stream. All profiles converge at a large value of transverse coordinate, again showing that a sufficiently large infinity boundary condition has been utilized in the numerical computations. Greater momentum slip substantially increases temperatures in the boundary layer and therefore also elevates thermal boundary layer thickness. The regime is therefore coolest when slip is absent (S_f =0 i.e. *no-slip classical case*) and hottest with strong hydrodynamic wall slip.

Figs. 6a and 6b illustrate the influence of thermal slip parameter (S_T) on the velocity and temperature. Both velocity and temperature are consistently suppressed with an increase in S_T. Temperatures are strongly depressed in particular at the cone surface. Greater thermal jump therefore decelerates the flow and cools the boundary layer.

(a)

(b)

Fig. 6 Effect of S_T on (a) velocity profiles and (b) temperature profiles

Figs. 7a-b illustrate the influence of the stream wise (tangential) coordinate, ξ, on the velocity and temperature distributions. A weak deceleration in the boundary layer flow is experienced with greater ξ, values i.e. with progressive distance along the cone surface from the lower stagnation point ($\xi=0$), as shown in fig. 7a. Momentum boundary

layer thickness is therefore increased marginally with ξ values. Conversely a weak enhancement in temperature is computed in fig. 7b, with increasing ξ values. Thermal boundary layer thickness is increased therefore as we progress from the lower stagnation point on the cone surface around the cone periphery upwards.

(a)

(b)
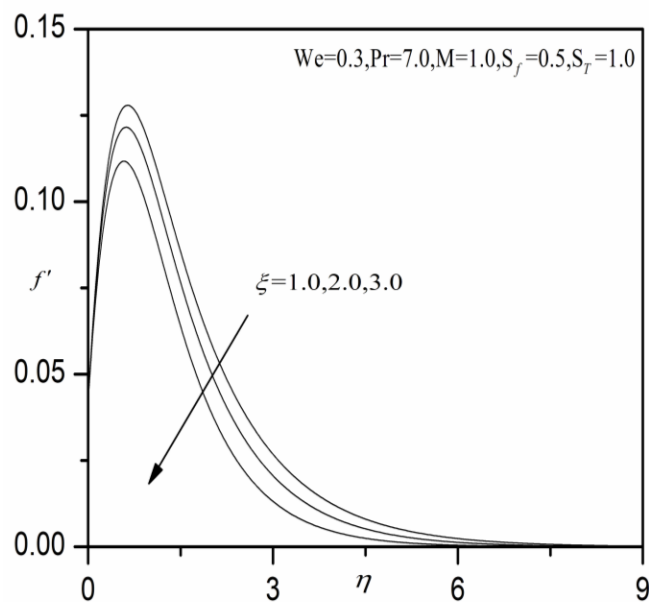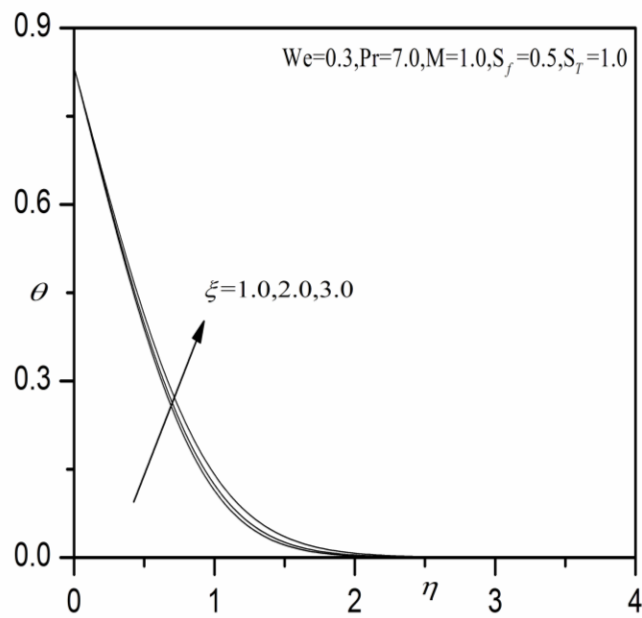
Fig. 7 Effect of ξ on (a) velocity profiles and (b) temperature profiles

6.   CONCLUSIONS

A theoretical study has been conducted of laminar incompressible free convection boundary layer flow of a viscoelastic (Williamson) fluid from a vertical truncated cone. Magnetic field, Momentum and thermal slip effects have been incorporated in the model. The transformed boundary layer equations for heat and momentum conservation have been solved using a finite difference method for the case of non-similar solutions present at the cone surface. Verification of the accuracy of the Keller box computational code has been achieved

via comparison with previous Newtonian solutions reported in the literature. The present investigation has shown that:

1) Increasing the velocity slip parameter (Sf) reduces the velocity near the cone surface and increases the temperature i.e. enhances momentum boundary layer thickness and decreases thermal boundary layer thickness. However, flow reversal is never computed.

2) Increasing thermal slip parameter, (S_T) consistently decelerates the flow and also decreases temperature (and thermal boundary layer thickness).

3) Increasing Williamson viscoelastic fluid parameter (We), decreases the velocity near the cone surface and also fractionally lowers the temperature throughout the boundary layer regime i.e. enhances thermal boundary layer thickness.

4) Increasing Prandtl number (Pr) decelerates the flow and also strongly depresses temperatures, throughout the boundary layer regime.

NOMENCLATURE

B_0	externally imposed radial magnetic field
C_f	skin friction coefficient
f	non-dimensional steam function
Gr	Grashof number
g	acceleration due to gravity
k	thermal conductivity of fluid
x_0	the leading edge distance of a truncated cone measured from the origin
K_0	thermal jump factor
Nu	local Nusselt number
M	magnetic body force parameter
Pr	Prandtl number
N_0	velocity (momentum) slip factor
S_f	non-dimensional velocity slip parameter
S_T	non-dimensional thermal jump parameter
T	temperature
u, v	non-dimensional velocity components along the x- and y-directions, respectively
We	Weissenberg (viscoelasticity) number
x	stream wise coordinate
y	transverse coordinate

Greek Symbols

α	thermal diffusivity
β	coefficient of thermal expansion
η	dimensionless transverse coordinate
γ	Half angle of the cone
ν	kinematic viscosity
θ	non-dimensional temperature
ρ	density of viscoelastic fluid
σ	electrical conductivity of viscoelastic fluid
ξ	dimensionless steam wise coordinate
ψ	dimensionless stream function
Γ	time-dependent material constant

Subscripts

w	conditions on the wall
∞	Free stream conditions

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