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SLIP EFFECTS ON BOUNDARY LAYER FLOW AND MASS TRANSFER WITH CHEMICAL REACTION OVER A PERMEABLE FLAT PLATE IN A POROUS MEDIUM

Krishnendu Bhattacharyya*

Department of Mathematics, The University of Burdwan, Burdwan-713104, West Bengal, India

ABSTRACT

A mathematical model is presented to analyse the steady boundary layer slip flow and mass transfer with n^{th} order chemical reaction past a porous plate embedded in a Darcy porous medium. Velocity as well as mass slips are considered at the boundary. The governing PDEs are transformed into self-similar nonlinear ODEs by similarity transformations. The reduced nonlinear equations are solved numerically. The momentum boundary layer thickness is reduced for increase of permeability and suction parameters, whereas it increases with blowing parameter. The increase of velocity slip parameter reduces the momentum boundary layer thickness and also enhances the mass transfer from the plate. Importantly, due to increase of mass slip the concentration and mass transfer decrease.

Keywords: *Boundary layer flow, slip condition, mass transfer, flat plate, porous medium, suction/blowing.*

1. INTRODUCTION

The development of boundary layer due to the flow of viscous fluid past a flat plate is very important in fluid dynamics. The flow and heat transfer over a flat plate has been widely studied from both theoretical and experimental standpoint in the past few decades. The formation of the velocity boundary layer due to the flow on a flat plate was first discussed by Blasius (1908) and the heat transfer for this problem was investigated by Pohlhausen (1921). Howarth (1938) numerically studied the various aspects of the Blasius flat plate flow problem. The existence of a solution for the flow past a flat plate was established by Abu-Sitta (1994). Further some important aspects of flat plate flow were studied by Wang (2004), Cortell (2005) and Batallar (2008).

Recently, considerable attention has been devoted to the study of boundary layer flow of a Newtonian fluid past a flat plate embedded in a fluid saturated porous medium because of its wide applications in engineering processes, especially in the enhanced recovery of petroleum resources and packed bed reactors and in chemical engineering. Cheng (1977) and Cheng and Minkowycz (1977) explained the free convective flow in a saturated porous medium. Vafai and Tien (1981) investigated the boundary and inertia effects on flow and heat transfer in porous media. Kumari et al. (1990) reported the non-Darcian effects on forced convective heat transfer over a flat plate in a highly porous medium. Mukhopadhyay and Layek (2009) presented the radiation effects on forced convective flow and heat transfer over a porous plate in a porous medium. Many important characteristics of Darcian and non-Darcian convection about a plate were discussed by Pop and Takhar (1983), Hsu and Cheng (1985), Hong et al. (1987) and Rashad (2008).

In addition to the heat transfer, the mass transfer phenomenon in porous medium is also grabbed attention of researchers due to its huge applications in chemical industries, reservoir engineering and many other technological processes. Lai and Kulacki (1991) discussed the coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium. Postelnicu (2007) described the influence of chemical reaction on heat and mass transfer by natural convection from vertical surface in porous media by taking into account the Soret and Dufour effects.

In every the investigations mentioned above, the no-slip condition at the boundary was assumed. The assumption of no-slip condition does no longer valid under certain circumstances and should be replaced by a partial slip boundary condition relating to the shear rate at the boundary. The partial slip condition had been used in studies of fluid flow past permeable wall by Beavers and Joseph (1967). Martin and Boyd (2006) considered the momentum and heat transfer in a laminar boundary layer flow over a flat plate with slip boundary condition. Aziz (2010) studied the boundary layer slip flow over a flat plate with constant heat flux condition at the surface and in this paper the local similarity was appeared in the slip boundary condition. Very recently, Bhattacharyya et al. (2011a) discussed the MHD slip flow over a flat plate. The effects of slip boundary condition on the flow of Newtonian fluid due to a stretching sheet ware explained by Andersson (2002) and Wang (2002).

The aim of this paper is to study the steady boundary layer slip flow and mass transfer with chemical reaction past a porous plate placed in porous medium using the Darcy model. Mass slip condition in addition to the velocity slip is also considered which gives interesting features regarding such flow. The slip model of Andersson (2002) is taken here in some modified form (2011). A complete self-similar set of equations are obtained. No local similarity is appeared at the boundary conditions. The equations with the boundary conditions are then solved numerically using shooting method. Computed numerical results are plotted and the characteristics of the flow and mass transfer are thoroughly analyzed.

* *Email: krish.math@yahoo.com; Fax: (+91)342 2530452, Tel: (+91)9474634200*

2. FLOW PROBLEM FORMULATION

Consider the steady two-dimensional boundary layer flow of a viscous incompressible fluid and mass transfer with n^{th} order chemical reaction past a porous flat plate in porous medium. The governing equations of motion and the concentration equation may be written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{k}(u-U_\infty) \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty)^n \tag{3}$$

where u and v are velocity components in x- and y-directions respectively, υ $(=\mu/\rho)$ is the kinematic fluid viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, U_∞ is the free stream velocity, k is the permeability of the porous medium, C is the concentration, D is the diffusion coefficient, C_∞ is the concentration in the free stream and n is the order of chemical reaction. $R(x)$ is the variable reaction rate and is given by $R(x)=R_0(L/x)$, L is the reference length and R_0 is a constant. The boundary conditions with partial slip for the velocity and the concentration are given by

$u=L_1(\partial u/\partial y)$, $v=v_w$ at $y=0$; $u\to U_\infty$ as $y\to\infty$ (4)

and $C=C_w+D_1(\partial C/\partial y)$ at $y=0$; $C\to C_\infty$ as $y\to\infty$. (5)

Here $L_1=L^*(Re_x)^{1/2}$ is the velocity slip factor and $D_1=D^*(Re_x)^{1/2}$ is the mass slip factor with L^* and D^* being initial values of velocity and mass slip factors having same dimension of length and Re_x being the local Reynolds number and $Re_x=U_\infty x/\upsilon$, C_w is the concentration of the plate assumed to be constants. Here v_w is prescribed distribution of suction or blowing through the porous plate and is given by $v_w=v_0/(x)^{1/2}$, v_0 being constant with $v_0<0$ suction and $v_0>0$ blowing.

We now introduce the stream function $\psi(x,y)$ as

$$u=\frac{\partial\psi}{\partial y}\text{ and }v=-\frac{\partial\psi}{\partial x}\,. \tag{6}$$

Now for relations in (6), the continuity equation (1) is satisfied automatically. Using (6), the momentum equation (2) and the concentration equation (3) take the following forms:

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\upsilon\frac{\partial^3\psi}{\partial y^3}-\frac{\upsilon}{k}\left(\frac{\partial\psi}{\partial y}-U_\infty\right), \tag{7}$$

$$\text{and }\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty)^n\,. \tag{8}$$

The boundary conditions in (4) for the velocity components reduce to

$$\frac{\partial\psi}{\partial y}=L_1\frac{\partial^2\psi}{\partial y^2},\ \frac{\partial\psi}{\partial x}=-v_w\text{ at }y=0;\ \frac{\partial\psi}{\partial y}\to U_\infty\text{ as }y\to\infty\,. \tag{9}$$

Next, we introduce the dimensionless variables for ψ and C as given below:

$$\psi=\sqrt{U_\infty\upsilon x}\,f(\eta)\text{ and }C=C_\infty+(C_w-C_\infty)\phi(\eta)\,, \tag{10}$$

where η is the similarity variable and is defined as $\eta=(y/x)(Re_x)^{1/2}$.

In view of relations in (10) we finally obtain following self-similar equations as

$$f'''+\frac{1}{2}ff''-\frac{1}{Da_xRe_x}(f'-1)=0 \tag{11}$$

$$\text{and }\phi''+\frac{1}{2}Sc\,f\phi'-Sc\beta\phi^n=0\,, \tag{12}$$

where $Da_x=k/x^2=k_0/x$ is the local Darcy number, $k=k_0x$, k_0 is a constant, $Sc=\upsilon/D$ is the Schmidt number and $\beta=LR_0(C_w-C_\infty)^{n-1}/U_\infty$ is the reaction rate parameter. It is to be noted that the chemical reaction is destructive if $\beta>0$ and the chemical reaction is constructive if $\beta<0$.

Now the equation (11) can be written as

$$f'''+\frac{1}{2}ff''-k^*(f'-1)=0\,, \tag{13}$$

where $k^*=1/(Da_xRe_x)$ is the permeability parameter of the porous medium [Mukhopadhyay and Layek (2009)].

The boundary conditions (9) and (5) reduce to the following forms:

$f(\eta)=S$, $f'(\eta)=\delta f''(\eta)$ at $\eta=0$; $f'(\eta)\to 1$ as $\eta\to\infty$ (14)

and $\phi(\eta)=1+\gamma\phi'(\eta)$ at $\eta=0$; $\phi(\eta)\to 0$ as $\eta\to\infty$, (15)

where $S=(-2v_w/U_\infty)(Re_x)^{1/2}=-2v_0/(U_\infty\upsilon)^{1/2}$ is the suction/blowing parameter, $S>0$ (i.e. $v_0<0$) corresponds to suction and $S<0$ (i.e. $v_0>0$) corresponds to blowing, $\delta=L^*U_\infty/\upsilon$ is the velocity slip parameter and $\gamma=D^*U_\infty/\upsilon$ is the mass slip parameter.

3. NUMERICAL METHOD FOR SOLUTION

The nonlinear coupled differential equations (13) and (12) along with the boundary conditions (14) and (15) form a two point boundary value problem (BVP) and are solved using shooting method [Bhattacharyya (2011a,b,c), Bhattacharyya et al. (2011b,c)], by converting it into an initial value problem (IVP). In this method we have to choose a suitable finite value of $\eta\to\infty$, say η_∞. We set following first-order system

$$f'=p,\ p'=q,\ q'=-\frac{1}{2}fq+k^*(p-1) \tag{16}$$

$$\text{and }\phi'=r,\ r'=-\frac{1}{2}Sc\,fr+Sc\beta\phi^n \tag{17}$$

with the boundary conditions

$f(0)=0$, $p(0)=\delta q(0)$, $\phi(0)=1+\gamma r(0)$. (18)

To solve (15) and (16) with (18) as an IVP we must need values for $q(0)$ i.e. $f''(0)$ and $r(0)$ i.e. $\phi'(0)$ but no such values are given. The initial guess values for $f''(0)$ and $\phi'(0)$ are chosen and applying fourth order Runge-Kutta method a solutions is obtained. We compare the calculated values of $f'(\eta)$ and $\phi(\eta)$ at $\eta_\infty(=20)$ with the given boundary conditions $f'(\eta_\infty)=1$ and $\phi(\eta_\infty)=0$ and adjust values of $f''(0)$ and $\phi'(0)$ using "secant method" to give better approximation for the solution. The step-size is taken as $\Delta\eta=0.01$. The process is repeated until we get the results correct up to the desired accuracy of 10^{-6} level.

4. RESULTS AND DISCUSSION

The numerical computations is performed for several values of dimensionless parameters viz., the permeability parameter k^*, suction/blowing parameter S, Schmidt number Sc, the reaction rate parameter β, order of reaction n, the velocity slip parameter δ and mass slip parameter γ. For illustrating obtained data, some figures are plotted and are explained in detailed.

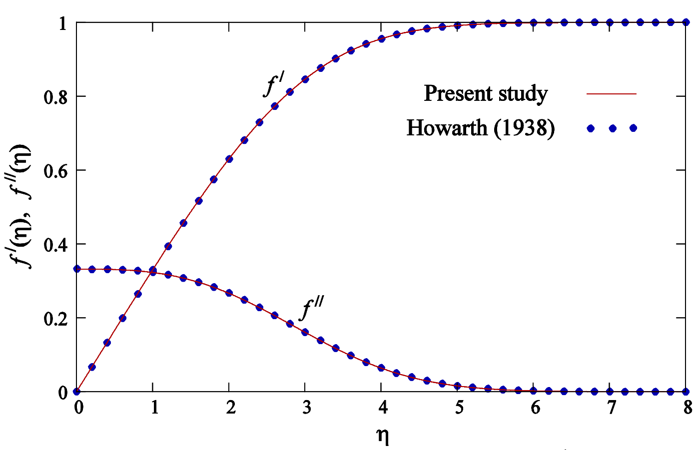

Fig. 1 Velocity $f'(\eta)$ and shear stress $f''(\eta)$ profiles for $k^*=0$ and $\delta=0$.

At first, for the verification of the accuracy of the applied numerical method we compare our results corresponding to the velocity and shear stress profiles for $k^*=0$ and $\delta=0$ (i.e. in non-porous medium and in absence of slip at the boundary) with the available published results of Howarth (1938) in Fig. 1 and are found in excellent agreement.

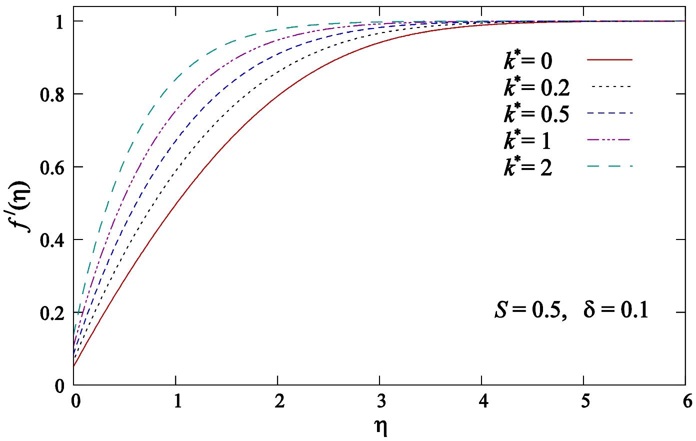

Fig. 2 Velocity profiles $f'(\eta)$ for various values of k^*.

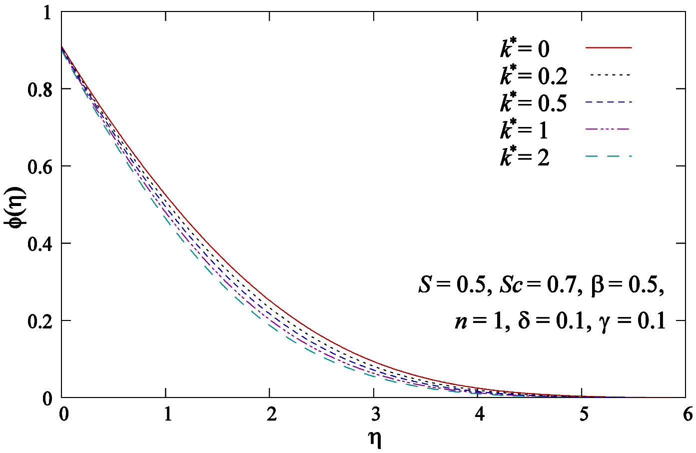

Fig. 3 Concentration profiles $\phi(\eta)$ for various values of k^*.

Now, we see the influence of the permeability parameter k^* on the velocity and concentration profiles. Fig. 2 shows the variation in velocity field for several values of k^* and corresponding concentration profiles are depicted in Fig. 3. With increase of k^* the dimensionless velocity $f'(\eta)$ along the plate increases and consequently the momentum boundary layer thickness decreases. With a rise in permeability of the medium, the regime becomes more porous. As a consequence, the Darcian body force decreases in magnitude (as it is inversely proportional to the permeability). The Darcian resistance acts to decelerate the fluid particles in continua. This resistance diminishes as permeability of the medium increases. So progressively less drag is experienced by the flow and flow retardation is thereby decreased. Thus the permeability parameter enhances the fluid motion inside the boundary layer. From Fig. 3, it is observed that the concentration $\phi(\eta)$ at a point decreases with k^*. Increase in the permeability parameter k^* causes to decrease the solute boundary layer thickness. The rate of mass transfer is enhanced with increase of k^*.

We now discuss deviation of velocity and concentration distribution for the variation of suction/blowing parameter S in porous medium. The velocity and concentration distributions for various values of S are shown in Fig. 4 and Fig. 5, respectively. With the increasing S (S>0), fluid velocity is found to increase [Fig. 4] i.e. suction causes to increase the velocity of the fluid inside the boundary layer. By sucking fluid particles through permeable plate the momentum boundary layer thickness is reduced and consequently, the velocity increases. But when fluid particles are injected i.e. for blowing case (S<0) the momentum boundary layer thickness becomes larger. Fig. 5 shows that the dimensionless concentration $\phi(\eta)$ decreases with the increasing suction parameter and the solute boundary layer thickness decreases with the suction parameter (S>0). Due to this the rate of mass transfer also increases. Whereas, the concentration increases with increase of blowing (S<0).

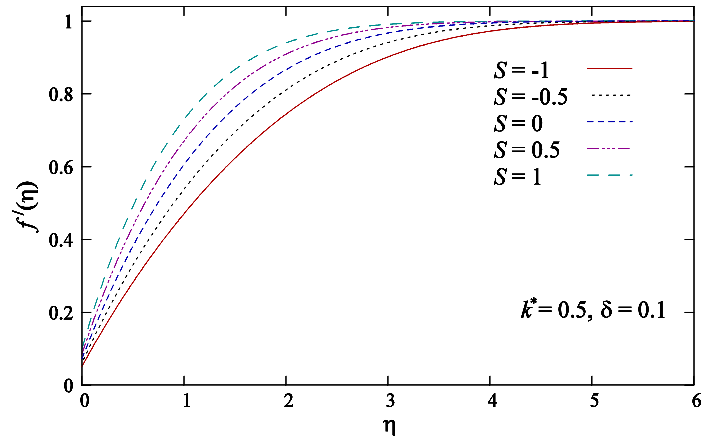

Fig. 4 Velocity profiles $f'(\eta)$ for various values of S.

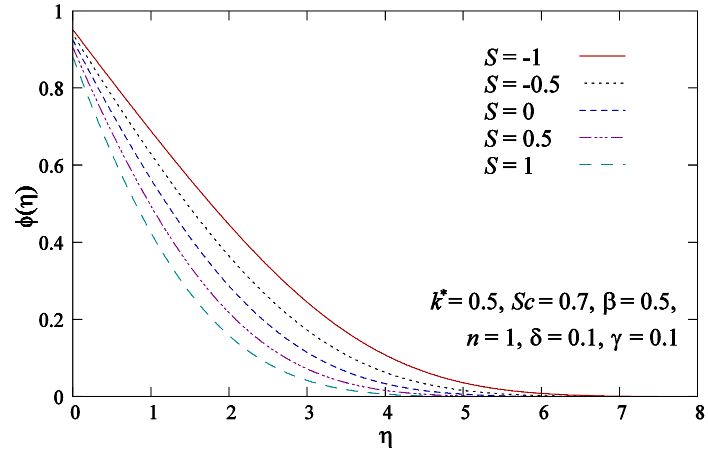

Fig. 5 Concentration profiles $\phi(\eta)$ for various values of S.

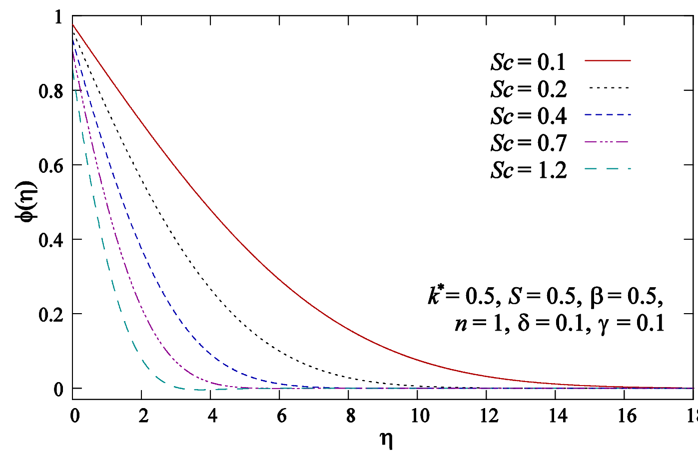

Fig. 6 Concentration profiles $\phi(\eta)$ for various values of Sc.

Figures 6 – 8 demonstrate the dimensionless solute profiles for various values of Schmidt number Sc, reaction rate parameter β and the order of reaction n in presence mass slip. The concentration at a point and the solute boundary layer thickness rapidly decrease with increasing values of Sc. Due to increase Schmidt number the diffusion coefficient decreases and consequently the thickness of solute boundary layer reduces. The destructive reaction (β>0) causes a decrease of concentration and reversely the constructive reaction (β<0) results an increase of concentration. On other hand, the outcome of increase of reaction order is very small increment of concentration and when n increases continuously the concentration increment almost died out after certain level.

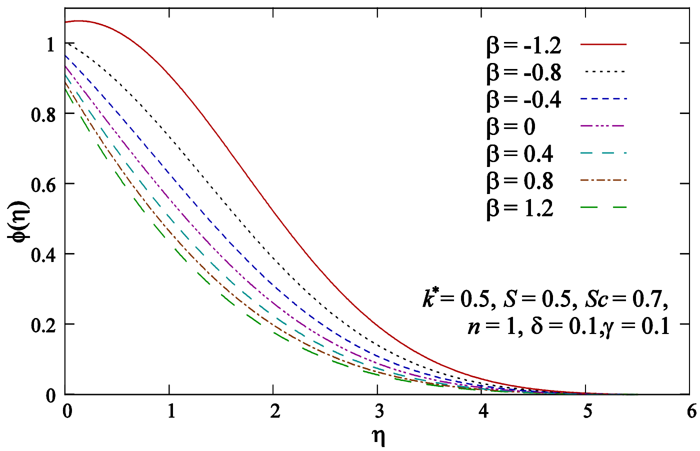

Fig. 7 Concentration profiles $\phi(\eta)$ for various values of β.

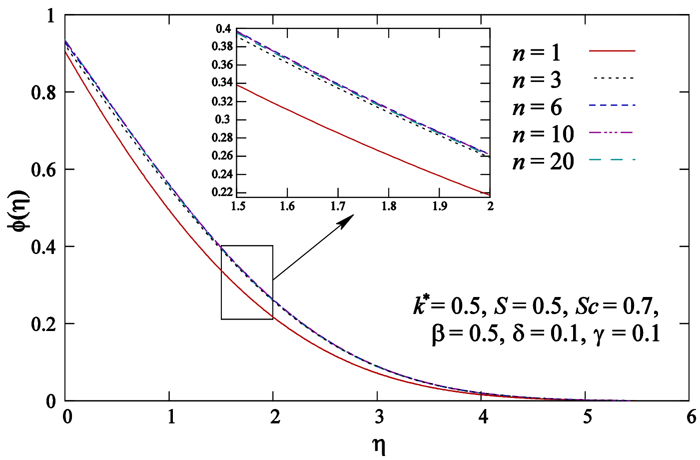

Fig. 8 Concentration profiles $\phi(\eta)$ for various values of n.

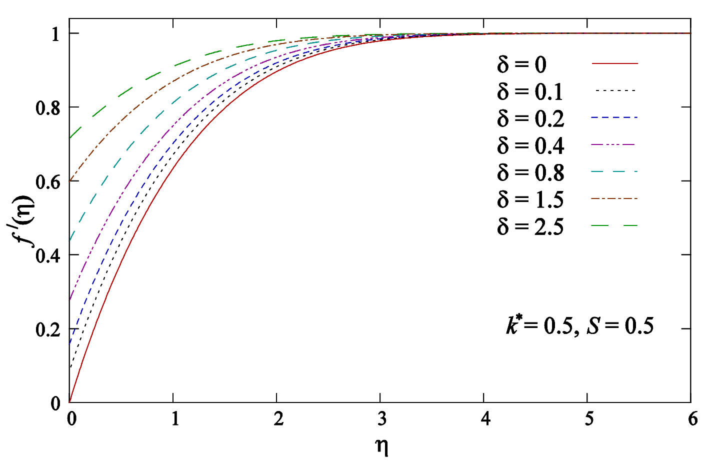

Fig. 9 Velocity profiles $f'(\eta)$ for various values of δ.

Next, we shall pay our attention to notice how the velocity slip parameter affects the velocity and the concentration profiles. The velocity profiles $f'(\eta)$ for various values of the velocity slip parameter δ are depicted in Fig. 9. With the increasing values of δ, the fluid velocity increases monotonically. Due to the slip condition at the plate the velocity of fluid adjacent to the plate has some positive value and accordingly the thickness of momentum boundary layer decreases.

Fig. 10 exhibits the dimensionless concentration profiles $\phi(\eta)$ for different values of δ. The concentration decreases with the increase in velocity slip parameter δ.

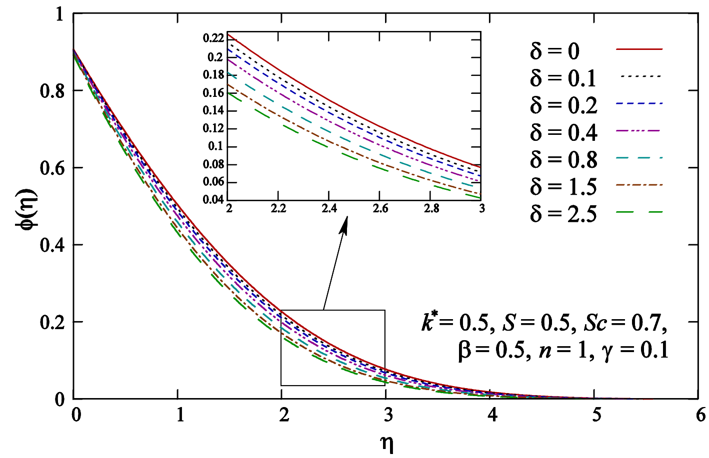

Fig. 10 Concentration profiles $\phi(\eta)$ for various values of δ.

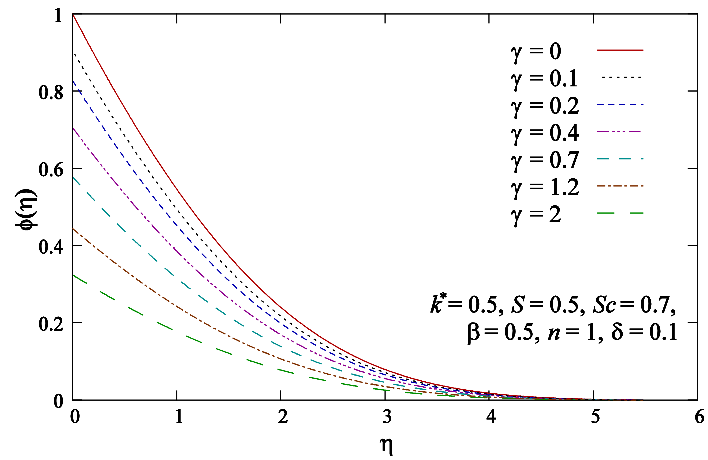

Fig. 11 Concentration profiles $\phi(\eta)$ for various values of γ.

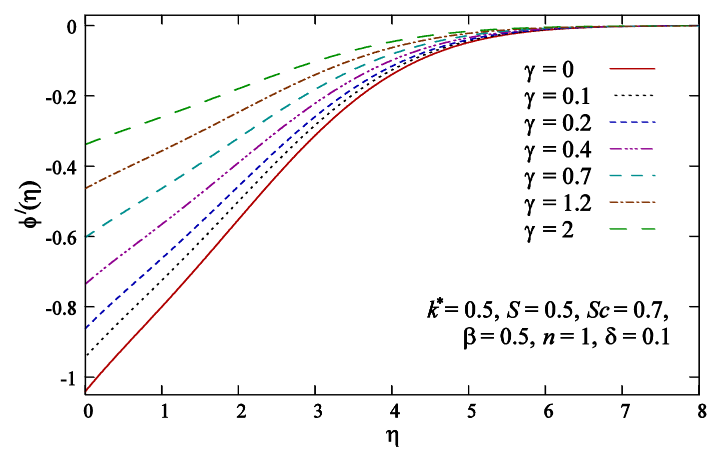

Fig. 12 Concentration gradient profiles $\phi'(\eta)$ for various values of γ.

In Fig. 11 and Fig. 12, the effect of mass slip on concentration and concentration gradient are displayed. Due the mass slip, the mass transfer from the plate to the fluid reduces and the concentration decreases. Whereas, the concentration gradient at a fixed point increases with increase of mass slip parameter γ. In Fig. 13, the negative value of temperature gradient at the plate $-\phi'(0)$ which is proportional to the rate of mass transfer from the plate is plotted against γ for various values of velocity slip parameter. It is observed that the mass transfer enhances with velocity slip and reduces with mass slip. These are physically realistic because in case of velocity slip, the fluid adjacent to the plate do not stick with the plate i.e. it has some positive velocity which boosts the mass transfer and in case of mass slip, the solute distribution adjacent to the plate slips which decelerated the mass transfer.

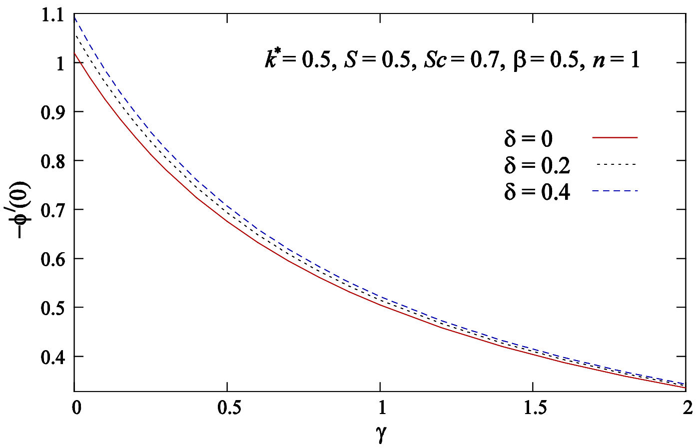

Fig. 13 Concentration gradient at the plate $-\phi'(0)$ against γ for various values of δ.

5. CONCLUSIONS

The mass transfer with chemical reaction in boundary layer flow over a permeable plate embedded in porous medium with slip conditions at the boundary is studied. Using similarity transformation, the nonlinear self-similar equations are obtained and are solved numerically by shooting method. In the analysis, the mass slip is introduced in addition with the velocity slip. The following observations can be made from the analysis:

(a) Due to increase of the permeability of the medium velocity inside the boundary layer increases and the concentration decreases.
(b) The concentration at a point decreases with the increase of velocity and mass slip parameter.
(c) The mass transfer from the plate enhances with velocity slip and it reduces with mass slip.

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NOMENCLATURE

C	concentration
C_w	concentration of the plate
C_∞	concentration in the free stream
D	diffusion coefficient
D_1	mass slip factor
Da_x	local Darcy number
f	dimensionless stream function
f'	dimensionless velocity
k	permeability of the porous medium
k^*	permeability parameter
L	reference length
L_1	velocity slip factor
n	order of chemical reaction
R	variable reaction rate
Re_x	local Reynolds number
S	suction/blowing parameter
Sc	Schmidt number
U_∞	free stream velocity
u	velocity component in x-direction
v	velocity component in y-direction
v_w	distribution of suction or blowing
x	distance along the plate
y	distance perpendicular to the plate

Greek Symbols

β	reaction rate parameter
δ	velocity slip parameter
η	similarity variable
γ	mass slip parameter
μ	coefficient of fluid viscosity
υ	kinematic fluid viscosity
ϕ	dimensionless concentration
ρ	fluid density
ψ	stream function

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