

# Robust adaptive fault tolerant control for nonlinear systems with actuator failure and mismatched disturbance

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## Abstract

In this paper, a class of nonlinear system with mismatched disturbance and actuator failure is investigated. A disturbance observer is proposed to estimate the disturbance first and the error of the estimation converges to zero exponentially. By introducing an integral sliding mode surface, the disturbance observer based integral sliding mode fault tolerant control scheme is proposed to attenuate the disturbance and to guarantee the stability of the system. Specially, the control law is designed for decoupling the partial disturbance and attenuating the disturbance that cannot be decoupled. Finally, two examples are given to illustrate the effectiveness of the proposed method. Index Terms—Actuator fault, Fault tolerant control (FTC), Disturbance observer, Adaptive Integral Sliding mode control, Nonlinear system.

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## 1. Introduction

Faults frequently occur in the engineering system because of the increasing complexity and scalability of industrial applications. Unexpected deviations of performance or system parameters can induce serious damage and even break down the system in the presence of a fault. With the growing demand for higher reliability, safety, and maintainability, it is desired that the fault can be detected at the early stage, determine the location and magnitude of the fault, identify the severity of the fault, and then accommodate the effects on the system and provide an acceptable performance. Abundant of results have been reported on the theme and many achievements have been applied to industrial systems such as aircraft systems [1], electric systems [2], and motor systems [3-4], etc. Many excellent methods were exploited such as robust control [5], sliding mode control [6-7], observer-based control [8], intelligent learning control [9], and adaptive control [10-11].

As aforementioned, the fault is vulnerable to occur in the practical system, which certainly reduces the nominal performance, results in vibration, destroys the stability of the system, and even causes catastrophic accidents. Actuator fault as the most common fault has caused wide attention. In Ye and Yang [12], the flight tracking control system with actuator fault was investigated; an adaptive controller was designed. In Zuo et al. [13], a class of singular systems with actuator saturation was developed; an adaptive controller was designed to compensate for the fault effects through the linear matrix inequality (LMI) technique. A class of nonlinear large-scale systems with actuator fault was considered and an observer-based fuzzy adaptive control was developed in Tong et al. [14]. The problem of part loss of effectiveness of the actuator was addressed in Li

and Yang [15]. In Van et al. [16], a class of nonlinear systems with partial loss of actuator was considered; a third-order sliding model control was developed to compensate for the actuator faults. These results have considered linear or nonlinear systems, however, the nonlinear function either satisfies the matched condition [16], i.e., the nonlinear function is in the control channel.

It is worth pointing out that disturbance exists widely in many industrial processes, which affects the stability of the system seriously. The basic characteristics of disturbance are its uncertainty, nonlinearity, and complexity. In some cases, the fault is considered as an additional uncertainty, disturbance, or nonlinear function in the system [16,17]. The common method is to make a compensator, i.e. construct an anti-disturbance mechanism to compensate for the disturbance. As the characteristics of the disturbance, observer design is necessary and popular in the existing literature, i.e., disturbance observer (DO). In Kempf and Kobayashi [18], a precision positioning table system was studied and a discrete time-tracking controller was designed based on the DO. In Chen et al. [19], the ball mill grinding circuits system was investigated and a DO was designed to estimate the strong disturbance, the controller was designed based on the observer to compensate for the disturbance. In Chen [20], a class of nonlinear systems was investigated, by introducing a disturbance generator; the DO-based control law was established. Yao and Guo [21] addressed a class of Markovian jump systems with multiple disturbances; the control law was designed by integrating the DO output information and state feedback control. Wu et al. [22] take an insight into a generic hypersonic vehicle system with modeled and unmodeled disturbances. In these results, the disturbance matrix  $B_d$  and the control matrix  $B$  are assumed to satisfy the

match condition, i.e.,  $\text{rank}(B) = \text{rank}([B \ B_d])$ , this limitation may not be applicable in some control processes.

It should be pointed out that the mismatched disturbance, that is, the disturbance enters the system in different channels, is more practical in industrial dynamic systems. In Zhang et al. [23], a class of nonlinear systems with actuator fault, sensor fault, and mismatched disturbance was considered. In Wei and Guo [24], the disturbance was considered as two parts, the matched disturbance was compensated by the DO while the mismatched disturbance was attenuated by variable structure control. In Yang et al. [25], by constructing a nonlinear disturbance observer (NDO), the sliding mode control scheme was developed to counteract the mismatched disturbance.

Based on the aforementioned analysis, this paper attempts to solve the FTC problem for a class of nonlinear systems with actuator fault and mismatched disturbance. The main works can be summarized as follows. First of all, an improved nonlinear disturbance observer is designed to estimate the mismatched disturbance. Then, the integral sliding mode controller is presented based on the observer, where the reachability of sliding motion is proved. Furthermore, the mismatched disturbance is divided into two parts, in which the matched part is compensated by the disturbance information while the remaining part is attenuated by the adaptive controller. Finally, the adaptive control law is proposed, which can adaptively adjust controller parameters to compensate for the fault and disturbance. The effectiveness of the method is verified through two examples.

The remaining part of this paper is organized as follows. In section 2, the system description and some assumptions are presented. In section 3, the construction of the observer and the design of the controller are presented. In section 4, two simulation examples are used to illustrate the effectiveness of the method, and some conclusions are obtained in section 5.

## 2. Problem formulation

Consider the following nonlinear system with actuator fault and mismatched disturbance as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + \Delta f(x, t) + \xi(x, t) \\ y = Cx(t) \end{cases} \quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $y(t) \in R^q$  is the output of the system,  $u(t) \in R^p$  is the control input,  $\Delta f(x, t)$  is a nonlinear function, which can be regarded as the un-modeled uncertainty,  $d(t) \in R^v$  is the unmatched external disturbance,  $\xi(x, t)$  is the mismatched nonlinearity,  $A, B, B_d$ , and  $C$  are known constant coefficient matrices with appropriate dimensions.

In this paper, the actuator failure problem is concerned. The actuators can be divided into two parts  $u_H$  and  $u_F$ , where  $u_H$  stands for the health actuator and  $u_F$  represent the actuator in a fault condition. Then system (1) becomes (2)

$$\begin{cases} \dot{x}(t) = Ax(t) + B_H u_H(t) + B_F u_F(t) + B_d d(t) + \Delta f(x, t) + \xi(x, t) \\ y = Cx(t) \end{cases}, t) \quad (2)$$

where  $B_H$  and  $B_F$  denote the healthy and fault matrices.

One can see that proposer designing of  $u_H$  can guarantee the stability of the system despite the existence of external disturbance.

*Assumption 1* [17]. The actuators in fault condition work

abnormally, and the remaining actuators work normally.

With assumption 1, one can use  $\bar{u}_F$  and  $u_F$  represent the actual control and designed control respectively for the actuators in fault condition. Then system (2) becomes (3)

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_F \Delta u_F(t) + B_d d(t) + \Delta f(x, t) + \xi(x, t) \\ y = Cx(t) \end{cases} \quad (3)$$

where  $\Delta u_F = \bar{u}_F - u_F$ .

*Assumption 2* [15]. There exists a known function  $\bar{f}(x)$  and two unknown constants  $\theta_0$  and  $v_0$ , such that the following inequality holds,

$$\Delta f(x, t) \leq \theta_0 \bar{f}(x) + v_0. \quad (4)$$

*Assumption 3.* The disturbance and the derivative of the disturbance are bounded, i.e.,  $\|d(t)\| \leq \epsilon_1$ ,  $\|\dot{d}(t)\| \leq \epsilon_d$ , where  $\epsilon_1$  and  $\epsilon_d$  are two positive constants.

*Assumption 4.* The nonlinearity function  $\xi(x, t)$  is bounded and satisfies the following condition

$$\tilde{\xi}(x, t)^T R \Delta \tilde{\xi}(x, t) \leq \tilde{x}(x, t)^T Q \tilde{x}(x, t) \quad (5)$$

where  $R$  and  $Q$  are two positive symmetry matrices,  $\tilde{\xi}(x, t) = \xi(x_1, t) - \xi(x_2, t)$ ,  $\tilde{x}(x, t) = x_1(x, t) - x_2(x, t)$ .

*Remark 1.* The assumption 1 is general for the actuator failure condition [16,17], in which the actuator fault can be treated as an additional uncertainty or disturbance. Compared with the results in [26], the assumption 2 in this study has been much more relaxed. Assumption 3 is common in FTC control results [15]. Assumption 4 is more general compared with the traditional Lipschitz condition [16], it should be noted that if  $R = I$ , and  $Q = l_f^2 I$ , then assumption 4 degenerates to the normal Lipschitz condition, where  $l_f$  is the Lipschitz constant.

With assumptions 1 and 2, the object of the paper is to design a control law to compensate for the effects of the disturbance and fault so that the stability and convergence of the system can be guaranteed in normal and fault conditions.

## 3. Main results

In this part, an observer will be applied to estimate the external disturbance, and an observer-based integral sliding mode fault colorant control scheme will be designed.

### 3.1 Observer design

For the feasibility of the observer, the following assumption is necessary.

*Assumption 5* [16]. The additional fault term  $\varphi(t) = B_F \Delta u_F(t)$  satisfies the following condition,  $\|B_F \Delta u_F(t)\| \leq \bar{\omega}_b$ , where  $\bar{\omega}_b$  is a positive scalar.

For system (3), the following observer is proposed in the form of

$$\begin{cases} \dot{p}(t) = -g(x) B_d p(t) - g(x) [B_d p(t) + Ax(t) + Bu(t) + \bar{\omega}_b + B_d d(t) + \Delta f(x, t) + \xi(x, t)] + T_r \\ \hat{d}(t) = p(t) + q(x) \end{cases} \quad (6)$$

where  $p(t)$  is the internal state of the observer,  $\hat{d}(t)$  is the

estimation of the disturbance  $d(t)$ ,  $q(x)$  is a nonlinear function to be designed,  $T_n$  is a parameter matrix with proper dimensions, and  $g(x)$  is the observer gain and satisfies the following condition

$$g(x) = \frac{\partial q(x)}{\partial x} \quad (7)$$

$\delta_n$  is the error compensator and is defined by

$$\delta_n = -\frac{T_n^T}{\|T_n\|^2} \delta_d \quad (8)$$

where  $\delta_d = \bar{\omega}_b - \|B_F \Delta u_F(t)\|$ .

**Lemma 1.** With the assumption 4 and the observer (6), the error of the observer converges to zero exponentially.

*Proof.* Define  $e_d(t) = d(t) - \hat{d}(t)$ . From the observer (6), it is easy to obtain that

$$\begin{aligned} \dot{\hat{d}}(t) = & -g(x)B_d p(t) - g(x)[B_d p(t) + Ax(t) + Bu(t) + \\ & \bar{\omega}_b \\ & + B_d d(t) + \Delta f(x, t) + \xi(x, t)] + \frac{\partial q(x)}{\partial x} \dot{x}(t) \\ = & g(x)B_d e_d(t) + g(x)(B_F \Delta u_F(t) - \bar{\omega}_b) + T_n \delta_n(t) \end{aligned} \quad (9)$$

It can be derivate that

$$\dot{e}_d(t) = -g(x)B_d e_d(t) + g(x)(B_F \Delta u_F(t) - \bar{\omega}_b) + \dot{\hat{d}}(t) - T_n \delta_n(t) \quad (10)$$

The solution of (9) is given by

$$\begin{aligned} e_d = & e^{-g(x)B_d t} e_d(0) + \int_0^t e^{-g(x)B_d(t-s)} \dot{\hat{d}}(s) ds - \\ & \int_0^t e^{-g(x)B_d(t-s)} g(x)(B_F \Delta u_F(s) - \bar{\omega}_b - T_n \delta_n(t)) ds \end{aligned} \quad (11)$$

From Eq.(10), one can deduce that

$$\begin{aligned} e_d \leq & \|e^{-g(x)B_d t} e_d(0)\| + \int_0^t \|e^{-g(x)B_d(t-s)} \dot{\hat{d}}(s)\| ds \\ & + \int_0^t \|e^{-g(x)B_d(t-s)} g(x)(\|B_F \Delta u_F(s)\| - \bar{\omega}_b - \\ & T_n \delta_n(t))\| ds \\ \leq & \|e^{-g(x)B_d t} e_d(0)\| + \int_0^t \|e^{-g(x)B_d(t-s)} e_d\| ds \end{aligned} \quad (12)$$

According to the inequality  $\|e^{At}\| \leq c e^{-\rho t}$  [3], where  $c$  and  $\rho$  are two positive constants. The one can obtain that

$$\begin{aligned} e_d \leq & \|e^{-g(x)B_d t} e_d(0)\| + \int_0^t \|e^{-g(x)B_d(t-s)} e_d\| ds \\ = & c e^{-\rho t} h_0 + c \int_0^t e^{-\rho(t-s)} ds \\ = & c e^{-\rho t} h_0 + \frac{c e_d}{\rho} (1 - e^{-\rho t}) \end{aligned} \quad (13)$$

where  $h_0$  is a constant bound satisfy  $\|e_d(0)\| \leq h_0$ . Define  $\bar{\kappa} = c e^{-\rho t} h_0 + \frac{c e_d}{\rho} (1 - e^{-\rho t})$ , then (13) can be written as  $e_d \leq \bar{\kappa}$ . This completes the proof.

### 3.2 Adaptive FTC design

In this section, the observer-based integral sliding mode fault tolerant control will be designed.  $\bar{\kappa}$  and  $v_0$  are supposed to be unknown, the adaptive controller is designed as follows:

$$u(t) = u_m(t) + u_n(t) \quad (14)$$

where

$$u_m(t) = (K_1(t) + K_2(t) + K_3(t))x(t) - (K_{d1} + K_{d2})\hat{d}(t) \quad (15)$$

$$u_n(t) = u_1(t) + u_2(t) + u_3(t) \quad (16)$$

where  $K_1(t)$ ,  $K_2(t)$ ,  $K_3(t)$ ,  $K_{d1}(t)$  and  $K_{d2}(t)$  are the parameter functions to be designed

$$u_1(t) = -\frac{(DB)^{-1}DB_d B_d^T D^T s(t)}{\|s^T(t)DB_d\|} (\hat{\kappa} + \lambda_1) \quad (17)$$

$$u_2(t) = -\frac{(DB)^{-1}DD^T s(t)}{\|s^T(t)D\|} (\delta_m + \hat{v}_0 + \lambda_2) \quad (18)$$

$$u_3(t) = -\frac{(DB)^{-1}D\hat{f}(x)\hat{f}(x)^T D^T s(t)}{\|s^T(t)D\hat{f}(x)\|} \gamma_1^* \quad (19)$$

where  $\lambda_1$  and  $\lambda_2$  are two positive scalars,  $v_1 \geq v_0$ ,  $\gamma_1^* \geq \gamma_1$  and  $\gamma_1$  is defined in Eq.(23),  $\delta_m$  is the bound of the nonlinear function  $\xi(x, t)$ ,  $\hat{\kappa}$ ,  $\hat{v}_0$  and  $\delta_m$  are the estimate of  $\bar{\kappa}$ ,  $v_0$  and  $\delta_m$ , and the adaptive control laws are designed in the form of

$$\dot{\hat{\kappa}} = \chi_q (\|s^T(t)DB_d\| + \beta_q) \quad (20)$$

$$\dot{\hat{v}}_0 = \chi_p (\|s^T(t)D\| + \beta_p) \quad (21)$$

$$\dot{\delta}_m = \chi_r (\|s^T(t)D\| + \beta_r) \quad (22)$$

where  $\chi_q$ ,  $\chi_r$ ,  $\beta_p$ ,  $\beta_q$  and  $\beta_r$  are positive parameters.

The control law will be used in the integral sliding mode control,  $u_m$  is used to make the system asymptotically stable,  $u_n$  is used to compensate for the effects of the actuator fault, disturbance estimation error, and nonlinear factors. In this paper, the sliding mode switching function is designed as follows

$$s(t) = Dx(t) - Dx(0) - D \int_0^t (Ax(\tau) + Bu_m(\tau) + B_d \hat{d}(\tau)) d\tau \quad (23)$$

where  $D \in R^{p \times n}$  is a designed matrix that  $DB$  is invertible. In the next section, the reaching ability will be verified.

**Theorem 1.** With the controller in Eq.(14), the state strategies of the system will drive onto the sliding surface  $s(t)$  in finite time.

*Proof.* Denote  $\tilde{\kappa} = \bar{\kappa} - \hat{\kappa}$ ,  $\tilde{v}_0 = v_0 - \hat{v}_0$ ,  $\tilde{\delta}_m = \delta_m - \hat{\delta}_m$ . Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2} s(t)^T s(t) + \frac{1}{2\chi_q} \tilde{\kappa}^2 + \frac{1}{2\chi_p} \tilde{v}_0^2 \quad (24)$$

From the expression (20), the time derivative of  $s(t)$  is

$$\begin{aligned} \dot{s}(t) = & D(Ax(t) + Bu(t) + B_F \Delta u_F(t) + B_d d(t)) - (25) \\ & D \Delta f(x, t) \\ & - DB u_m(t) - DB_d \dot{d}(t) + D(\Delta f(x, t) + \xi(x, t)) \\ & = DB u_n(t) + DB_F \Delta u_F(t) + DB_d d(t) \\ & - DB_d \dot{d}(t) + D \Delta f(x, t) + D \xi(x, t) \end{aligned}$$

From assumption 2, one has

$$\theta_0 \leq \gamma_1 \quad (26)$$

From Eq.(23), one has

$$\begin{aligned} \dot{s}(t) = & DB u_n(t) + DB_F \Delta u_F(t) + DB_d d(t) - DB_d \dot{d}(t) + (27) \\ & D \Delta f(x, t) + D \xi(x, t) \\ \leq & - \frac{DB_d B_d^T D^T s(t)}{\|s^T(t) DB_d\|} (\bar{\kappa} + \lambda_1) - \frac{DD^T s(t)}{\|s^T(t) D\|} (\delta_m + v_0 + \\ & \lambda_2) - \frac{D \bar{f}(x) \bar{f}^T(x) D^T s(t)}{\|s^T(t) D \bar{f}(x)\|} \gamma_1 \\ & + DB_d e_d(t) + D(\gamma_1 \bar{f}(x) + \xi(x, t) + B_F \Delta u_F(t) + v_0) \end{aligned}$$

The time derivative of Eq.(35) can be determined as

$$\dot{V}(t) = s(t)^T \dot{s}(t) + \frac{1}{\chi_q} \dot{\tilde{\kappa}} \tilde{\kappa} + \frac{1}{\chi_p} \dot{\tilde{v}}_0 v_0 + \frac{1}{\chi_r} \dot{\tilde{\delta}}_m \delta_m \quad (28)$$

By introducing Eqs.(17)-(27), Eq.(28) can be rewritten as

$$\begin{aligned} \dot{V}(t) \leq & - \frac{s(t)^T DB_d B_d^T D^T s(t)}{\|s^T(t) DB_d\|} (\bar{\kappa} + \lambda_1) + (29) \\ & \|s(t)^T DB_d\| \bar{\kappa} - \frac{s(t)^T DD^T s(t)}{\|s^T(t) D\|} \\ & (\delta_m + v_0 + \lambda_2) + \|s(t)^T D\| (\delta_m + v_0) \\ & - \frac{s(t)^T D \bar{f}(x) \bar{f}^T(x) D^T s(t)}{\|s^T(t) D \bar{f}(x)\|} \gamma_1 + \gamma_1 \|s(t)^T D \bar{f}(x)\| - \tilde{\kappa} \\ & (\|s^T(t) DB_d\| + \beta_q) - \tilde{v}_0 (\|s^T(t) D\| + \beta_p) \\ & - \tilde{\delta}_m (\|s^T(t) D\| + \beta_r) < -\lambda_1 \|s^T(t) DB_d\| - \\ & \lambda_2 \|s^T(t) D\| < 0 \end{aligned}$$

Thus, the reaching ability is satisfied, this completes the proof.

*Remark 2.* The controller proposed in (14) is discontinuous, to reduce chattering in the practical implementation, the discontinuous function can be replaced, for example,  $u_1(t)$  can be replaced by  $-\frac{(DB)^{-1} DB_d B_d^T D^T s(t)}{\|s^T(t) DB_d\| + \alpha} (\bar{\kappa} + \lambda_1)$ , where  $\alpha$  is a small positive constant.

### 3.3 Stability analysis

In this section, the stability of the closed-loop system will be analyzed. By solving the equation  $\dot{s}(t) = 0$  in Eq.(25), the equivalent control law can be obtained as

$$u_{eq}(t) = - (DB)^{-1} DB_F \Delta u_F(t) - (DB)^{-1} DB_d e_d(t) - (DB)^{-1} D \Delta f(x, t) - (DB)^{-1} D \xi(x, t) \quad (30)$$

Substituting Eqs.(26) and (30) into the system (3) yields

$$\begin{aligned} \dot{x}(t) = & (A + B(K_1(t) + K_2(t) + K_3(t)))x(t) - (31) \\ & BK_d \dot{d}(t) + B_F \Delta u_F(t) + \Delta f(x, t) + \xi(x, t) + B_d d(t) - \\ & B(DB)^{-1} DB_F \Delta u_F(t) - B(DB)^{-1} DB_d e_d(t) - \\ & B(DB)^{-1} D(\Delta f(x, t) + \xi(x, t)) \end{aligned}$$

By defining  $K_{d1} = (DB)^{-1} DB_d$ ,  $\bar{B}_d = B_d - B(DB)^{-1} DB_d$ ,  $\bar{M} = I - B(DB)^{-1} D$ , Eq.(31) can be rewritten as

$$\dot{x}(t) = (A + B(K_1(t) + K_2(t) + K_3(t)))x(t) + \bar{B}_d d(t) - BK_{d2} \dot{d}(t) + \bar{M} B_F \Delta u_F(t) + \bar{M}(\Delta f(x, t) + \xi(x, t)) \quad (32)$$

As before mentioned, the disturbance-matched condition is not satisfied, i.e.  $rank(B) < rank(B, B_d)$ . From the definition of  $\bar{B}_d$ , we can easily check that  $rank(B) < rank(B, \bar{B}_d)$ . In this paper, the disturbance is divided into two parts, i.e.,

$$\bar{B}_d = [\bar{B}_{d1} \quad \bar{B}_{d2}], \quad d = [d_1 \quad d_2]^T,$$

then  $\bar{B}_d d(t)$  can be written as

$$\bar{B}_d d(t) = \bar{B}_{d1} d_1(t) + \bar{B}_{d2} d_2(t),$$

where

$$\begin{aligned} \bar{B}_{d1} = & [\bar{B}_{d\kappa 1} \quad \bar{B}_{d\kappa 2} \quad \dots \quad \bar{B}_{d\kappa m_1}] \in R^{n \times m_1}, \bar{B}_{d2} = \\ & [\bar{B}_{d\theta 1} \quad \bar{B}_{d\theta 2} \quad \dots \quad \bar{B}_{d\theta m_2}] \in R^{n \times m_2}, \\ d_1 = & [d_{\kappa 1} \quad d_{\kappa 2} \quad \dots \quad d_{\kappa m_1}]^T \in R^{m_1 \times 1}, \\ d_2 = & [d_{\theta 1} \quad d_{\theta 2} \quad \dots \quad d_{\theta m_2}]^T \in R^{m_2 \times 1}, \\ & m_1 + m_2 = v \end{aligned}$$

Let  $rank(B) = rank(B, \bar{B}_{d1})$ ,  $rank(B) < rank(B, \bar{B}_{d2})$  and the parameter  $K_{d2}$  in Eq.(15) can be chosen as  $K_{d2} = [K_{s1} \quad K_{s1} \quad \dots \quad K_{sv}] \in R^{p \times v}$ , where  $BK_{si} = \bar{B}_{dki}$  for  $i \leq m_1$  and  $K_{si} = 0$  for  $i > m_1$ . Then the equation  $BK_{d2} = \bar{B}_{d1}$  is solvable. Note that  $\bar{M} = B\tilde{M}$  and  $\tilde{M} = (B^\dagger - (DB)^{-1} D)$ , where  $B^\dagger = B^T (BB^T)^{-1}$ . Then (32) can be rewritten as

$$\dot{x}(t) = (A + B(K_1(t) + K_2(t) + K_3(t)))x(t) + \bar{B}_{d2} d_2(t) + \bar{B}_{d1} e_{d1}(t) + B\tilde{M} B_F \Delta u_F(t) + B\tilde{M}(\Delta f(x, t) + \xi(x, t)) \quad (33)$$

*Remark 3.* We can see that if  $\bar{B}_{d2} = 0$ , then  $rank(B) = rank(B, \bar{B}_d)$ , so the matched condition is a special case, i.e., this paper considers a more general case. In addition, we can also obtain that one of the solutions of  $K_{d2}$  is  $K_{si} = B^\dagger \bar{B}_{dki}$  and  $B^\dagger$  is the general inverse of  $B$ .

The object of the next part is to design  $K_1(t)$ ,  $K_2(t)$  and  $K_3(t)$  such that the stability of the system can be guaranteed. The control laws are designed as follows

$$K_1(t) = - \frac{\hat{\omega} B^T P \|x^T P\|}{\|x^T P B\|^2} \quad (34)$$

$$K_2(t) = - \frac{K_{d2} \hat{w}_1 \bar{B}_{d1}^T P}{\|x^T P \bar{B}_{d1}\|} \quad (35)$$

$$K_3(t) = - \frac{\tilde{M} \hat{\gamma}_1^2 \bar{f}^2(x) \tilde{M}^T B^T P}{\|x^T P B \tilde{M}\| \bar{f}(x) \hat{\gamma}_1 + \sigma(t)} \quad (36)$$

where  $\hat{w}_1$ ,  $\hat{\omega}$  and  $\hat{\gamma}_1$  are estimations of  $w_1$ ,  $\varpi$  and  $\gamma_1$ , respectively,  $w_1$  and  $\varpi$  are designed in Eqs.(59) and (60),  $P$  is a positive symmetry matrix, and  $\sigma(t)$  is a continuous function, and satisfies

$$\int_0^{\infty} \sigma(t) dt \leq \sigma^* < \infty. \quad (37)$$

where  $\sigma$  is a positive scalar.

The adaptive updating control laws are given by

$$\dot{\hat{\omega}} = -\beta_1 \hat{\omega} \sigma(t) + 2\beta_1 \|x^T P\| \quad (38)$$

$$\dot{\hat{w}}_1 = -\beta_2 \hat{w}_1 \sigma(t) + 2\beta_2 \|x^T P B_{d1}\| \quad (39)$$

$$\dot{\hat{\gamma}}_1 = -\beta_3 \hat{\gamma}_1 \sigma(t) + 2\beta_3 \|x^T P B \tilde{M}\| \tilde{f}(x) \quad (40)$$

where  $\beta_1, \beta_2$  and  $\beta_3$  are three positive constants.

Denote  $\tilde{\omega} = \omega - \hat{\omega}$ ,  $\tilde{w}_1 = w_1 - \hat{w}_1$ ,  $\tilde{\gamma}_1 = \gamma_1 - \hat{\gamma}_1$ , we can obtain the following dynamics

$$\dot{\tilde{\omega}} = -\beta_1 \tilde{\omega} \sigma(t) + \beta_1 \omega \sigma(t) - 2\beta_1 \|x^T P\| \quad (41)$$

$$\dot{\tilde{w}}_1 = \beta_2 w_1 \sigma(t) - \beta_2 \tilde{w}_1 \sigma(t) - 2\beta_2 \|x^T P B_{d1}\| \quad (42)$$

$$\dot{\tilde{\gamma}}_1 = \beta_3 \gamma_1 \sigma(t) - \beta_3 \tilde{\gamma}_1 \sigma(t) - 2\beta_3 \|x^T P B \tilde{M}\| \tilde{f}(x) \quad (43)$$

**Theorem 2.** With the controller (34)-(36) and the adaptive control laws (38)-(40), the closed-loop system is stable if there exist two positive symmetry matrices  $P$  and  $Q$ , such that the following condition holds

$$\begin{bmatrix} PA + A^T P + Q & P_1 \\ * & -I \end{bmatrix} < 0 \quad (44)$$

where  $P_1 = P B \tilde{M}$ ,  $Q$  is defined in Eq.(5).

*Proof.* Design the Lyapunov function candidate as

$$V(t) = x(t)^T P x(t) + \frac{1}{2\beta_1} \tilde{\omega}^2 + \frac{1}{2\beta_2} \tilde{w}_1^2 + \frac{1}{2\beta_3} \tilde{\gamma}_1^2 \quad (45)$$

Then the time derivative of Eq.(51) can be obtained as

$$\begin{aligned} \dot{V}(t) &= 2x(t)^T P \dot{x}(t) + \frac{1}{\beta_1} \tilde{\omega} \dot{\tilde{\omega}} + \frac{1}{\beta_2} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\beta_3} \tilde{\gamma}_1 \dot{\tilde{\gamma}}_1 \quad (46) \\ &= 2x(t)^T P ( (A + B(K_1(t) + K_2(t) + K_3(t)))x(t) + \\ &\quad B_{d1} e_{d1} ) + 2x(t)^T P ( B \tilde{M} B_F \Delta u_F(t) + B_{d2} d_2(t) ) \\ &\quad + 2x(t)^T P B \tilde{M} \Delta f(x, t) + 2x(t)^T P B \tilde{M} \xi(x, t) + \frac{1}{\beta_1} \tilde{\omega} \dot{\tilde{\omega}} + \\ &\quad \frac{1}{\beta_2} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\beta_3} \tilde{\gamma}_1 \dot{\tilde{\gamma}}_1 \end{aligned}$$

From Eqs.(41)-(46), it can be derivative that

$$2x^T(t) P B K_1(t) x(t) = -2x^T(t) P B \frac{\hat{\omega} B^T P \|x^T P\|}{\|x^T P B\|^2} = -2\hat{\omega} \|x^T P\| \quad (47)$$

$$2x^T(t) P B K_2(t) x(t) = -2x^T(t) P B \frac{K_{d2} \hat{w}_1 B_{d1}^T P}{\|x^T P B_{d1}\|} = -2\hat{w}_1 \|x^T P B_{d1}\| \quad (48)$$

$$2x^T(t) P B K_3(t) x(t) = -2 \frac{\hat{\gamma}_1^2 \tilde{f}^2(x) \|x^T P B \tilde{M}\|^2}{\|x^T P B \tilde{M}\| \tilde{f}(x) \hat{\gamma}_1 + \sigma(t)} \quad (49)$$

$$\begin{aligned} \frac{1}{\beta_1} \tilde{\omega} \dot{\tilde{\omega}} &= \frac{1}{\beta_1} \tilde{\omega} ( -\beta_1 \tilde{\omega} \sigma(t) + \beta_1 \omega \sigma(t) - 2\beta_1 \|x^T P\| ) \quad (50) \\ &= -\tilde{\omega}^2 \sigma(t) + \tilde{\omega} \omega \sigma(t) - 2\tilde{\omega} \|x^T P\| \end{aligned}$$

$$\begin{aligned} \frac{1}{\beta_2} \tilde{w}_1 \dot{\tilde{w}}_1 &= \frac{1}{\beta_2} \tilde{w}_1 ( \beta_2 w_1 \sigma(t) - \beta_2 \tilde{w}_1 \sigma(t) - \\ &\quad 2\beta_2 \|x^T P B_{d1}\| ) \quad (51) \\ &= \tilde{w}_1 w_1 \sigma(t) - \tilde{w}_1^2 \sigma(t) - 2\tilde{w}_1 \|x^T P B_{d1}\| \end{aligned}$$

$$\begin{aligned} \frac{1}{\beta_3} \tilde{\gamma}_1 \dot{\tilde{\gamma}}_1 &= \frac{1}{\beta_3} \tilde{\gamma}_1 ( \beta_3 \gamma_1 \sigma(t) - \beta_3 \tilde{\gamma}_1 \sigma(t) - 2\beta_3 \|x^T P B \tilde{M}\| ) \quad (52) \\ &= \tilde{\gamma}_1 \gamma_1 \sigma(t) - \tilde{\gamma}_1^2 \sigma(t) - 2\tilde{\gamma}_1 \|x^T P B \tilde{M}\| \end{aligned}$$

By assumptions 2, 4, 5, and lemma 1, we can derivative that the following inequalities hold

$$\|B \tilde{M} B_F \Delta u_F(t)\| + \|\bar{B}_{d2}\| \|d_2(t)\| + \|B \tilde{M} v_0\| \leq \varpi \quad (53)$$

$$e_{d1} \leq w_1 \quad (54)$$

$$\begin{aligned} 2x(t)^T P B \tilde{M} \xi(x, t) &\leq 2\|x(t)^T P B \tilde{M}\| \|\xi(x, t)\| \quad (55) \\ &\leq \|x(t)^T P B \tilde{M}\|^2 + \|\xi(x, t)\|^2 = x(t)^T (P_1^T P_1 + Q) x(t) \end{aligned}$$

where  $\varpi$  and  $w_1$  are positive scalars.

Substituting Eqs.(47)-(55), Eq.(46) can be rewritten as

$$\begin{aligned} \dot{V}(t) &\leq x(t)^T (PA + A^T P + P_1^T P_1 + Q) x(t) - \quad (56) \\ &\quad 2\hat{\omega} \|x^T P\| + \|2x(t)^T P \bar{B}_{d1}\| w_1 \\ &\quad - 2 \frac{\hat{\gamma}_1^2 \tilde{f}^2(x) \|x^T P B \tilde{M}\|^2}{\|x^T P B \tilde{M}\| \tilde{f}(x) \hat{\gamma}_1 + \sigma(t)} - 2\tilde{\gamma}_1 \|x^T P B \tilde{M}\| + \\ &\quad 2\gamma_1 \|x^T P \tilde{M}\| \tilde{f}(x) \\ &\quad - 2\|x^T P \bar{B}_{d1}\| \hat{w}_1 + \|2x(t)^T P\| \varpi - 2\tilde{\omega} \|x^T P\| - \\ &\quad 2\tilde{w}_1 \|x^T P B_{d1}\| \end{aligned}$$

Note that the following equalities

$$\|2x(t)^T P\| w_1 - 2\|x^T P \bar{B}_{d1}\| \hat{w}_1 + 2\tilde{w}_1 \|x^T P B_{d1}\| = 0 \quad (57)$$

$$\|2x(t)^T P\| \varpi - 2\hat{\omega} \|x^T P\| - 2\tilde{\omega} \|x^T P\| = 0 \quad (58)$$

$$2\gamma_1 \|x^T P \tilde{M}\| \tilde{f}(x) - 2\tilde{\gamma}_1 \|x^T P B \tilde{M}\| \tilde{f}(x) - 2 \frac{\hat{\gamma}_1^2 \tilde{f}^2(x) \|x^T P B \tilde{M}\|^2}{\|x^T P B \tilde{M}\| \tilde{f}(x) \hat{\gamma}_1 + \sigma(t)} = \frac{2\hat{\gamma}_1 \|x^T P \tilde{M}\| \tilde{f}(x) \sigma(t)}{\|x^T P \tilde{M}\| \tilde{f}(x) \hat{\gamma}_1 + \sigma(t)} \quad (59)$$

Substituting Eqs.(57)-(59) into (56) yields

$$\begin{aligned} \dot{V}(t) &\leq x(t)^T (PA + A^T P + P_1^T P_1 + Q) x(t) - \sigma(t) (\tilde{\omega}^2 - \quad (60) \\ &\quad \tilde{\omega} \omega) - \sigma(t) (\tilde{w}_1^2 - \tilde{w}_1 w_1 + \tilde{\gamma}_1^2 - \tilde{\gamma}_1 \gamma_1) + \\ &\quad \frac{2\hat{\gamma}_1 \|x^T P \tilde{M}\| \tilde{f}(x) \sigma(t)}{\|x^T P \tilde{M}\| \tilde{f}(x) \hat{\gamma}_1 + \sigma(t)} \end{aligned}$$

For any positive scalar  $\lambda_m$  and  $\lambda_n$ , the following equality holds  $0 \leq \frac{\lambda_m \lambda_n}{\lambda_m + \lambda_n} \leq \lambda_n$ . Then we can obtain that

$$\dot{V}(t) \leq x(t)^T (PA + A^T P + P_1^T P_1 + Q) x(t) + \zeta_\kappa \sigma(t) \quad (61)$$

where  $\zeta_\kappa = 2 + \frac{1}{4}(\varpi^2 + w_1^2 + \gamma_1^2)$ .

According to Eq.(45), by integrating Eq.(61) yields

$$V(t) \leq V(t_0) - \int_{t_0}^t \lambda_{\min}(Q_1) \|x(s)\|^2 ds + \int_{t_0}^t \zeta_{\kappa} \sigma(s) ds \quad (62)$$

$$\leq V(t_0) - \int_{t_0}^t \lambda_{\min}(Q_1) \|x(s)\|^2 ds + \zeta_{\kappa} \sigma$$

which means the system described in Eq.(33) is bounded.  $\lambda_{\min}(Q^1)$  denotes the minimum eigenvalue of  $Q_1$ , and  $-Q_1 = A^T P + PA + P_1^T P_1 + Q$ . Eq.(68) also implies

$$\int_{t_0}^t \lambda_{\min}(Q_1) \|x(s)\|^2 ds \leq V(t_0) + \zeta_{\kappa} \sigma \quad (63)$$

According to Barbalat Lemma, we have  $\lim_{t \rightarrow \infty} \|x(t)\|^2 = 0$ . This completes the proof.

*Remark 4.* Compared with the results in [15], where the nonlinear function is matched, i.e.,  $\Delta f(x, t)$  is in the control channel. In this paper,  $\Delta f(x, t)$  exists in the different channel from the control input, i.e., which means that  $\Delta f(x, t)$  is more general.

#### 4. Numerical examples

In this section, two examples are simulated to illustrate the effectiveness of the proposed method.

*Example 1.* In this example, the linearized longitudinal dynamic of the VTOL aircraft which is borrowed from [23] is considered. It is assumed that the system is subjected to unmodeled dynamics, actuator fault, and external disturbance. Then, the system can be described as system (1), where  $x(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$ ,  $x_1(t)$  is the horizontal velocity (*knot*),  $x_2(t)$  represents the vertical velocity (*knot*),  $x_3(t)$  is the pitch rate (*degree/s*), and  $x_4(t)$  expresses the pitch angle (*degree*). The parameters of the system are given as follows

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}, B_d = \begin{bmatrix} 0.4422 & 0.116 \\ 3.5446 & -0.102 \\ -5.5200 & 0 \\ 0 & 0 \end{bmatrix}, B_F = [0.1767 \ 5.924 \ 4.900].$$

The nonlinear unmodeled uncertainty and mismatched nonlinearity are assumed to be:  $\Delta f(x, t) = \sin(x_4)$ ,  $\xi(x, t) = \sin(x_1)$ . The actuator fault and external are supposed to be as follows

$$f(t) = \begin{cases} 0 & (0 \leq t < 8) \\ 0.5\sin(2t + 1) & (8 \leq t < 10) \\ 0 & (10 \leq t < 15) \end{cases}$$

$$\begin{cases} d_1(t) = 2\sin(2t + 1) & (0 \leq t < 15) \\ d_2(t) = 0.5\sin(2t - 3) & (0 \leq t < 15) \end{cases}$$

Note that the mismatched disturbance and the condition  $rank(B) \neq rank(B, B_d)$  hold, hence that the traditional method will be failed in this example. Choosing the matrix  $D =$

$\begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ , we can check that  $DB$  is invertible. By solving (50), we can obtain that

$$P = \begin{bmatrix} 14843 & 161.7 & 226.9 & -745.0 \\ 161.7 & 25.50 & 12.80 & -101.8 \\ 226.9 & 12.80 & 74.80 & -47.40 \\ 745.0 & -101.8 & -47.40 & 896.5 \end{bmatrix}$$

Choose  $\sigma(t) = 2e^{-t}$ , the results of the simulation are as follows.

Figure 1 shows the trajectory of the system. Figure 2 illustrates the estimation of the disturbance signals, the solid line is the original signal and the dashed line is the estimated value. From Figure 1, we can know that the states of the system have a fast response with the proposed method. In addition, the controller can ensure the stability of the system in the presence of the actuator fault and mismatched disturbance. Figure 2 characterizes that the observer has a good performance of the disturbance.

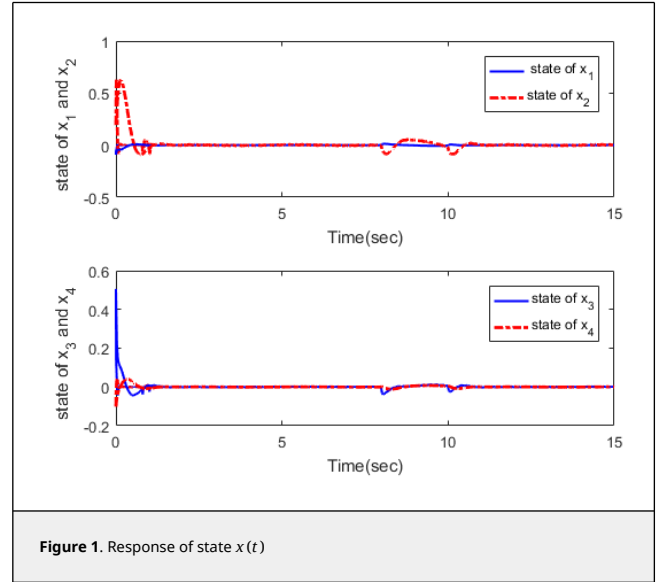


Figure 1. Response of state  $x(t)$

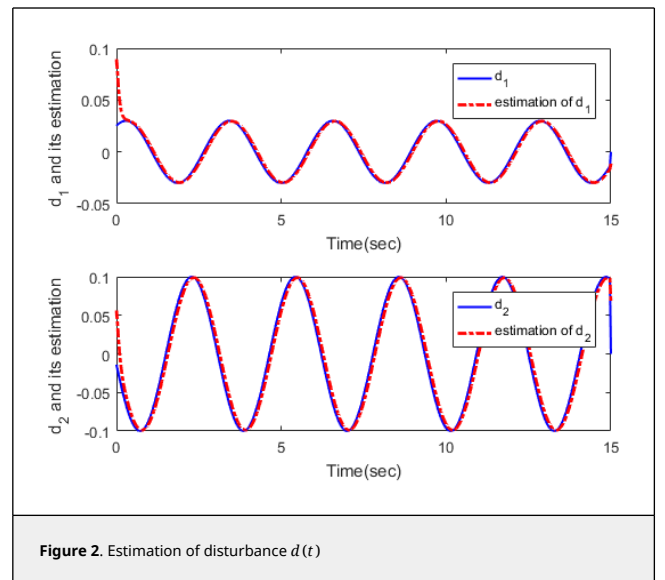


Figure 2. Estimation of disturbance  $d(t)$

In order to illustrate the importance of the disturbance

observer, the responses of the system are shown in Figure 3 without the disturbance observer. From the figure, we can see that the closed-loop system becomes unstable when removing the disturbance observer and compensator.

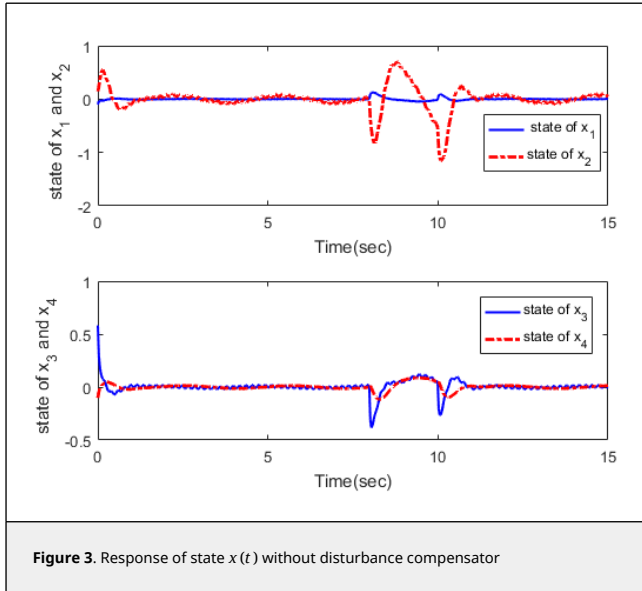


Figure 3. Response of state  $x(t)$  without disturbance compensator

Example 2. In this section, the two-cart system which borrowed form is provided to illustrate the effectiveness of the proposed method [27].

As shown in Figure 4, the first cart is connected to a rigid wall via a damper, and is connected to a second cart by a spring. The external force is applied to a second cart via an actuator. Both carts have a nominal mass of  $a = 1\text{kg}$ , the damper has a constant of  $b_0 = 1\text{N/m}$ , and the spring constant  $c_0 = 1\text{N/m}$ . The time constant of the actuator  $\tau = 0.2$ . The states are the force, velocities, and positions of the two carts. The actuator fault and mismatched disturbance are considered. The system parameters are given as follows

$$A = \begin{bmatrix} -\frac{1}{\tau} & 0 & 0 & 0 & 0 \\ 0 & -\frac{b_0}{a} & 0 & -\frac{c_0}{a} & \frac{c_0}{a} \\ \frac{1}{a} & 0 & 0 & \frac{c}{a} & -\frac{c_0}{a} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{\tau} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_F = \begin{bmatrix} \frac{1}{\tau} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B_d = \begin{bmatrix} 1 & 1 \\ 0 & 0.2 \\ 0 & 0.2 \\ 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}.$$

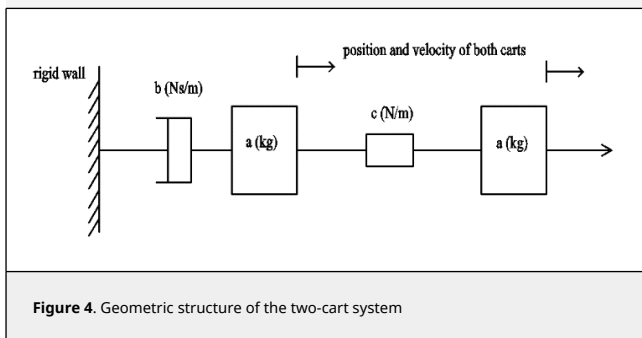


Figure 4. Geometric structure of the two-cart system

The nonlinear unmodeled uncertainty and mismatched nonlinearity are assumed to be:  $\Delta f(x, t) = \sin(x_4)$ ,  $\xi(x, t) = \sin(x_1)$ . The actuator fault and external are supposed to be as follows

$$f(t) = \begin{cases} 0 & (0 \leq t < 25) \\ 0.5\sin(0.2t) & (25 \leq t < 40) \\ 1 & (40 \leq t < 80) \end{cases},$$

$$d_1(t) = \begin{cases} 0 & (0 \leq t < 15) \\ 0.5\sin(t) & (15 \leq t < 50) \\ 1 & (50 \leq t < 80) \end{cases},$$

$$d_2(t) = \begin{cases} 0.5\sin(3t) & (0 \leq t < 20) \\ 0.5\sin(3t) & (20 \leq t < 50) \\ 1 & (50 \leq t < 80) \end{cases}.$$

Choosing the matrix  $D = [1 \ 0 \ 0 \ 0 \ 0]$ , we can check that  $DB$  is invertible. By solving Eq.(50), we can obtain that

$$P = \begin{bmatrix} 0.0799 & -0.000 & -0.000 & -0.000 & -0.000 \\ -0.000 & 1.1046 & 0.000 & -0.000 & -0.000 \\ -0.000 & 0.000 & 1.1046 & 0.000 & -0.000 \\ -0.000 & -0.000 & 0.000 & 1.1046 & 0.000 \\ -0.000 & -0.000 & -0.000 & 0.000 & 1.1046 \end{bmatrix}.$$

Choose  $\sigma(t) = 2e^{-t}$ , the results of the simulation are as follows.

Figures 5 and 6 express the trajectories of the system. From Figures 5 and 6, we can see that the stability of the positions and velocities of the first and second carts can be guaranteed. Figure 7 shows the estimation of the external disturbance, we can check that the proposed method performs better than the intermediate method proposed in [27], precisely, the method proposed responds faster than the method in [27], and the proposed method has less chattering.

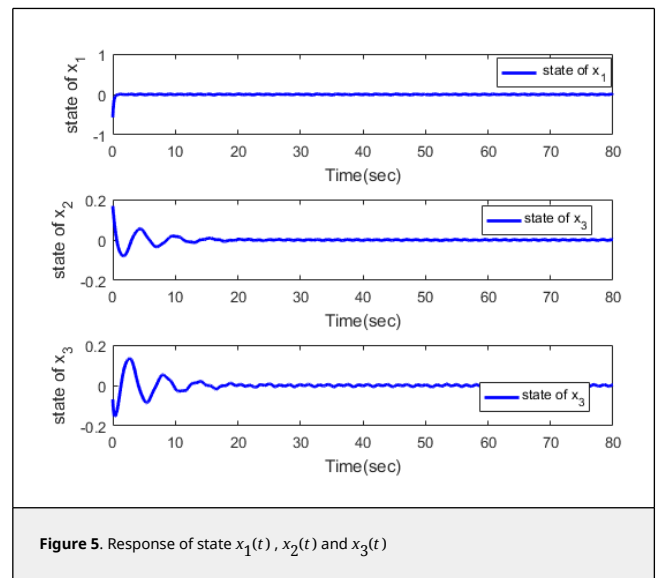
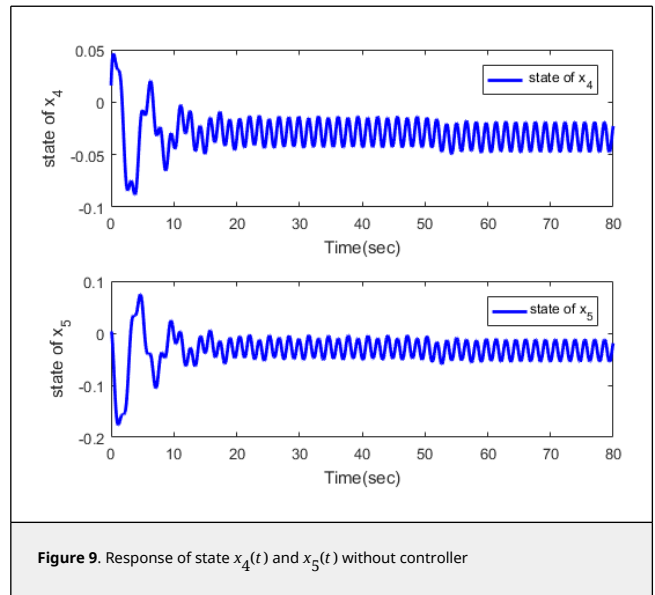
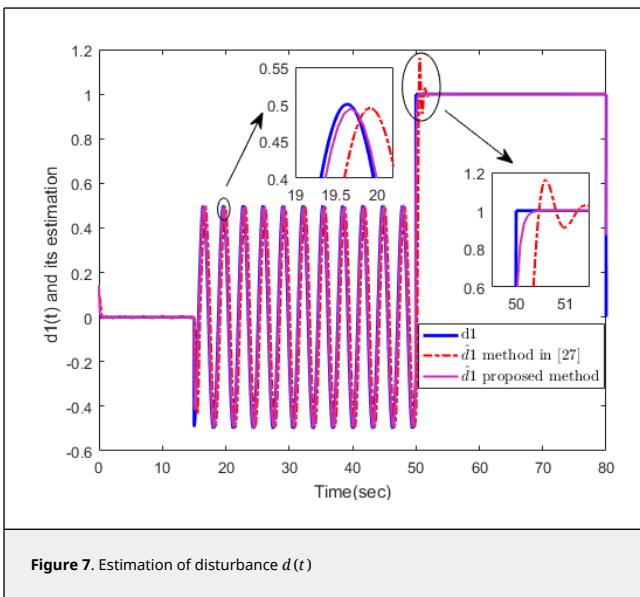
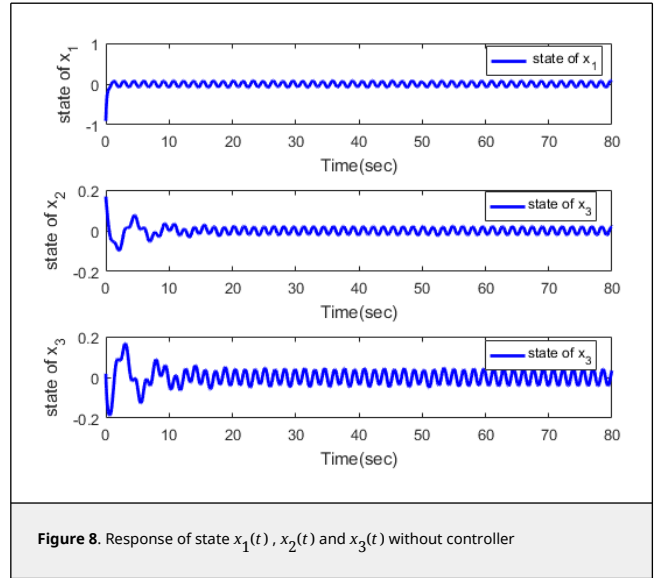
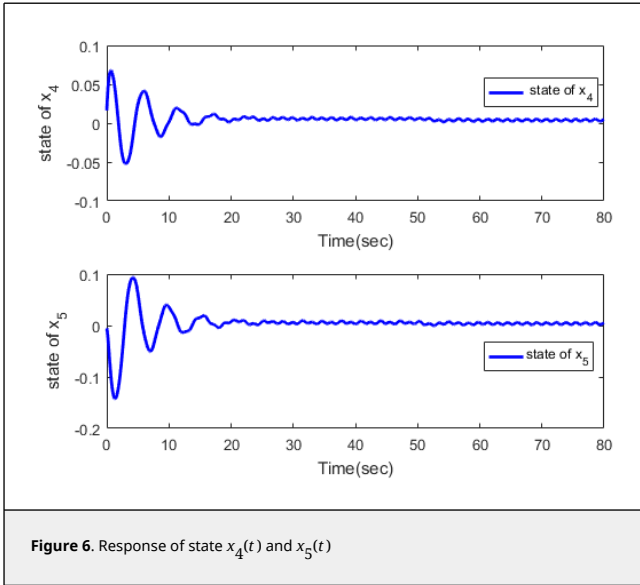


Figure 5. Response of state  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$



In order to illustrate the effectiveness of the proposed method, the responses of the system are shown in Figures 8 and 9 without the controller. From the figure, we can infer from this that the closed-loop system becomes unstable when removing the disturbance observer and compensator.

### 5. Conclusion

In this paper, the problem of a general Lipschitz nonlinear system with actuator fault and unmatched disturbance is investigated. Specifically, a disturbance observer is designed to estimate the mismatched disturbance first. Then, an observer-based integral sliding mode fault tolerant control scheme is proposed. In order to guarantee the stability of the system, three adaptive control laws are constructed because of the unknown nonlinear function parameters and the unmodeled uncertainty. Finally, two examples are given to illustrate the effectiveness of the proposed method. In our future work, we would like to focus on the fault-tolerant control methods for multiple faults and disturbances and their applications.

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